CHAPTER 1
NUMBER CONCEPTS

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1.1 Rational and Irrational Numbers

Rational Numbers

A **rational number** is any number that can be written in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$. This includes the natural numbers (the counting numbers: 1, 2, 3, …), the whole numbers (add 0 to the counting numbers to get 0, 1, 2, 3, …), and integers (add the negatives of counting numbers to the set to get ..., -2, -1, 0, 1, 2, …). Natural, whole, and integer numbers are rational since each can be written in the form $\frac{a}{b}$ ($1 = \frac{1}{1} = \frac{2}{2}$, $-3 = \frac{-3}{1}$, $0 = \frac{0}{4}$).

Examples of Rational Numbers:

All fractions and mixed numbers, both positive and negative

Examples: $\frac{2}{3}$, $\frac{-3}{4}$, $\frac{5}{2}$, $-3\frac{1}{4}$ (note: $\frac{0}{7} = 0$ is rational, but $\frac{7}{0}$ is **not rational** since it is not defined when the denominator equals 0).

All integers

Examples: -11, -3, 0, 1, 5, 68

All terminating and repeating decimals, both positive and negative

Examples: 0.8, -0.32, $0.\overline{3}$, $7.\overline{12}$

Irrational Numbers

A number that cannot be written as the quotient ($\frac{a}{b}$) of two integers is called an **irrational number**.

Examples of Irrational Numbers:

Numbers that are roots of whole numbers that cannot be simplified to obtain a rational number

Examples: $\sqrt{2}$, $\sqrt{5}$, $\sqrt{11}$ ($\sqrt{9}$ is rational since it is equal to 3)

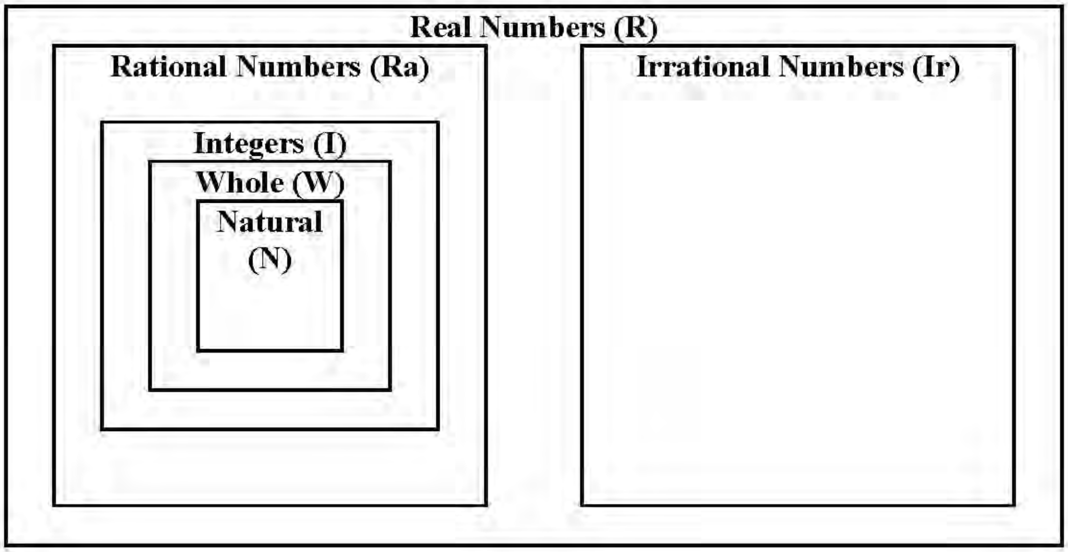
Numbers whose decimal representation does not repeat in a pattern

Examples: 0.1357421… ($0.\overline{3}$ is rational since it repeats a pattern and is equal to $\frac{1}{3}$)

Special numbers such as π

Real Numbers

Real numbers consist of the set of all rational and all irrational numbers. The diagram below shows the relationship among the sets of numbers discussed so far.



Sets of rational numbers include the following:

Natural numbers: N = $\{1, 2, 3, 4, \ldots\}$

Whole numbers: W = $\{0, 1, 2, 3, 4, \ldots\}$

Integers: I = $\{\ldots -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$

Rational numbers: Ra = All of the above plus any other number that can be written in the form $\frac{a}{b}$, b $\neq 0$

Identifying Rational and Irrational Numbers

Rational numbers can be shown in several different formats.

1. Natural numbers, whole numbers, and integers

   Examples: 2, -23, 0, 5001, -673

2. Fractions, mixed numbers, or improper fractions

   Examples: $\frac{2}{7}$, $-\frac{3}{5}$, $1\frac{1}{4}$, $\frac{7}{5}$, $-2\frac{1}{10}$, $-\frac{8}{3}$

3. Decimals (terminating or repeating)

   Examples: 0.8, -0.25, $0.22\overline{3}$, $2.\overline{61}$

Examples with Solutions

1. Put a check mark (√) if the number belongs to the set of numbers

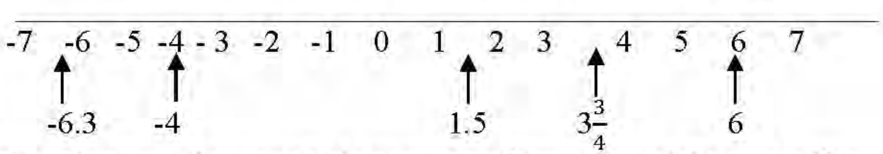
Set of Numbers

	Number	**N**	**W**	**I**	**Ra**	**R**
1.	3	√	√	√	√	√
2.	-10			√	√	√
3.	$\frac{11}{3}$				√	√
4.	0.9				√	√
5.	$0.\overline{7}$				√	√
6.	π					√
7.	$1\frac{5}{8}$				√	√
8.	1.25				√	√
9.	0		√	√	√	√
10.	$\sqrt{9}$	√	√	√	√	√

Note: N = Natural Numbers, W = Whole Numbers, I = Integers, Ra = Rational Numbers, R = Real Numbers

Comparing and Ordering Rational Numbers

Each rational number corresponds to a point on the number line. Several examples are shown below.



Numbers **increase** in magnitude as you go from left to right on the line.

Examples: 1 < 3, 2.1 < 4; -7 < -6; and 2 > 1.8; –3 > -5; -1 > -10.5

To compare the magnitudes of rational numbers where one is written in decimal and the other in common fraction form, write both either in decimal or else in common fraction form and then compare.

Examples with Solutions

1. Compare 0.1 with $\frac{3}{20}$.

   Convert both to fractions first. Change 0.1 to $\frac{1}{10}$. The common denominator is 20, so $\frac{1}{10} = \frac{2}{20}$.

   $\frac{2}{20} < \frac{3}{20}$ or $0.1 < \frac{3}{20}$

2. Compare 3.15 with $3\frac{1}{11}$.

   Convert both to decimals first. Change $3\frac{1}{11}$ to a decimal → $3.\overline{09}$.

   $3.15 > 3.\overline{09}$ or $3.15 > 3\frac{1}{11}$.

Exercises 1.1

Put a check mark (√) if the number belongs to the set of numbers

Set of Numbers

	Number	N	W	I	Ra	R
1.	7					
2.	-18					
3.	$\frac{7}{8}$					
4.	0.36					
5.	$0.\overline{5}$					
6.	$\sqrt{6}$					
7.	$2\frac{3}{4}$					
8.	-5.6					
9.	$\frac{0}{8}$					
10.	$\sqrt{16}$					

11. Locate the following numbers on the number line: 3.1, $2\frac{5}{8}$, $-\frac{13}{12}$, $-\sqrt{6}$, $-\sqrt{16}$

-4 -3 -2 -1 0 1 2 3 4

12. Arrange the following numbers from smallest to largest.

a. –0.57, -0.507, -5.07, -5.70

b. 3.4, $-\frac{11}{3}$, -3.4, -3.5

c. $-\frac{3}{8}$, $-\frac{2}{3}$, - 0.6, -0.4

13. Put the correct symbol (>, =, <) between each pair of numbers.

a. 0.15 ☐ $\frac{7}{40}$

b. -1.8 ☐ $-\frac{9}{5}$

c. -2.8 ☐ $-\frac{13}{5}$

Extra for Experts

14. Express each term in common fraction form (as a quotient of two integers).

a. 0.17

b. $-0.\overline{5}$

c. $-1\frac{2}{3}$

d. 3.07

15. Which rational number is greater?

a. $-0.\overline{6}$ or -0.6?

b. -0.25 or $-\frac{1}{3}$?

c. $-\frac{2}{3}$ or $-\frac{4}{5}$?

1.2 Two and Three Term Ratios

Ratios and Proportions

When you make orange juice from concentrate, the instructions usually ask you to combine 3 cans of water with 1 can of frozen concentrate. In this case, the ratio of cans of water to cans of concentrate is 3 to 1. In reverse, the ratio of cans of concentrate to cans of water is 1 to 3.

Equivalent ratios are pairs of numbers, written as ratios, that are equal to each other.

Example: The ratio of 3 to 1 is equivalent to the ratio of 6 to 2 since $\frac{3}{1} = \frac{6}{2}$. This could also be written as 3:1 = 6:2.

Equivalent ratios can be formed by multiplying or dividing the terms by the same non-zero number.

Example: $\frac{2}{5} = \frac{6}{15}$ (multiplying by 3); $\frac{2}{5} = \frac{-4}{-10}$ (multiplying by -2); $\frac{2}{5} = \frac{1}{10}$ (dividing by 2)

When one ratio is equal to another, it forms a proportion.

Examples with Solutions

1. Write a ratio equivalent to $\frac{3}{4}$.

   $\frac{3}{4} = \frac{top}{bottom}$, multiply the top and the bottom by the same non-zero number.

   $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20} \ldots$

2. Write a ratio equivalent to $\frac{36}{20}$ in lowest terms.

   Divide the top and the bottom by the greatest common factor (GCF), which is 4.

   $\frac{36}{20} = \frac{9}{5}$, in lowest terms.

Finding Missing Terms in Proportions

A **proportion** consists of one ratio that is set equal to another.

Example: $\frac{3}{4}$, $\frac{15}{20}$, and 18:24 are all equivalent ratios.

When one ratio is set equal to an equivalent ratio, it forms a proportion.

Examples: $\frac{3}{4} = \frac{15}{20}$; $\frac{5}{x} = \frac{30}{12}$

Note that $\frac{3}{4} = \frac{15}{20} \rightarrow 3 \times 20 = 4 \times 15$ (the numerator of the first ratio times the denominator of the second ratio is equal to the denominator of the first ratio times the numerator of the second ratio.

Example: $\frac{2}{5} = \frac{x}{15} \rightarrow 2(15) = 5x$. Solving, $x = 6$.

If more than one term is missing in a proportion consisting of three ratios, break the question into two simpler ones. Sometimes these are shown as numerator ratios equal to denominator ratios as shown below.

Example: $3 : 6 : y = x : 14 : 28$

$$\frac{3}{x} = \frac{6}{14} = \frac{y}{28} \rightarrow \frac{3}{x} = \frac{6}{14} \text{ and } \frac{6}{14} = \frac{y}{28}$$

$$\rightarrow 6x = 3(14) \text{ and } 14y = 6(28)$$

$$\rightarrow x = 7 \quad \text{and} \quad y = 12$$

Examples with Solutions

1. Find the missing term in the proportion $\frac{x}{4} = \frac{15}{12}$.

   $12x = 4(15)$

   $x = \frac{4(15)}{12} = 5$

2. Find the missing term in the proportion $\frac{8}{x} = \frac{5}{6}$.

   $6(8) = 5x$

   $x = \frac{6(8)}{5} = \frac{48}{5} = 9.6$

3. Solve for x and y in the proportion $\frac{3}{x} = \frac{9}{12} = \frac{y}{60}$.

   $9x = 3(12)$ and $12y = 9(60)$

   $x = 4$ and $y = 45$

Exercises 1.2

1. Write an equivalent ratio in lowest terms.

   a. 8 : 24

   b. $\frac{14}{35}$

   c. 24 to 9

   d. 100 : 50 : 25

   e. $\frac{0.5}{0.4}$

2. Solve for the missing term in each proportion.

   a. $4 : 5 = x : 25$

   b. $\frac{20}{16} = \frac{5}{x}$

   c. $\frac{x}{5} = \frac{7}{1}$

   d. $x : 2 = 2 : 1$

   e. $\frac{x}{2.5} = \frac{4}{10}$

3. Determine the missing term.

   a. $\frac{48}{72} = \frac{x}{9}$

   b. $x : 7 = 192 : 84$

   c. $1.2 : 2.0 = x : 3.0$

   d. $\frac{0.9}{0.6} = \frac{x}{2}$

e. $\frac{6}{11} = \frac{20}{x}$

4. Find the unknown values in each proportion.

a. $\frac{3}{x} = \frac{9}{15} = \frac{y}{45}$

b. $\frac{x}{4} = \frac{5}{y} = \frac{20}{16}$

c. $9 : 8 : 2 = x : 40 : y$

d. $15 : 10 : 25 = 45 : x : y$

e. $x : y : 49 = 4 : 5 : 7$

<u>Extra for Experts</u>

5. Find the unknown values in each proportion.

a. $x : 3 : 5 = 6 : y : 15$

b. $x : 6 : 8 = 36 : y : 24$

c. $3x : 5 = 6 : 10$

d. $7 : 4 : 3 = x : 18 : y$

e. $9 : 7 : x = 15 : y : 2$

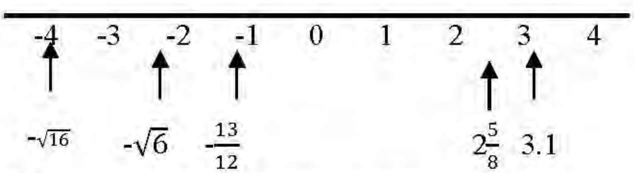
ANSWERS TO EXERCISES AND CHAPTER TESTS

CHAPTER 1

Exercises 1.1 (page 5)

		Set of Numbers				
	No.	**N**	**W**	**I**	**Ra**	**R**
1.	7	√	√	√	√	√
2.	-18			√	√	√
3.	$\frac{7}{8}$				√	√
4.	0.36				√	√
5.	$0.\overline{5}$				√	√
6.	$\sqrt{6}$					√
7.	$2\frac{3}{4}$				√	√
8.	-5.6				√	√
9.	$\frac{0}{8}$		√	√	√	√
10.	$\sqrt{16}$	√	√	√	√	√

11.



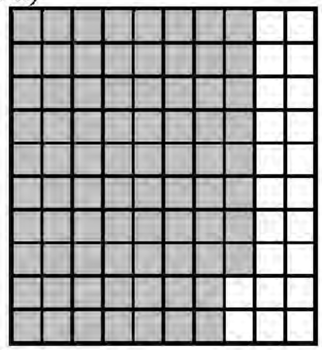
12. a) -5.70, -5.07, -0.57, -0.507
b) $-\frac{11}{3}$, -3.5, -3.4, 3.4 **c)** $-\frac{2}{3}$, -0.6, -0.4, $-\frac{3}{8}$
13. a) < **b)** = **c)** < **14. a)** $\frac{17}{100}$ **b)** $-\frac{5}{9}$ **c)** $-\frac{5}{3}$
d) $3\frac{7}{100} = \frac{307}{100}$ **15. a)** -0.6 It is to the right of $-0.\overline{6}$ on the number line. b) -0.25
c) $-\frac{2}{3}$ (change to $-\frac{10}{15}$ and $-\frac{12}{15}$)

Exercises 1.2 (page 9)

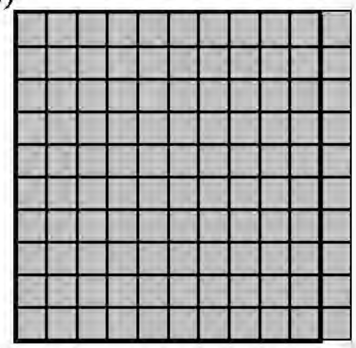
1. a) 1 : 3 **b)** $\frac{2}{5}$ **c)** 8 to 3 **d)** 4 : 2 : 1 **e)** $\frac{5}{4}$
2. a) $x = 20$ **b)** $x = 4$ **c)** $x = 35$ **d)** $x = 4$
e) $x = 1$ **3. a)** $x = 6$ **b)** $x = 16$ **c)** $x = 1.8$
d) $x = 3$ **e)** $x = 36$ **4. a)** $x = 5, y = 27$
b) $x = 5, y = 4$ **c)** $x = 45, y = 10$
d) $x = 30, y = 75$ **e)** $x = 28, y = 35$
5. a) $x = 2, y = 9$ **b)** $x = 12, y = 18$ **c)** $x = 1$
d) $x = 31.5, y = 13.5$ **e)** $x = 1.2, y = 11$

Exercises 1.3 (page 14)

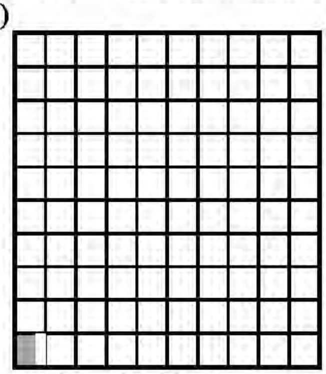
1. a)



b)



c)



2. a) 4% **b)** 118%

3.

	Ratio	Fraction	Decimal	Percent
a)	3 : 8	$\frac{3}{8}$	0.375	37.5%
b)	6 : 5	$\frac{6}{5}$	1.2	120%
c)	39 : 100	$\frac{39}{100}$	0.39	39%
d)	7 : 100	$\frac{7}{100}$	0.07	7%
e)	11 : 3	$\frac{11}{3}$	$3.\overline{6}$	$366.\overline{6}$%
f)	17 : 6	$2\frac{5}{6}$	$2.8\overline{3}$	$283.\overline{3}$%
g)	51 : 10	$5\frac{1}{10}$	5.1	510%