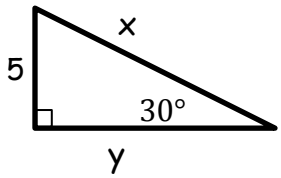
Lesson 12: Sine

How did you do on that last worksheet? Is finding the opposite side and adjacent side of an angle super-duper easy for you now? Good, now I can show you why I wanted you to learn that first. If it's still not clear, read that last lesson again.

You will need a Scientific Calculator for this lesson.

Below is a 5" tall right triangle with one $30°$ angle. Can you tell me the lengths of x and y with only this much information? I bet you can't. But I'm going to teach you a math skill that will make it possible. Isn't that incredible? I know!



The skill you are going to learn is a Trigonometric Ratio called *Sine.* It is pronounced *sign* and it is often abbreviated *sin*, although it is still pronounced *sine*, even when abbreviated.

This is the second hardest part of geometry. Many people struggle to learn this concept, but once your realize it is just a division problem it gets a lot easier. Look at the division problem below. It reads, "Five equals fifteen divided by three."

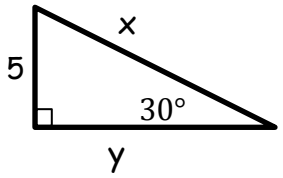
$$5 = \frac{15}{3}$$

The formulas you are about to learn in this lesson, and the next two, are in the same format as the division problem above. Look at the formula below. It is saying, "The sine of Angle A is equal to the Opposite side divided by the Hypotenuse."

$$\sin A = \frac{o}{h}$$

This is just a division problem. The "o" stands for the "opposite side," the "h" stands for the "Hypotenuse," and "sin A" can be found with a calculator. By filling

in this formula we can find the two missing lengths from the triangle at the beginning of this lesson. I'll bring back that problem and show you how to find the missing lengths.



Focus your attention on the $30°$ angle. Now find the side opposite that angle. What is the length of that side? It is 5, so I will replace the "o" in the formula with 5.

$$\sin A = \frac{5}{h}$$

Opposite Side

Hypotenuse

We don't know the length of the hypotenuse, so it will be "x."

$$\sin A = \frac{5}{x}$$

So here is the tricky part. This side of the equation means the "sine of the angle." Our angle is $30°$, so let's replace that side with $\sin 30°$. Now it says, "The sine of thirty degrees equals five divided by x."

$$\sin 30° = \frac{5}{x}$$

Do you see how this is a division problem? Now all we have to do is find the sine of $30°$. But don't worry, that's a piece of cake! You just use a calculator. Some people just memorize the answer, but for me the easiest way is to use a scientific calculator, so you'll want to get one now.

I am using a TI T-30x II s, but you can use any scientific calculator. There is probably one on your computer, but you might have to change the "view" to "scientific" before you start.

Each calculator is different, but basically most are set on "degrees" to begin with and that's what we want. Next, type in 30 (you don't have to say degrees; that is already set) and then press the [SIN] button. The calculator on my computer wants me to press the [SIN] button first and then "30." Try for yourself with your calculator until you get the answer .5 (point five) and then remember how you got that answer.

Now that we know the sine of 30 degrees is .5, we can replace that part of the equation.

$$\sin 30° = \frac{5}{x}$$

$$\downarrow$$

$$.5 = \frac{5}{x}$$

Now we just do some basic algebra. You could do it the long way; multiply both sides by x and then divide both sides by .5, but in volume VI, I showed you the "el switch-a-roo" trick. It works for division equations like that one. According to "el switch-a-roo," we can just switch the .5 with the x and we'll end up with same answer as if you had solved it the old fashioned way. I'll switch them around now.

$$x = \frac{5}{.5}$$

If that confuses you, it's no different than switching the 5 and the 3 in the division problem below. Either way, the equation will still be true.

$$3 = \frac{15}{5}$$

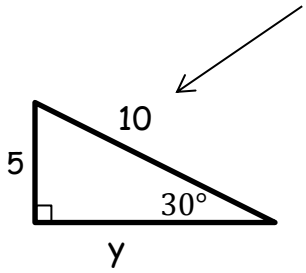
But let's get back to our problem. We left it at "x equals 5 divided by .5."

$$x = \frac{5}{.5}$$

All that's left to do is the math. I'll divide 5 by .5 and get the answer to the length of the hypotenuse.

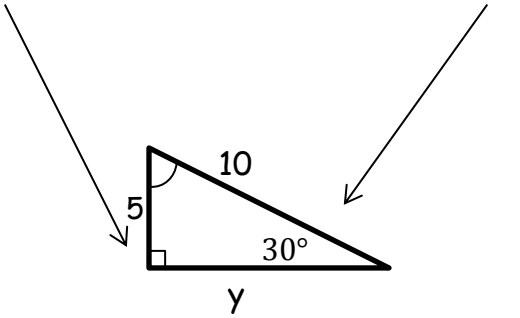
$$x = 10$$

OK, now we have the answer to x. I'll fill in the length of the hypotenuse for our triangle.

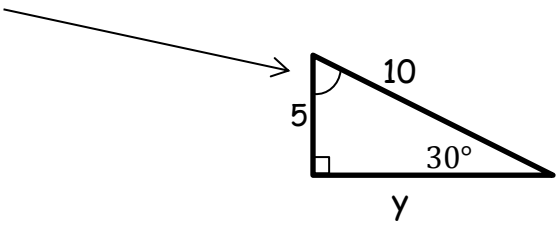


Now it is time to solve for y. I could use the Pythagorean Theorem to solve for y, but this lesson is about *sine*, so let's use that.

We need to find the sine of the other acute angle. Do you know the size of the other angle? Well let's see, all triangles have a total of $180°$, so I'll just subtract the angles we already know from 180 and then whatever is left is the answer. This is a right triangle, so of course this angle is $90°$. I'll add that to $30°$, which equals 120.

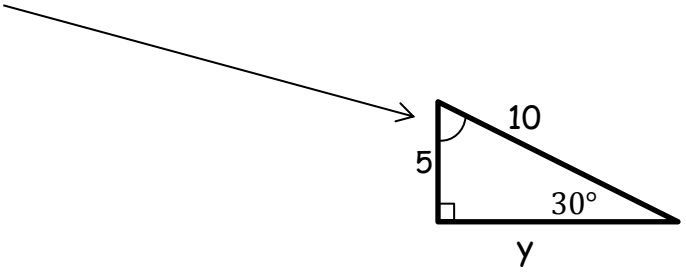


So now do you know the size of the third angle? I'll do the math, $180 - 120 = 60$. The third angle is $60°$.



We are trying to solve for y. Is that side opposite or adjacent to our $60°$ angle? It is opposite because the "flashlight" is "shining" on that side. Which side is the hypotenuse? The side that measures 10 is the hypotenuse.

OK, so let's solve for y. Some super smart mathematician figured out that the sine of this angle will be equal to the side opposite of that angle divided by the hypotenuse.



Alright then, I'll write out that equation.

$$\sin 60° = \frac{y}{10}$$

OK, so what is the sine of $60°$? I can't really remember, so I'm going to use my Scientific Calculator to find out. My son is an engineer. He uses this kind of math nearly every day, so he just memorizes some of these answers. I still use a calculator. So let's see, I will enter [SIN] 60 = and the screen says .08660254, so that is the sine of $60°$. I will round that off and then plug that into our equation.

$$0.8660 = \frac{y}{10}$$

The sine of an angle will always be a number between 0 and 1, and often times it will be several digits long. But mathematicians across the world agree that rounding the number down to the first four digits is close enough.

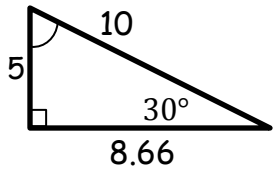
Now back to our equation. Since this side is saying, "y divided by 10," I will multiply both sides by 10, to get y by itself.

$$0.8660 = \frac{y}{10}$$

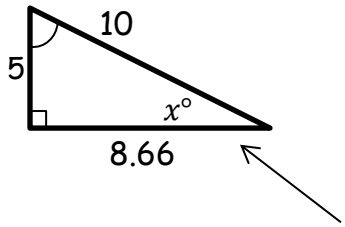
$$.8660 \times 10 = y$$

$$8.66 = y$$

There we go. The third side of the triangle is 8.66 units long. Pretty cool, huh?



Now I'll switch it up a bit. This time I'll erase the size of the angle above and then show you how to figure out it's size based on the length of the sides. Look at the drawing below.



In this problem, we are trying to find the measurement of this angle, so we need to solve for x. No problem, there is a button for that on our calculators. It is called the sine INVERSE and it is written a little differently; look at the equation below. The "negative 1" in superscript makes it read, "the inverse sine of x equals five over ten." That little negative 1 looks like an exponent, but it's not.

$$\sin^{-1} x = \frac{5}{10}$$

Since it looks so much like an exponent, often times the word *Arcsin* (pronounced Arc Sign) is used instead of $\sin^{-1}$. Either way is acceptable and they mean the same thing. I'll write it that way below.

$$\arcsin x = \frac{5}{10}$$

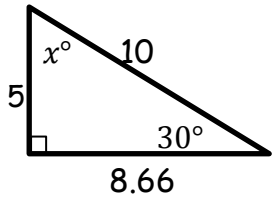
Good news! Your calculator is going to do all the work, so all you need to learn is how to enter the equation into the calculator. In Trigonometry and Pre-Calculus, you will learn about the math that the calculator is doing, but not in this book.

When you are trying to find the inverse sine with a calculator, you'll need to convert the fraction into a decimal number. So let's start by turning $\frac{5}{10}$ in a decimal number. I don't need a calculator for that one; I know that $\frac{5}{10}$ equals $\frac{1}{2}$ which is .5.

$$\sin^{-1} x = .5$$

OK, now I need a calculator. In order to find the inverse sine of x, you need to press two buttons. Each calculator is a little different, so look at yours and try to find a button labeled [2ND] or [INV] for Inverse, or it could be [SHIFT]. You have to press this button before you hit [SIN] and then press [x] [=] [.5].

My calculator works like this: I press [2nd] [SIN] [=] .5 and magically the screen comes up with 30. On my computer's calculator I press [.5] [INV] and instantly the "sin" key turns into a "sin^{-1}" key. Once I hit the "sin^{-1}" key, the calculator displays 30. That is the measurement of angle x! Try it on your calculator and see if you can get it to display 30. Now let's find out the measurement of the other angle. Solve for x.



The inverse sine of x will be equal to "opposite over hypotenuse."

$$\sin^{-1} x = \frac{8.66}{10}$$

First, turn that fraction into a decimal number.

$$\sin^{-1} x = .8660$$

I entered into my scientific calculator: [2nd] [SIN] .8660 [=] and the screen says, "59.9970891" which is pretty darn tootin' close to $60°$. I'll round it up to 60. Had I not rounded the sine of 60 degrees down to four decimal points, that side would have measured .08660254. Then, when you enter [2nd] [SIN] .8660254 = into your calculator, the answer is even closer to 60 degrees. Give it a try.

In this lesson, you learned how to use the sine function to find the length of a right triangle's side when all you know is the size of one angle plus the length of either the opposite side or hypotenuse.

Then you learned how to use the inverse sine function. This trigonometric function allows you to find the size of an angle when all you know is the length of the hypotenuse and the opposite side.

You'll get a chance to practice on the next worksheet. Just remember that "sine equals opposite over hypotenuse" or:

$$\sin = \frac{opposite}{hypotenuse}$$

OR

$$s = \frac{o}{h}$$

OR

SOH,
(pronounced as SEW)

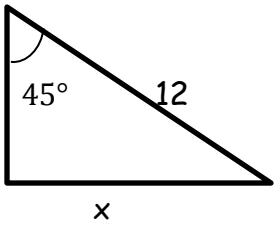
Complete the next worksheet. If you miss more than two or if you are confused, read this lesson again; you'll get it.

Name _____ Date _____

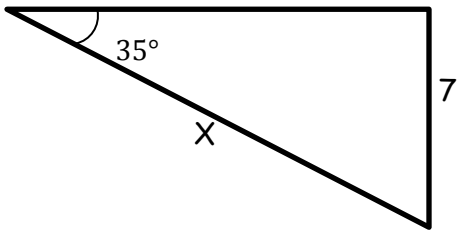
Worksheet 12 page 1 of 2

Solve for x in the Right Triangles below.

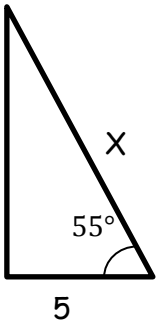
1.



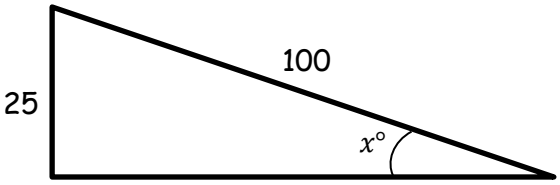
2.



3.

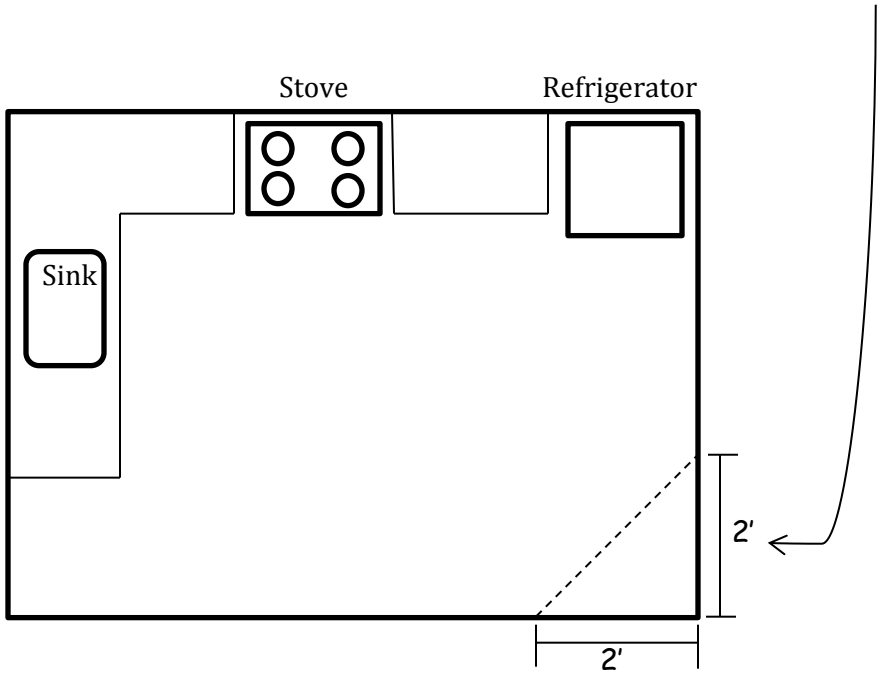


4.



Worksheet 12 page 2 of 2

5. Below is a sketch of Becky's Kitchen. She would like to add a corner pantry here.



Leah is going to design the face of the cabinet, which will be placed on the dotted line in the sketch above. All she needs to know is the width of the cabinet face. Use Sine to find the width of the cabinet for Becky and Leah.