(d) (i) The y-intercept is 8.

(ii) Gradient = $\frac{-8}{40} = -\frac{1}{5}$

(iii) $y = -\frac{1}{5}x + 8$

22. An open rectangular container of depth 10 cm is 20% filled with water initially.
Water is dripped into the container so that the water level in it increases at a constant rate of 0.5 cm every 30 seconds.
Let y cm be the depth of water in the container after x minutes.

(a) Copy and complete the following table.

x	0	2	4	6	8
y					

(b) Taking 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw the graph of y against x for $0 \leqslant x \leqslant 8$.

(c) Use the graph in **(b)** to estimate

(i) the depth of the water after 7 minutes,

(ii) the time taken to fill the container to a depth of 5 cm.

(d) **(i)** Write down the y-intercept of the graph.

(ii) Find the slope of the graph.

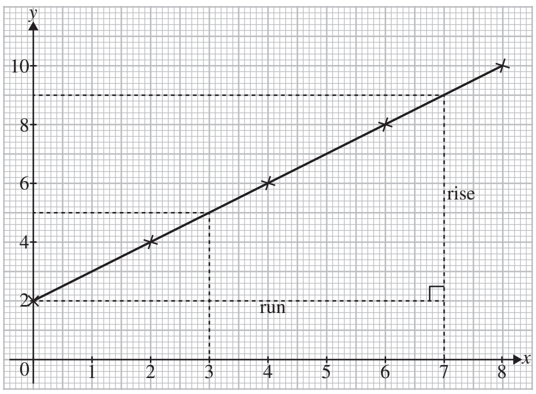
(iii) Hence, express y as a function of x.

Solution

(a)

x	0	2	4	6	8
y	2	4	6	8	10

(b)



(c) **(i)** After 7 min, the depth is 9 cm.

(ii) 3 min

(d) **(i)** y-intercept is 2

(ii) Slope = $\frac{\text{rise}}{\text{run}}$

$= \frac{7}{7}$

$= 1$

(iii) $y = x + 2$

23. The altitude y metres of a mini aircraft above ground level at time t seconds is given by $y = t^2 - 7t + 50$ for $0 \leqslant t \leqslant 7$. The following table shows some corresponding values of t and y.

t	0	1	2	3	4	5	6	7
y	50	p	40	38	q	40	44	50

(a) Calculate the values of p and q.

(b) Taking 2 cm to represent 1 unit on the t-axis and 2 cm to represent 5 units on the y-axis, draw the graph of $y = t^2 - 7t + 50$ for $0 \leqslant t \leqslant 7$.

(c) Estimate from the graph, the minimum altitude of the mini aircraft during its flight. Write down the corresponding time.

(d) Estimate from the graph, the time interval for which the mini aircraft is at most 45 cm above ground level.

Solution

(a) When $t = 1$,

$y = (1)^2 - 7(1) + 50$

$= 44$

$\therefore p = 44$

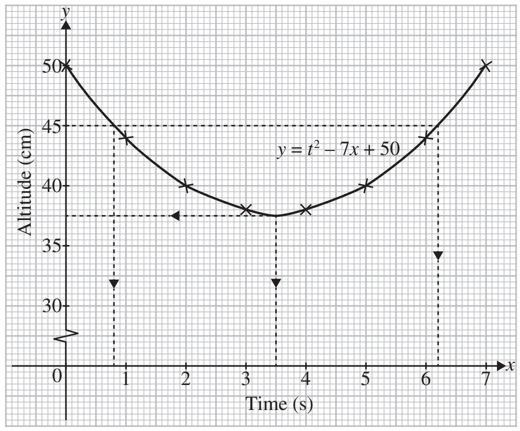
When $t = 4$,

$y = (4)^2 - 7(4) + 50$

$= 38$

$\therefore q = 38$

(b)



(c) From the graph, the minimum point is (3.5, 37.5).
Hence, the minimum altitude is 37.5 m and the corresponding time is 3.5 s.
Note: by calculation, the minimum altitude is 37.75.

(d) From the graph, the estimated time interval is
$0.8 \text{ s} \leqslant t \leqslant 6.2 \text{ s}$.

Solution

(a) The revenue is greater than the expenditure in the third and fourth quarters. Hence, the company made a profit in the third and fourth quarters.

(b) Total revenue = $[(220 + 307 + 425 + 400) × 1,000]
= $1,352,000
Total expenditure = $[(265 + 310 + 374 + 352) × 1,000]
= $1,301,000
Since the total revenue is greater than the total expenditure, the company made a profit for the year.

2. The prices, in dollars, of some stationery items that are sold in 3 bookstores are shown in the table below.

	Unit prices of stationery items ($)		
	Pen	**Marker**	**Ruler**
Bookstore *A*	0.45	1.35	0.60
Bookstore *B*	0.50	1.30	0.50
Bookstore *C*	0.40	1.50	0.55

(a) **(i)** Which bookstore's marker is the cheapest?
(ii) Which bookstore's ruler is the most expensive?

(b) A student wants to buy 5 pens, 4 markers, and 3 rulers from the same bookstore. Which bookstore should he buy the items from? Explain your answer.

Solution:

(a) **(i)** Bookstore *B*
(ii) Bookstore *A*

(b) Total cost in Bookstore *A*
= 5 × $0.45 + 4 × $1.35 + 3 × $0.60
= $9.45
Total cost in Bookstore *B*
= 5 × $0.50 + 4 × $1.30 + 3 × $0.50
= $9.20
Total cost in Bookstore *C*
= 5 × $0.40 + 4 × $1.50 + 3 × $0.55
= $9.65
He should buy the items from bookstore *B* as they cost the least in bookstore *B*.

3. The following table shows the arrival time and departure time of 5 visitors to a school on a particular day.

Visitor	Arrival time	Departure time
Mr. Parler	08:30	10:00
Ms. Rose	09:10	13:15
Mrs. Smith	10:25	11:00
Mr. Dylan	11:45	12:30
Mr. James	14:55	16:40

(a) How many visitors came to the school between 9 AM and 1 PM?

(b) How many of the visitors stayed in the school for
(i) at most an hour,
(ii) at least 1 hour and 30 minutes.

(c) Find the mean duration of the visits. Give your answer in hours and minutes.

Solution

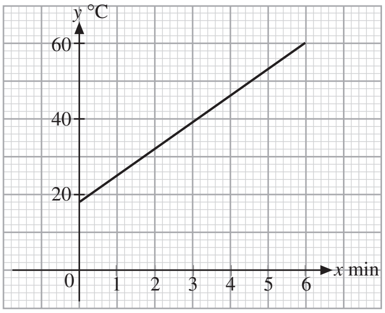
(a) 3 visitors came to the school between 9 AM and 1 PM.

(b) **(i)** 2 visitors stayed in the school for at most an hour.
(ii) 3 visitors stayed in the school for at least 1 hour 30 minutes.

(c) Total number of hours the visitors stayed
= 1 hr 30 min + 4 hr 5 min + 35 min + 45 min + 1 hr 45 min
= 6 hr 160 min
= 520 min

∴ mean duration of the visits = $\frac{520}{5}$
= 104 min
= 1 hr 44 min

4. The diagram below shows the temperature *y* °C of a mug of water at time *x* minutes as it is warmed over a 6-minute interval.



(a) State the initial temperature of the water.

(b) Find, from the diagram,
(i) the temperature of the water after 4 minutes of warming,
(ii) the time taken to warm the water to 32 °C.

(c) Find the rate of increase in the temperature of the water.

(d) Hence, express *y* in terms of *x*.

Solution

(a) 18 °C

(b) **(i)** 46 °C
(ii) 2 min

(c) Rate of increase = $\frac{60 - 18}{6}$
= 7 °C/min

(d) $y = 7x + 18$

5. Instructions on the use of medications in liquid form are usually quoted in metric teaspoons. The following is a conversion table between metric teaspoons and cubic centimeters (cm^3).

Metric teaspoons (*x*)	2	4	6	8	10
Cubic centimeters (*y* cm^3)	10	20	30	40	50

(a) Draw the graph of *y* against *x*.

(b) Read from the graph,
(i) the number of cubic centimeters in 5 metric teaspoons,
(ii) the number of metric teaspoons in 38 cubic centimeters.

(c) Find an equation connecting *x* and *y*.

(b) $QX^2 = XY^2 + QY^2$ (Pythagorean Theorem)

$= 24^2 + 30^2$

$= 1,476$

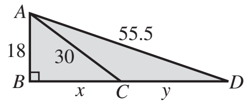
$QX = \sqrt{1,476}$

$= 38.4$ cm (correct to 3 sig. fig.)

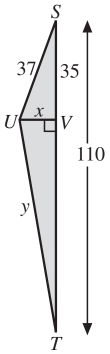
Further Practice

11. Calculate the values of x and y in each of the following diagrams given that the measurements are in cm.

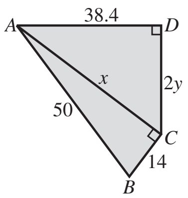
(a) BCD is a straight line.



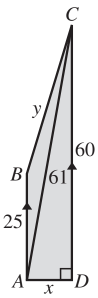
(b) SVT is a straight line.



(c)



(d) AB // DC.



Solution

(a) In $\triangle ABC$,

$AC^2 = AB^2 + BC^2$ (Pythagorean Theorem)

$BC^2 = AC^2 - AB^2$

$x^2 = 30^2 - 18^2$

$= 576$

$x = \sqrt{576}$

$= 24$

In $\triangle ABD$,

$AD^2 = AB^2 + BD^2$ (Pythagorean Theorem)

$55.5^2 = 18^2 + (24 + y)^2$

$(24 + y)^2 = 2,756.25$

$24 + y = 52.5$

$y = 28.5$

(b) In $\triangle USV$,

$US^2 = SV^2 + UV^2$ (Pythagorean Theorem)

$UV^2 = US^2 - SV^2$

$x^2 = 37^2 - 35^2$

$= 144$

$x = \sqrt{144}$

$= 12$

In $\triangle UVT$,

$UT^2 = UV^2 + VT^2$ (Pythagorean Theorem)

$y^2 = 12^2 + (110 - 35)^2$

$= 5,769$

$y = \sqrt{5,769}$

$= 76.0$ (correct to 3 sig. fig.)

(c) In $\triangle ABC$,

$AB^2 = AC^2 + BC^2$ (Pythagorean Theorem)

$AC^2 = AB^2 - BC^2$

$x^2 = 50^2 - 14^2$

$= 2,304$

$x = \sqrt{2,304}$

$= 48$

In $\triangle ADC$,

$AC^2 = AD^2 + DC^2$ (Pythagorean Theorem)

$48^2 = 38.4^2 + (2y)^2$

$4y^2 + 38.4^2 = 48^2$

$4y^2 = 829.44$

$y^2 = 207.36$

$y = \sqrt{207.36}$

$= 14.4$

(d) In $\triangle ACD$,

$AC^2 = AD^2 + CD^2$ (Pythagorean Theorem)

$AD^2 = AC^2 - CD^2$

$x^2 = 61^2 - 60^2$

$= 121$

$x = \sqrt{121}$

$= 11$

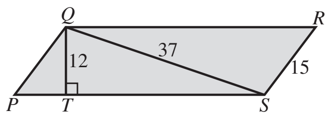
$y^2 = 11^2 + (60 - 25)^2$ (Pythagorean Theorem)

$= 1,346$

$y = \sqrt{1,346}$

$= 36.7$ (correct to 3 sig. fig.)

12. In the figure, $PQRS$ is a parallelogram, T is a point on PS, and QT is perpendicular to PS. $QT = 12$ cm, $QS = 37$ cm, and $SR = 15$ cm.



(a) Find the length of QR.

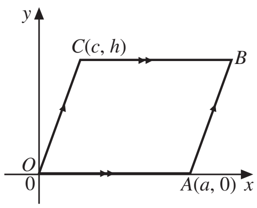
(b) Calculate

(i) the perimeter of $PQRS$,

(ii) the area of $PQRS$.

Enrichment

26.



In the figure, $OABC$ is a parallelogram, where O is the origin, A is $(a, 0)$, and C is (c, h).

(a) Express the coordinates of B in terms of a, c, and h.

(b) Show that $2(OA^2 + OC^2) = OB^2 + AC^2$.

Solution

(a) Let (p, q) be the coordinates of B.

$CB = OA$

$\therefore p - c = a$

$p = a + c$

Since $BC \parallel OA$, $q = h$.

$\therefore$ the coordinates of B is $(a + c, h)$.

(b) $2(OA^2 + OC^2) = 2[a^2 + (c - 0)^2 + (h - 0)^2]$

$= 2(a^2 + c^2 + h^2)$

$OB^2 + AC^2$

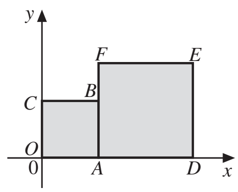
$= [(a + c - 0)^2 + (h - 0)^2] + [(c - a)^2 + (h - 0)^2]$

$= a^2 + 2ac + c^2 + h^2 + c^2 - 2ac + a^2 + h^2$

$= 2(a^2 + c^2 + h^2)$

$\therefore 2(OA^2 + OC^2) = OB^2 + AC^2$ (shown)

27.



In the diagram, $OABC$ and $ADEF$ are squares. The sum of the areas of $ADEF$ and $OABC$ is 34 units². The difference of the areas of $ADEF$ and $OABC$ is 16 units². Find

(a) the areas of $ADEF$ and $OABC$,

(b) the distance CF,

(c) the equation of the line CE.

Solution

(a) Let p units² and q units² be the areas of $ADEF$ and $OABC$ respectively.

$p + q = 34$ ------------ (1)

$p - q = 16$ ------------ (2)

(1) + (2):

$2p = 50$

$p = 25$

Substituting $p = 25$ into (1),

$25 + q = 34$

$q = 9$

$\therefore$ the area of $ADEF$ = 25 units² and area of $OABC$ = 9 units².

(b) Length of a side of $ADEF = \sqrt{25}$

$= 5$ units

Length of a side of $OABC = \sqrt{9}$

$= 3$ units

$\therefore$ the coordinates of C is $(0, 3)$ and the coordinates of F is $(3, 5)$.

$CF = \sqrt{(3 - 0)^2 + (5 - 3)^2}$

$= \sqrt{13}$ units

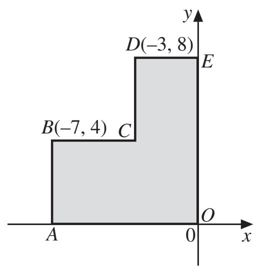
(c) The coordinates of E is $(8, 5)$.

Slope of $CE = \dfrac{5 - 3}{8 - 0}$

$= \dfrac{1}{4}$

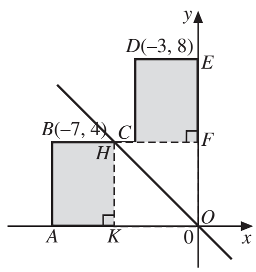
$\therefore$ the equation of the line CE is $y = \dfrac{1}{4}x + 3$.

28.



The figure shows a L-shaped region $OABCDE$, where B is $(-7, 4)$ and D is $(-3, 8)$. Find the equation of the line passing through the origin O that divides the region into two parts of equal area.

Solution



Produce BC to meet the y-axis at F.

Area of $CDEF = 3 \times (8 - 4)$

$= 12$ units²

Construct the vertical line HK such that the area of $ABHK$ is equal to the area of $CDEF$.

i.e. $AK \times AB = 12$

$AK \times 4 = 12$

$AK = 3$

Hence, the coordinates of H is $(-4, 4)$.

We see that OH divides the square $OFHK$ into two parts of equal area.

Thus, area of $ABHK$ + area of $\triangle OHK$ = area of $CDEF$ + area of $\triangle OFH$.

$\therefore$ OH is the line that divides the L-shaped region into two parts of equal area.

Slope of $OH = \dfrac{4 - 0}{-4 - 0}$

$= -1$

$\therefore$ the equation of OH is $y = -x$.