Solutions

Chapter 1 Exponents and Scientific Notation

Basic Practice

1. Evaluate the following without using a calculator.

Solution

(a) $(4^5 + 7^9)^0 = 1$

(b) $6^8 \times 6^7 \div 6^{13} = 6^{8+7-13}$
$= 6^{15-13}$
$= 6^2$
$= 36$

(c) $2^{15} \times 2^5 \div (4)^8 = 2^{15+5} \div (2^2)^8$
$= 2^{20} \div 2^{16}$
$= 2^{20-16}$
$= 2^4$
$= 16$

(d) $(3^2)^2 = 3^4$
$= 81$

(e) $(5^0 + 5^1) \times 5^2 = (1 + 5) \times 25$
$= 6 \times 25$
$= 150$

(f) $(2^3)^2 + (2^3 \times 2^2) = [2^6 + (2^3 \times 2^2)]$
$= 64 + (8 \times 4)$
$= 64 + 32$
$= 96$

(g) $9^{\frac{1}{2}} + 9^2 + 9^{-1} = 3 + 81 + \frac{1}{9}$
$= 84\frac{1}{9}$

(h) $27^{\frac{2}{3}} = (3^3)^{\frac{2}{3}}$
$= 3^2$
$= 9$

2. Evaluate the following without using a calculator.

Solution

(a) $16^{10} \times 16^{-8} \div \sqrt{16} = 16^2 \div 16^{\frac{1}{2}}$
$= 16^{\frac{3}{2}}$
$= (4^2)^{\frac{3}{2}}$
$= 4^3$
$= 64$

(b) $5^{\frac{5}{2}} \times 5^2 \div 5^{\frac{3}{2}} = 5^{\frac{5}{2}+2-\frac{3}{2}}$
$= 5^3$
$= 125$

(c) $25^{\frac{3}{2}} = (5^2)^{\frac{3}{2}}$
$= 5^3$
$= 125$

(d) $\left(\sqrt{64}\right)^{\frac{2}{3}} = 8^{\frac{2}{3}}$
$= (2^3)^{\frac{2}{3}}$
$= 2^2$
$= 4$

(e) $2^{-1} - 2^{-2} + 2^{-3} = \frac{1}{2^1} - \frac{1}{2^2} + \frac{1}{2^3}$
$= \frac{1}{2} - \frac{1}{4} + \frac{1}{8}$
$= \frac{3}{8}$

(f) $\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3$
$= \frac{27}{8}$
$= 3\frac{3}{8}$

(g) $(-3)^2 + 3^{-2} = 9 + \frac{1}{9}$
$= 9\frac{1}{9}$

(h) $\left(8^{\frac{2}{3}}\right)^{-2} = \left[(2^3)^{\frac{2}{3}}\right]^{-2}$
$= (2^2)^{-2}$
$= 4^{-2}$
$= \frac{1}{16}$

3. Simplify the following and express your answers with positive exponents.

Solution

(a) $(3x^4)^2 = 9x^8$

(b) $5x^3 \times 3x^2 = (5 \times 3) \times (x^3 \times x^2)$
$= 15x^5$

(c) $24y^3 \div 8y^2 = \frac{24}{8} y^{3-2}$
$= 3y$

(d) $(3x^5)^0 = 1$

(e) $\left(a^{\frac{2}{3}}\right)^6 = a^4$

(f) $\left(\frac{z^8}{z^2}\right)^{\frac{1}{3}} = \left(z^6\right)^{\frac{1}{3}}$
$= z^2$

(g) $w^{-4} = \frac{1}{w^4}$

(h) $(c^4)^{-2} = c^{-8}$
$= \frac{1}{c^8}$

4. Simplify the following and express your answers with positive exponents.

Solution

(a) $15x^3y \div 3xy^4 = \dfrac{5x^3y}{xy^4}$

$= \dfrac{5x^2}{y^3}$

(b) $(p^2)^{-1} \times (q^3)^2 = p^{-2} \times q^6$

$= \dfrac{q^6}{p^2}$

(c) $\sqrt[3]{27s^6t^9} = 3s^{\frac{6}{3}}t^{\frac{9}{3}}$

$= 3s^2t^3$

(d) $(3x^3y^2)^2 \times x^2y^4 = 9x^6y^4 \times x^2y^4$

$= 9x^8y^8$

(e) $(2a^4b^{-3})^2(a^{-1}b)^5 = 4a^8b^{-6}a^{-5}b^5$

$= \dfrac{4a^3}{b}$

(f) $\dfrac{(n^2)^2}{m^6 \times n^7} = \dfrac{n^4}{m^6n^7}$

$= \dfrac{1}{m^6n^3}$

(g) $\dfrac{6p^4 \times 7q^3}{14q^6 \times 3p^2} = \dfrac{2p^4q^3}{2q^6p^2}$

$= \dfrac{p^2}{q^3}$

(h) $\left(\dfrac{x^4}{9y^6}\right)^{\frac{1}{2}} = \dfrac{x^2}{3y^3}$

5. Solve the following equations.

Solution

(a) $6^x = 1$

$6^x = 6^0$

$x = 0$

(b) $3^x = 27$

$3^x = 3^3$

$x = 3$

(c) $2^x = \dfrac{1}{16}$

$2^x = \dfrac{1}{2^4}$

$2^x = 2^{-4}$

$x = -4$

(d) $4^x = 2^{15}$

$2^{2x} = 2^{15}$

$2x = 15$

$x = 7\dfrac{1}{2}$

(e) $5^x = 25^{-8}$

$5^x = (5^2)^{-8}$

$5^x = 5^{-16}$

$x = -16$

(f) $\sqrt{7^x} = 49$

$7^{\frac{x}{2}} = 7^2$

$\dfrac{x}{2} = 2$

$x = 4$

6. Solve the following equations.

Solution

(a) $3^{4x} = 9^{12}$

$3^{4x} = (3^2)^{12}$

$3^{4x} = 3^{24}$

$4x = 24$

$x = 6$

(b) $6^{2-x} = 36^4$

$6^{2-x} = (6^2)^4$

$6^{2-x} = 6^8$

$2 - x = 8$

$x = -6$

(c) $5^2 \times 5^{2x} = 5^2$

$5^{2+2x} = 5^2$

$2 + 2x = 2$

$2x = 0$

$x = 0$

(d) $2^x \div 32 = 2^{-x}$

$2^x \div 2^5 = 2^{-x}$

$2^{x-5} = 2^{-x}$

$x - 5 = -x$

$2x = 5$

$x = 2\dfrac{1}{2}$

(e) $\sqrt[x]{7^2} = 7^6$

$7^{\frac{2}{x}} = 7^6$

$\dfrac{2}{x} = 6$

$x = \dfrac{1}{3}$

(f) $4^x - 1 = 0$

$4^x = 1$

$4^x = 4^0$

$x = 0$

7. Express each of the following in scientific notation correct to 3 significant figures.

Solution

(a) $3{,}245 = 3.245 \times 10^3$

$= 3.25 \times 10^3$ (correct to 3 sig. fig.)

(b) $6{,}782{,}450 = 6.78245 \times 10^6$

$= 6.78 \times 10^6$ (correct to 3 sig. fig.)

(c) $0.03463 \times 10^7 = 3.463 \times 10^{-2} \times 10^7$

$= 3.46 \times 10^5$ (correct to 3 sig. fig.)

(d) $279{,}825 \div 10^2 = 2{,}798.25$

$= 2.79825 \times 10^3$

$= 2.80 \times 10^3$ (correct to 3 sig. fig.)

(e) $0.006752 = 6.752 \times 10^{-3}$

$= 6.75 \times 10^{-3}$ (correct to 3 sig. fig.)

(f) $0.0000464 = 4.64 \times 10^{-5}$

(g) $0.03463 \times 10^{-5} = 3.463 \times 10^{-2} \times 10^{-5}$

$= 3.46 \times 10^{-7}$ (correct to 3 sig. fig.)

(h) $4{,}295 \div 10^{-8} = 4.295 \times 10^3 \div 10^{-8}$

$= 4.295 \times 10^{11}$

$= 4.30 \times 10^{11}$ (correct to 3 sig. fig.)

22. **(a)** Simplify by factorization,

(i) $(x + 1)^2 - (x - 1)^2$,

(ii) $(x + 2)^2 - (x - 2)^2$,

(iii) $(x + 3)^2 - (x - 3)^2$.

(b) Hence, simplify $(x + n)^2 - (x - n)^2$.

(c) Use the answer in **(b)** to

(i) evaluate $(345 + 29)^2 - (345 - 29)^2$,

(ii) solve the equation $(x + 100)^2 - (x - 100)^2 = 640$.

Solution

(a) **(i)** $(x + 1)^2 - (x - 1)^2$

$= [x + 1 + x - 1][x + 1 - (x - 1)]$

$= 2x(2)$

$= 4x$

(ii) $(x + 2)^2 - (x - 2)^2$

$= (x + 2 + x - 2)[x + 2 - (x - 2)]$

$= 2x(4)$

$= 8x$

(iii) $(x + 3)^2 - (x - 3)^2$

$= (x + 3 + x - 3)[x + 3 - (x - 3)]$

$= 2x(6)$

$= 12x$

(b) By inspection of the answers in **(a)**,

$(x + n)^2 - (x - n)^2 = 2x(2n)$

$= 4nx$

(c) **(i)** $(345 + 29)^2 - (345 - 29)^2$

$= 4(29)(345)$

$= 40{,}020$

(ii) $(x + 100)^2 - (x - 100)^2 = 400$

$\therefore\ 4(100)x = 640$

$400x = 640$

$x = 1.6$

Challenging Practice

23. **(a)** The diameter of a circle is $(6r + 16s)$ cm, where r and s are positive numbers.

Find and simplify, in terms of r, s, and π,

(i) the circumference of the circle,

(ii) the area of the circle.

(b) Suppose that the circle is the base of a solid prism and the height of the prism is twice the diameter of its base. Find and simplify, in terms of r, s, and π,

(i) the volume of the prism,

(ii) the total surface area of the prism.

Solution

(a) **(i)** Diameter $= (6r + 16s)$ cm

Circumference $= (6r + 16s)\pi$ cm

(ii) Radius $= (3r + 8s)$ cm

$\therefore$ area $= \pi(3r + 8s)^2$

$= (9r^2 + 48rs + 64s^2)\pi$ cm^2

(b) **(i)** Volume of prism

= base area × height

$= \pi(9r^2 + 48rs + 64s^2) \times 2(6r + 16s)$

(ii) Total surface area of prism

= 2 × base area + circumference × height

$= 2(9r^2 + 48rs + 64^2)\pi + (6r + 16s)\pi \times 2(6r + 16s)$

$= (18r^2 + 96rs + 128s^2)\pi + 2\pi(6r + 16s)^2$

$= [18r^2 + 96rs + 128s^2 + 2(36r^2 + 192rs + 256s^2)]\pi$

$= (90r^2 + 480rs + 640s^2)\pi$ cm^2

24. **(a)** Evaluate $(997 - w)^2 + (993 - w)^2$ if $(997 - w)(993 - w) = 21$.

(b) Find the value of $(1 + x)(2 + x)(3 + x)^2(4 + x)(5 + x)$ if $x^2 + 6x = 2$.

Solution

(a) $(997 - w)^2 + (993 - w)^2$

$= [(997 - w)^2 - 2(997 - w)(993 - w) + (993 - w)^2] + 2(997 - w)(993 - w)$

$= [(997 - w) - (993 - w)]^2 + 2(21)$

$= (997 - 993)^2 + 42$

$= 16 + 42$

$= 58$

(b) $(1 + x)(2 + x)(3 + x)^2(4 + x)(5 + x)$

$= (1 + x)(5 + x)(2 + x)(4 + x)(3 + x)^2$

$= (x^2 + 6x + 5)(x^2 + 6x + 8)(x^2 + 6x + 9)$ --------- (1)

Since $x^2 + 6x - 2 = 0$

$x^2 + 6x = 2$ -------------------------------- (2)

Putting (2) into (1), $(2 + 5)(2 + 8)(2 + 9) = 770$

$\therefore\ (1 + x)(2 + x)(3 + x)^2(4 + x)(5 + x) = 770$

25. A stone is tossed from a point W into the air. The height y meters above ground level of the stone at time t seconds is given by $y = -t^2 + 4t + 5$.

(a) How far above the ground level is W?

(b) **(i)** If $-t^2 + 4t + 5$ can be expressed as $-(t - 2)^2 + P$, find the value of P.

(ii) Hence, deduce the maximum vertical distance of the stone above point W and the time taken to reach this distance.

Solution

(a) When $t = 0$,

$y = -(0)^2 + 4(0) + 5$

$= 5$

$\therefore$ W is 5 m above the ground level.

(b) **(i)** Method 1

$-(t - 2)^2 + P = -(t^2 - 4t + 4) + P$

$= -t^2 + 4t - 4 + P$

Given $-t^2 + 4t - 4 + P = -t^2 + 4t + 5$

$-4 + P = 5$

$\therefore\ P = 9$

Method 2

Given $-t^2 + 4t + 5 = -(t - 2)^2 + P$

$-t^2 + 4t + 5 = -(t^2 - 4t + 4) + P$

$-t^2 + 4t + 5 = -t^2 + 4t - 4 + P$

$\therefore\ P = 9$

(ii) $y = -t^2 + 4t + 5$

$= -(t - 2)^2 + 9$

$(t - 2)^2 \geqslant 0$

$\therefore\ -(t - 2)^2 \leqslant 0$

$\therefore$ y is maximized when $t = 2$.

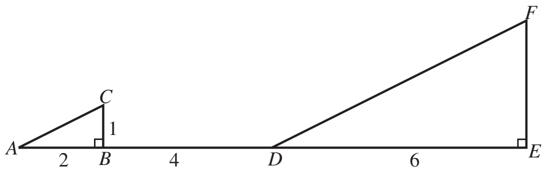
maximum value of $y = 9$

$\therefore$ maximum vertical distance of the stone above point $W = 9 - 5$

$= 4$ m

Time taken to reach this distance = 2 s

33. In the diagram, the right-angled triangle *DEF* is an enlargement of the triangle *ABC* about the centre *P* which is not shown. *B* and *D* are points on the line *AE*, *AB* = 2 units, *BC* = 1 unit, *BD* = 4 units, and *DE* = 6 units.



(a) State the scale factor of the enlargement.
(b) Find the length of *EF*.
(c) By calculation or accurate scale drawing on a sheet of graph paper, find the distance of *A* from the centre of enlargement *P*.

Solution

(a) Scale factor $= \dfrac{DE}{AB}$

$= \dfrac{6}{2}$

$= 3$

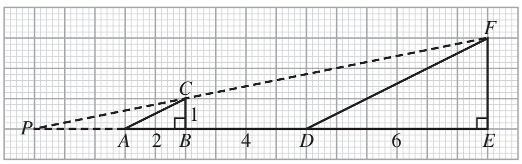
(b) $\dfrac{EF}{BC} = \dfrac{DE}{AB}$

$= 3$

$\therefore \dfrac{EF}{1} = 3$

$EF = 3$ units

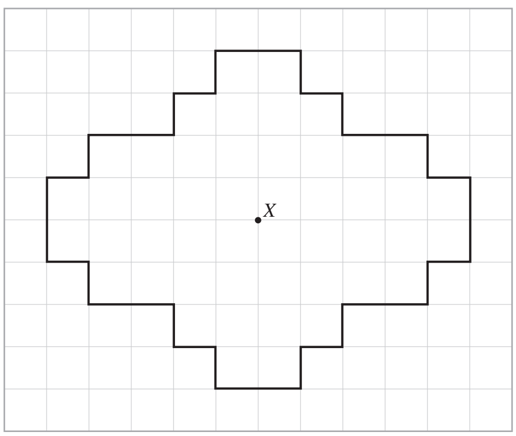
(c)



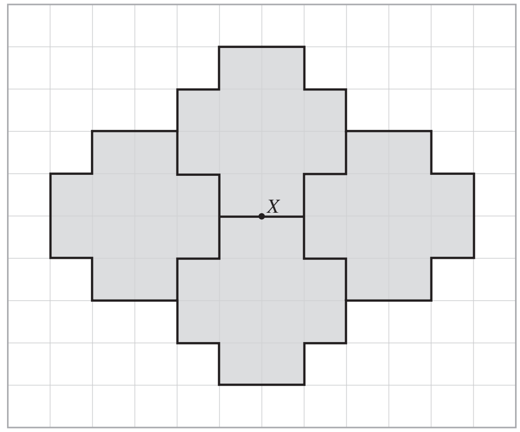
From the diagram, the distance of *A* from the center of enlargement *P* is 3 units.

Enrichment

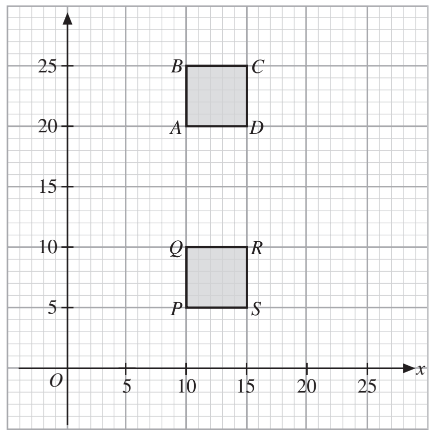
34. The diagram shows a figure and a point *X* drawn on the grid lines.
Divide the figure into 4 congruent figures along the grid lines only. The four congruent figures must not all meet at the point *X*.



Solution



35. Two squares, *ABCD* and *PQRS*, are drawn in the diagram.



Describe completely the transformation that moves
(a) *ABCD* to *PQRS*,
(b) *ABCD* to *QPSR*,
(c) *ABCD* to *QRSP*,
(d) *ABCD* to *RSPQ*.

Solution

(a) *ABCD* is translated to *PQRS* by 15 units down.
(b) *ABCD* is reflected in the line $y = 15$ to *QPSR*.
(c) *ABCD* is rotated through 90° clockwise about the point (5, 15) to *QRSP*.
(d) *ABCD* is rotated 180° about the point (12.5, 15) to *RSPQ*.