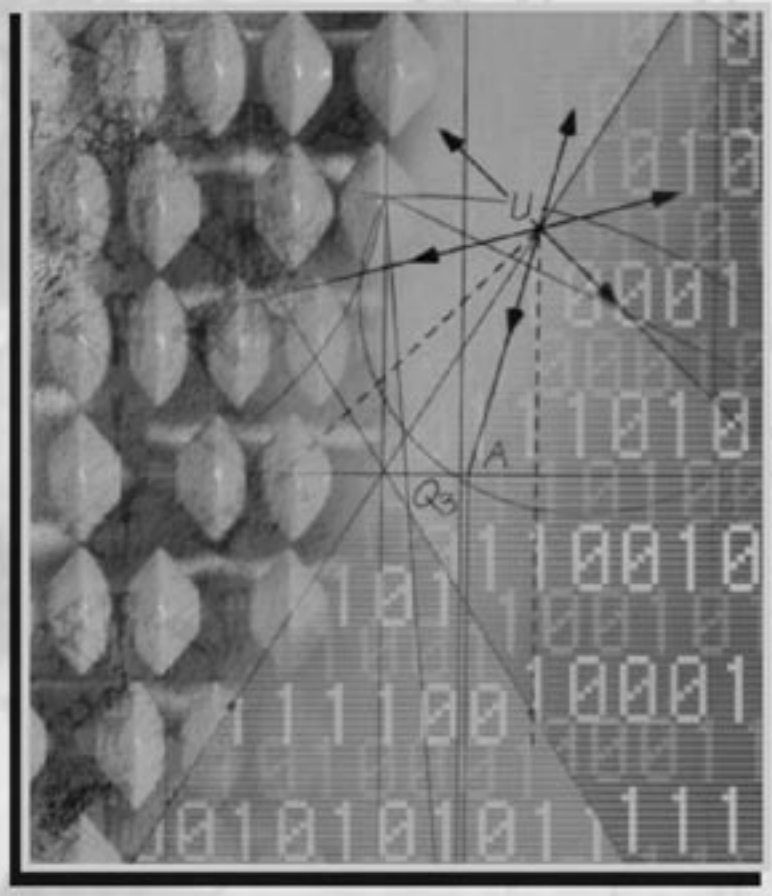
EXPLORING

THE WORLD OF MATHEMATICS



From the Checker Board to the Super Computer

JOHN HUDSON TINER

EXPLORING

THE WORLD OF MATHEMATICS

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Dedication

To Paul Conner Stephens, again.

How to Use Exploring Mathematics

Students of several different ages and skill levels can use *Exploring Mathematics*. Children in elementary grades can grasp many of the concepts, especially if given parental help.

Middle school students can enjoy the book independently and can quickly test their understanding and comprehension by the challenge of answering the questions at the end of each chapter.

Junior high and high school students can revisit the book as a refresher course. The sidebars marked "For More Study" are intended for students with more experience in mathematics. These sidebars can be a springboard for additional study by advanced students.

Thought-provoking questions and problems are found throughout the book, and the student should work through them. The activities illustrate mathematical principles. Whenever possible, solve them mentally or with pencil and paper. In some cases, those methods are too tedious. A calculator will remove the burden of involved calculations. The reader can focus on the important principles that the problems illustrate.

Thinking is preferred over rote memorization. Approach the study as a fun endeavor.

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Chapter One 1

Counting the Years

People have been numbering and counting since ancient times. Counting days is mentioned in the first chapter of Genesis: "And there was evening, and there was morning — the first day." The seven days of the creation week are counted in Genesis 1:5–2:2.

For measuring time, people in the ancient world needed an event that repeated itself exactly. It also had to be clearly visible. The sun made a perfect choice for measuring time. Its motion followed a regular path in the sky that anyone could track. The sun rose, wheeled across the sky, and set. Then after a period of darkness it rose again. The time from sunrise to sunrise made one day.

Although people followed the sun to count days, the motion of the sun that they saw was actually due to the earth spinning on its axis. The sun did not go around the

PROBLEMS

1. How can time be measured without a clock or calendar?
2. How did farmers know the best time to plant and harvest crops?
3. How did 11 days vanish?

Can You Propose Solutions?

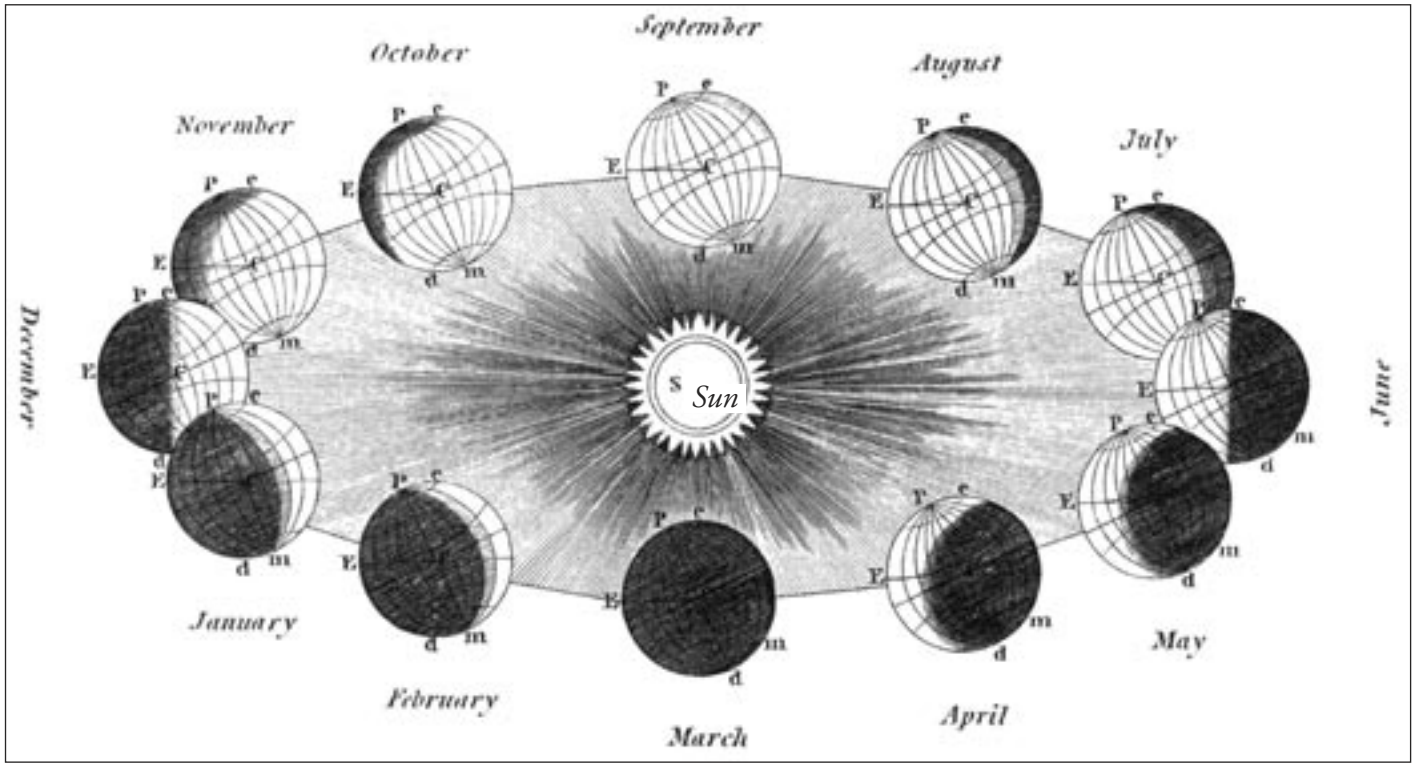
The earth spins on its axis; each complete rotation takes one day.

earth. Instead, the earth rotated on its axis like a top. The sun appeared to move. The time for the earth to make a complete spin gave the sun the apparent motion that marked the passage of one day.

Keeping track of time by days can become cumbersome. Except for small babies, listing a person's age in days gives rise to large numbers. How old is a student who is 4,380 days old? A year is 365 days (for simplification, don't include leap years), so divide 4,380 days by 365 days per year to change days to years. A student who is 4,380 days old is 12 years old: 4,380 days ÷ 365 days per year = 12 years. A person who has lived to be 72 years old would be 26,280 days old: 72 years x 365 days per year = 26,280 days.

Ancient people quickly saw that the moon could be a celestial timekeeper. The moon passed from a particular phase, such as full moon, back to the same phase in about 29.5 days. The word *month* comes from the word "moon."

A month was convenient for measuring longer periods of time. The United States and Canada have four distinct seasons: winter, spring, summer, and autumn. The time from one season to the next was about three moons, or three months. People could also plan events for those nights when the moon was full. The harvest moon was the full moon falling closest to the first day of autumn (about September 21). It gave farmers light to gather crops at night.



The earth's orbit around the sun and the tilt of the axis create the seasons and the imaginary lines of the equator and the tropics. Each complete revolution takes one year.

From George Adams Astronomical and Geographical Essays, London 1795

A lunar calendar is based upon the phases of the moon.

Keeping track of seasons was not an idle pastime for ancient people. Sailors needed to winter in ports to avoid seasonal storms. The Romans sailed the Mediterranean Sea. It was known for stormy weather in the winter. The New Testament describes how a storm destroyed a ship carrying Paul to Rome for trial. The ship's master had refused to stay in the safe harbor for the winter. (Read Acts 27:12–44.)

The seasons are actually caused by the tilt of the earth's axis and its motion around the sun. The tilt makes days and nights of different lengths. During summer, the sun's rays strike the Northern Hemisphere more directly. The length of daylight is longer and the weather is warmer. During winter, the sun is more to the south and its rays strike the Northern Hemispheres at an angle and give less light. The start of winter is the shortest day of the year.

On the first day of summer in the Northern Hemisphere, the sun is at its highest at noon. Shadows of buildings are at their shortest. Ancient people measured a year from the first day of summer to the next.

A year is actually due to the motion of the earth. During a year, the earth makes a complete trip around the sun. A person who is 12 years old has gone around the sun 12 times. A person with an average life expectancy will travel around the sun 72 times in his or her lifetime.

The motions of the earth and moon in relation to the sun give the three easiest ways of measuring time: day, month, and year. A day is the time it takes for the earth to spin once on its axis. The month is the time for the moon to go once around the earth. A year is the time for the earth to go once around the sun.

But all of these measures do not come out evenly when compared with one another. A lunar month is not a whole number of days. It is about 29.5 days long. A solar year is not a whole number of days, either. It is about 365.25 days. Nor is it a whole number of lunar months. Twelve months of 29.5 days gives a year of 354 days: 12 months x 29.5 days per month = 354 days. A lunar year of 12 months is 11 days short of a solar year: 365 days - 354 days = 11 days.

Suppose a farmer enjoyed a bountiful harvest by planting crops on a certain day one year. The farmer would want to repeat the success from one year to the next. He needed a calendar that kept itself in step with the seasons. A calendar that gave the wrong date for the start of spring could lead to disaster. Crops planted too early or too late would die. The farmer's family would face starvation.

A lunar calendar with 12 months of 29.5 days quickly got out of step with the seasons. After one year, it was off by 11 days. After three years, farmers would be planting a month too soon. Over the years, a date in spring would slowly drift into winter. One solution was to insert an extra month about every three years into the calendar.

The Jewish calendar had months based on the moon, but years based on the sun. To keep them in step with one another they put in an extra month seven times in 19 years.

Some countries ignored the problem entirely. The Babylonians lived along the Tigris and Euphrates Rivers in what is now

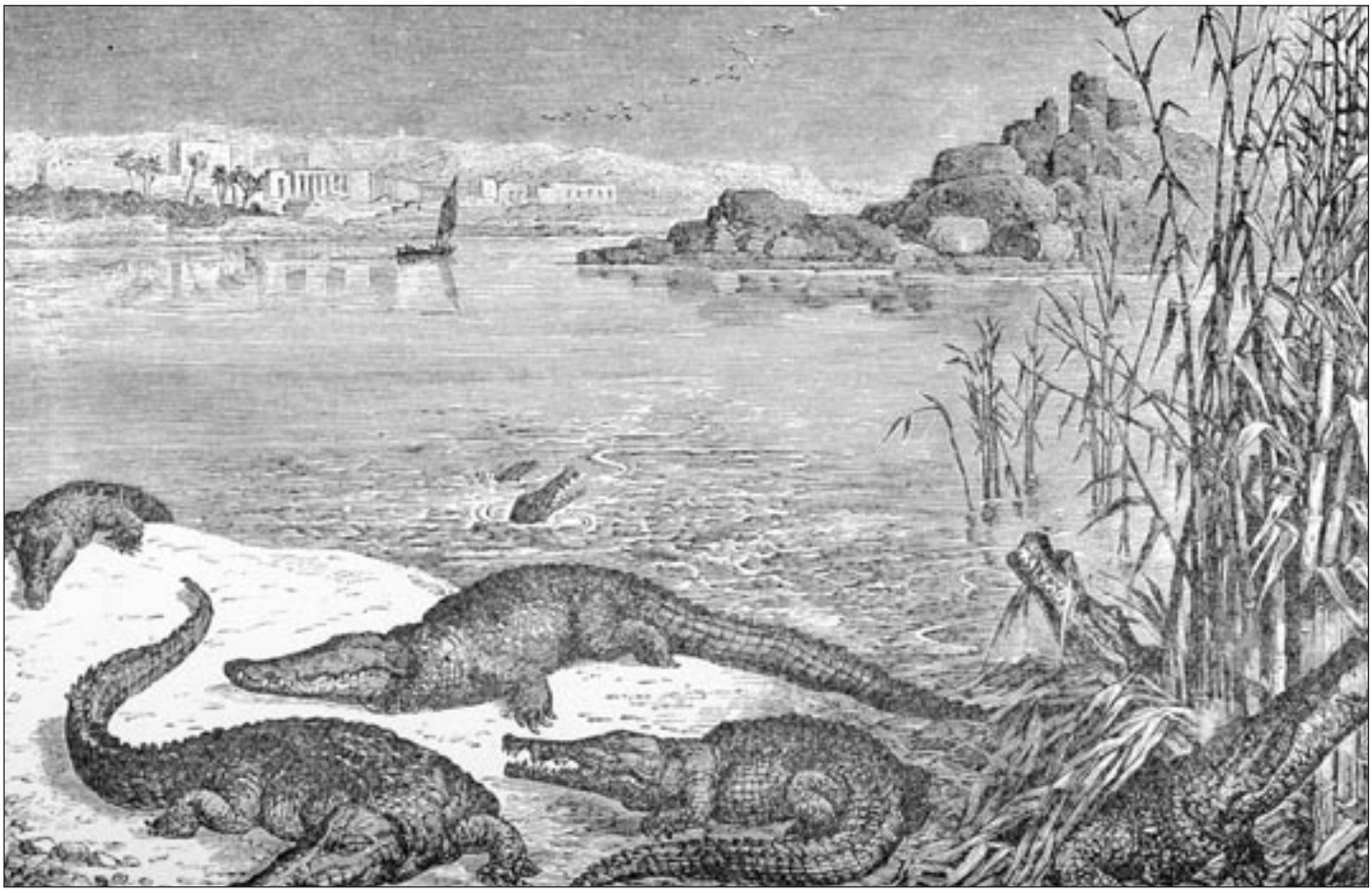
Iraq. The Old Testament mentions Babylon. Daniel, for instance, served in the palace of the king of Babylon (Daniel 1:1–7). The Babylonians used 30 days rather than 29.5 days for a month. Their calendar had 12 months of 30 days to give a year of 360 days: 12 months x 30 days per month = 360 days.

All three ways of measuring time — day, month, and year — came out even in the Babylonian calendar. Even parts of years came out as whole number days. A half of a year was 180 days: 360 days per year ÷ 2 = 180 days. A fourth of a year was 90 days: 360 days per year ÷ 4 = 90 days. A sixth of a year was 60 days: 360 days per year ÷ 6 = 60 days. A twelfth of a year was 30 days: 360 days per year ÷ 12 = 30 days.

The Babylonian calendar missed the right time to plant crops by five days every year: 365 days - 360 days = 5 days. Within six years, the calendar slipped when compared to the seasons by a full month. However, the Babylonians were mainly shepherds and people who gathered crops that grew wild. They did not view with alarm the difference between the calendar and the seasons.

Their neighbors, the Egyptians, planted crops. The Egyptians required a calendar that matched the seasons. Farmers in Egypt planted crops right after the Nile River flooded. The flood was very dependable. The river left a rich layer of soil on the farmland after it returned to its banks.

Egyptians predicted the Nile flood by watching the stars. The brightest star in the sky was Sirius, also known as the "Dog Star." When it appeared in the east just as the sun was setting in the west, then the flooding of the Nile could be expected. Egyptian astronomers carefully measured the time from the appearing



Living along the Nile River, the Egyptians needed an accurate calendar to predict the time of spring floods.

Why Is a Leap Year So Named?

The years and week days in which the leap day falls for the years 1900–2100.

Sun.	1920, 1948, 1976, 2004, 2032, 2060, 2088
Mon.	1904, 1932, 1960, 1988, 2016, 2044, 2072
Tues.	1916, 1944, 1972, 2000, 2028, 2056, 2084
Wed.	1928, 1956, 1984, 2012, 2040, 2068, 2096
Thurs.	1912, 1940, 1968, 1996, 2024, 2052, 2080
Fri.	1924, 1952, 1980, 2008, 2036, 2064, 2092
Sat.	1908, 1936, 1964, 1992, 2020, 2048, 2076

A year with an extra day is called a "leap year." Here's why.

A week has seven days. A year of 365 days has 52 full weeks and one day left over: 365 days ÷ 7 days per week = 52 weeks with a remainder of 1 day.

Because of the remainder of one day, a month that starts on one day of the week this year, will start one day later the next year. For instance, March 1, 2009, is a Sunday; March 1, 2010, is a Monday; March 1, 2011, is a Tuesday, and so on.

Or rather it would be "and so on" if it were not for the extra day in February. The year 2012 is a leap year. Rather than 52 full weeks and one extra day, the leap year has 52 weeks and two extra days: 366 days ÷ 7 days per week = 52 weeks with a remainder of 2 days.

Rather than March 1, 2012, falling on Wednesday, the extra day on February 29 causes March 1, 2012, to fall on Thursday. It skips or leaps over Wednesday. Hence, the name "leap year."

of Sirius to the next appearance of Sirius. They realized that a calendar should have 365 days in a year.

The Egyptians retained the Babylonian calendar of 360 days. To make up for the five lost days, they added five extra days at the end of the year. It was a five-day holiday.

However, the Egyptian calendar had an error that slowly threw the calendar off. A solar year — the time for the earth to go around the sun — is actually 365 days, 5 hours, 48 minutes, 46 seconds. So the Egyptian calendar had a year that was too short by almost six hours, or about ¼ of a day. In four years, the Egyptian calendar would be off by one full day: 4 years x ¼ lost day per year = 1 lost day. In 120 years, the calendar would slip behind the seasons by a full month. It lost a day every four years, and four divided into 120 years gives 30 days: 120 years ÷ 4 years per lost day = 30 lost days.

The Roman calendar was based on the Egyptian calendar, so it lost days against the seasons. When Julius Caesar (100 B.C.–44 B.C.) came into power, the Roman calendar was 80 days behind the seasons. The winter months were falling in the autumn, the autumn months were in the summer and so on. It was a mess. Julius Caesar's astronomers suggested that he remake the calendar. He agreed.

First, he decreed that the next year (what we call 46 B.C.) would be 445 days long to make up the 80 days that the calendar was behind the seasons: 365 days + 80 days = 445 days. It was the longest year in civilized history. After that, years would follow a new calendar.

He set the start of the year as January 1. Before then, different countries started the year at the beginning of different months.

Rather than having five extra days at the end of the year, he spread them throughout the year. The next Roman leader, Augustus Caesar (63 B.C.–B.C. 14), made a few other changes. The final result was seven months with 31 days each, four months with 30 days each. The Romans considered February an unlucky month for some reason. Days were taken away from it so the unlucky month would be only 28

Names for Months from the Roman Calendar

Most names for months in our calendar are from the Roman calendar. The ancient Roman calendar originally had only 10 months and 304 days. The year began with the month of March. Later, the months of January and February were inserted before March, and the new year began with January.

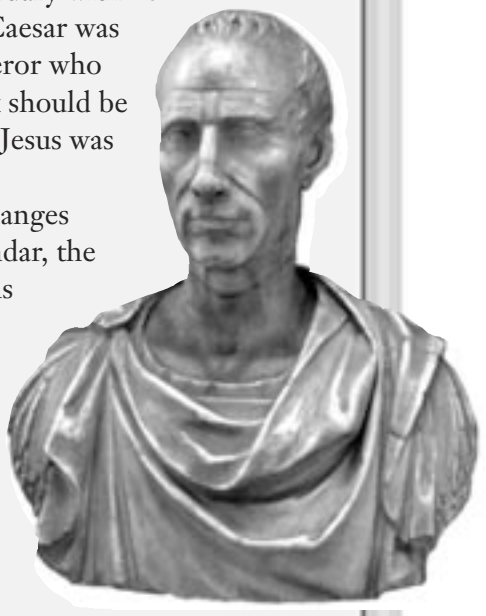
January was named for Janus. In Roman mythology, he was the keeper of doorways. January was the entrance to the new year. February was from a Roman word meaning "festival." March was named after Mars, the Roman god of war. April came from a Roman word meaning "to open," probably because buds opened in April. May was named after Maia, the mother of Mercury. June was named for Juno, the queen of the gods in Roman mythology. She was portrayed as the protector of women.

In the Roman calendar, months after June had names based on their positions in the original calendar before January and February were inserted: Quintilis (quin, "fifth"), Sextilis (sex, "sixth"), September (sep, "seventh"), October (oct, "eighth"), November (non, "ninth"), and December (dec, "tenth").

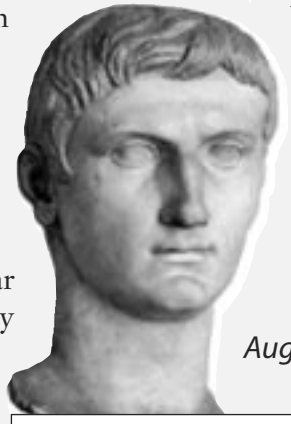
Julius Caesar took the month Quintilis and named it "July" after himself. The next Roman ruler, Augustus Caesar took the month Sextilis and named it "August" after himself.

August had only 30 days but July had 31 days. Augustus took another day from February and added it to August so his month would be as long as the one for Julius Caesar. This left February with 28 days. (Augustus Caesar was the Roman Emperor who decreed that a tax should be taken at the time Jesus was born [Luke 2:1].)

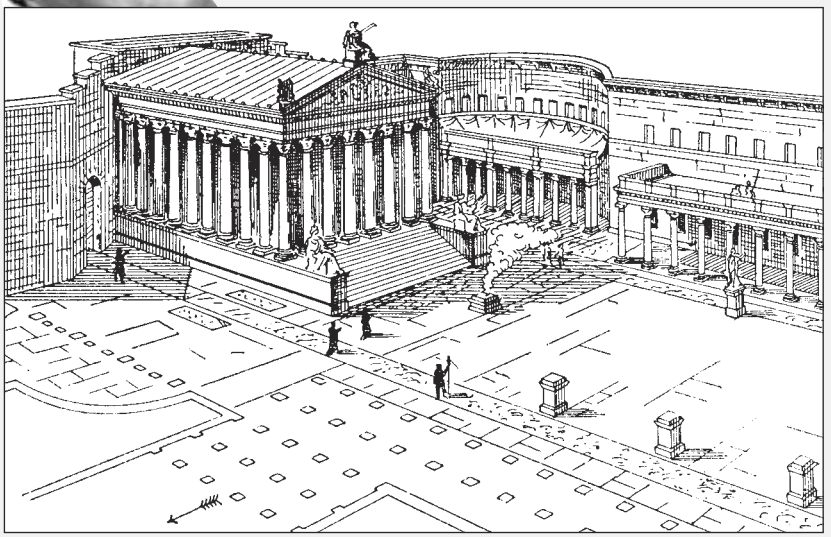
Because of changes made to the calendar, the prefixes of months after August are off by two. September is the 9th month, not the 7th, October is the 10th month, not the 8th, and so on.



Julius Caesar — July is named for him.
courtesy of Justin D. Paola



Augustus Caesar — August is named for him.
(courtesy of Justin D. Paola)



Forum of Augustus

days long. This gave a year of 365 days.

Most important of all, Julius Caesar found a way to take care of the annoying ¼ of a day. He decreed that February would have an extra day every four years. It would have 29 days rather than 28 days. This gave the year an average length of 365¼ days. Written as a decimal, ¼ = 0.25. So the year by the new calendar had an average of 365.25 days.

Pope Gregory XIII
(Courtesy of the Smithsonian)

The calendar with the extra day every four years became known as the "Julian Calendar." Many countries began using it. The extra day was called a "leap day," and the year with the longer February was called a “leap year.”

A person born on February 29 has an interesting problem when it comes to celebrating a birthday — the birthday comes but once every four years. One day out of every 1,461 days is a leap day: 365 days + 365 days + 365 days + 366 days = 1,461 days. So about one person out of every 1,461 will have a birthday on February 29. A town of 5,000 people is likely to have three or four people with birthdays on a leap day: 5,000 ÷ 1,461 = 3.4.

The Julian calendar did a good job of keeping the seasons and calendar together. However, it still had a slight error. The Julian year was 365¼ days. One-fourth of a day is six hours: 24 hours ÷ 4 = 6 hours. However, the earth takes 365 days, 5 hours, 48 minutes, 46 seconds to orbit the sun once. The Julian calendar was too long by about 11 minutes 14 seconds each year. Although slight, it added up over time. In 128 years, it would be off by one day.

By A.D. 1500, the calendar had moved ahead of the actual seasons by 10 days. The calendar did not give the correct date for the start of the seasons. The shortest day of the year, the official start of winter, fell on the calendar's date of December 12. The shortest day of the year should fall on December 21. Christmas, according to the calendar, actually fell before the seasonal start of winter. If something wasn’t done, Christmas would slowly gain until it occurred in summer.

A new calendar was designed to remove the error. The new calendar was named the "Gregorian calendar" after Pope Gregory XIII. He approved the changes for use by the Catholic Church in 1582.

First, the calendar had to be put back in step with the seasons. Thursday, October 4 on the Julian calendar became Friday, October 15 on the new Gregorian calendar. The calendar skipped the days from October 5 through October 14. Ten whole days vanished.

Next, the Julian calendar had put in too many leap days. Every year evenly divisible by four received a leap day. The Julian calendar had 100 leap days every 400 years. The new Gregorian calendar removed three of those leap days. The year 1600 had a leap day. But

the new calendar removed leap days from the years 1700, 1800 and 1900.

In other words, the Gregorian calendar had a leap day every four years, except every 100 years it took one out, but every 400 years it put one back in. The years 2000 and 2100 are both evenly divisible by four. The year 2000 had a leap day, but 2100 will not have the extra day.

Protestant countries delayed changing to the Gregorian calendar. But in 1752, Great Britain and most of the English-speaking countries switched over to the Gregorian calendar. Because of the delay, the error had grown to 11 days. England matched the seasons with the calendar by skipping 11 days from September 3 through 13. The day after September 2 became September 14.

People who went to sleep on the evening of September 2, 1752, woke up on the morning of September 14. Many people became upset because 11 days had disappeared overnight. It did cause confusion. Some people had reason to complain. Landlords demanded to be paid a month's rent during September although it had only 20 days. Workers, in turn, demanded that their employers pay wages for the days that had disappeared.

It caused confusion in other ways, too. How should birthdays be celebrated? George Washington was born on February 11 in 1732

HEBREW/JEWISH CALENDAR OF THE OLD TESTAMENT

Hebrew Month	Our Month	Farm Season	Climate	Festivals
Nisan	March/April	Barley Harvest	Latter rains	Religious New Year Passover First Fruits
Iyyar	April/May	General Harvest	Latter rains	
Sivan	May/June	Wheat Harvest Vine Dressing	Dry Season	Pentecost
Tammuz	June/July	Early Grape Harvest	Dry Season	
Ab	July/August	Harvest: Grapes, Figs, Olives	Dry Season	
Elul	August/Sept	Summer Fruit	Dry Season	
Tishri	Sept/October	Plowing, Olive Harvest		Civil New Year Day of Atonement Tabernacles
Marchesvan	Oct/Nov	Olive Harvest, Grain Planting	Early Rains	
Chislev	Nov/Dec	Grain Planting	Early Rains	Dedication of Temple
Tebeth	Dec/January	Late Planting, Spring Growth	Rainy Season	
Shebat	January/Feb	Late Planting, Winter Figs	Rainy Season	
Adar	Feb/March	Pulling Flax, Almonds Bloom	Rainy Season	
Adar Sheni (Second Adar)	This additional month was added to the end of the year 7 times every 19 years to to keep in step with calendars based on the sun.			

The Hebrew calendar

A solar year is 365.2422 days or 365 days, 5 hours, 48 minutes, 46 seconds in length.

Type	Average Length	Approximate Error
Babylonian	360 days	5.25 days too short every year
Egyptian	365 days	1 day too short every 4 years
Julian	365.25 days	3 days too long every 400 years
Gregorian	365.2425 days	1 day too long every 3,323 years

under the Julian calendar. The British colonies in America switched to the new calendar in 1752. Washington changed his birthday to February 22. This matched his true age of 20 years. For a time, people gave dates on both calendars. For instance, Washington listed his birthday as February 11 Old Style and February 22 New Style.

Some religious groups chose to observe religious events on the old calendar. The Greek Orthodox Church (also known as the Eastern Orthodox Church) continues to use the Julian calendar. They observe Christmas on December 25 of their calendar, although it is January 6 by the Gregorian calendar. The Jewish religion continues to follow a lunar calendar in which an extra month is added about every three years.

However, countries around the world use the Gregorian calendar for most commercial activities. It is the one used in the United States.

It is very exact, but not perfect. It, too, is slightly too long. Every 400 years it has 303 regular years with a total of 110,595 days (303 years x 365 days per year = 110,595 days) and 97 leap years with a total of 35,502 days (97 years x 366 days per year = 35,502 days). In 400 years it has 146,097 days: 110,595 days + 35,502 days = 146,097 days. Dividing the total days, 146,097, by the total years, 400, gives the average number of days per year: 146,097 days ÷ 400 years = 365.2425 days per year.

The Gregorian year of 365.2425 days works out to be 365 days, 5 hours, 49 minutes, 12 seconds. But the year measured by how long the earth takes to orbit the sun is 365.2422 days or 365 days, 5 hours, 48 minutes, 46 seconds. The Gregorian calendar is still too long by 26 seconds per year.

How long will it take for the 26 seconds to make a difference? In about 3,323 years, the Gregorian calendar will have run ahead of the seasons by a full day. Around the year A.D. 5329 something will need to be done to correct the Gregorian calendar.

SOLUTIONS

1. The sun, moon, stars, and rotating earth measured time.
2. Farmers used calendars to help ensure a bountiful harvest.
3. In 1752, England changed the calendar to match the seasons, resulting in 11 lost days.

Calendars measure long periods of time.

Questions

T F 1. The extra day, or leap day, every four years was put in the calendar to honor Augustus Caesar.

T F 2. The Gregorian calendar has 100 leap days every 400 years.

3. What is the main reason to have leap days?

A B C D 4. The first calendar with a leap day every four years was the one (A. authorized by Julius Caesar B. used by the American colonies after 1752 C. used by the Babylonians D. used by the Egyptians).

Matching

5.	_____ day	a. due to the tilt of the earth's axis, equal to three months
6.	_____ week	b. earth revolves around the sun once
7.	_____ month	c. earth rotates on its axis once
8.	_____ season	d. moon revolves around the earth once
9.	_____ year	e. seven days

Try Your Math

10. The Bible says that Methuselah died at age 969 years (Genesis 5:27). What would be that age in days?

11. Using the Babylonian calendar of 360 days in a year, how many days are in one-third of a year; one-fifth of a year; one-twentieth of a year; one-sixtieth of a year?

12. Find the population of your city and calculate how many people are likely to have a birthday on February 29.