NOTES ON TEACHING

Chapter 1 Exponents and Scientific Notation

Suggested Approach

The idea of exponents has been introduced in Grade 7. In this chapter, students will study the laws of indices, and the meaning of negative and rational indices. Activities to discover the laws of indices for numbers should be provided prior to stating the laws to students.

It is advisable to introduce the laws one at a time; each law should be supported by immediate simple examples. This would make it easier for students to master them individually. After teaching rational indices, some examples involving the successive use of the laws will help in reinforcement.

Students should recognize that all measurements are approximations owing to the limitation of measuring instruments. Further, we sometimes use approximated numbers because they are easier to remember. To reinforce the idea of scientific notation and significant figures, students should be given exposure to use them in real-life situations and problems.

Estimation techniques can be explored to ask students to work in groups to estimate quantities (numbers and measures) to an appropriate degree of accuracy in a variety of contexts, compare the estimates and share the estimation strategies.

Students should compare follow-through errors arising from intermediate values that are rounded in different degrees of accuracy in some calculations. It is important to highlight to students that estimation can be used to check the validity of an answer and also to encourage them to make estimates and check that their answers obtained from calculators are correct.

1.1 Positive Exponents and the Laws of Exponents

It would be better if simple numerical examples are used to illustrate the laws. Students should not go into tedious manipulations of complicated expressions.

Some student may be confused with the addition and multiplication of indices. A common error could be $(2x)^3 = 2x^3$ or $(a^3)^4 = a^7$.

1.2 Zero and Negative Integer Exponents

It should be emphasized that zero to the power of zero is undefined. Students should develop the habit of expressing the answer in positive indices. Care must be taken to distinguish between $(-3)^{-2}$ and -3^{-2}.

1.3 Fractional Exponents

In general, for fractional indices, the base should be positive.

1.4 Comparing Exponents

Students should be reminded to compare the indices of two numbers only when the numbers are expressed with the same base. Only simple equations are required in this section.

Class Activity 4

Objective: To understand the rounding of a number to a desired degree of accuracy.

Questions

1. The exact thickness of a piece of glass is 0.004503 m.

(a) State the thickness of the piece of glass in meters correct to 2 decimal places. 0.00 m

(b) If you give the rounded figure in **(a)** to a handyman, will it make any sense to him? No.

2. The price of a condominium is $208,175.

(a) If you are discussing to buy the condominium, how would you round the price to discuss with the property agent?

$200,000

(b) If you are trying to sell this condominium, what rounded price will you use to discuss the sale price?

$210,000

Class Activity 5

Objective: To find out some uses of estimation of quantities.

Questions

1. Marie wants to buy three items that cost $39.80, $11.30, and $52.50 from a supermarket. She has $120 in her purse. How can she know if she has enough money to buy these items?

To help her gauge if she has enough money for the three items, she can estimate their total price.

(a) **(i)** Round the price of each item to the nearest ten dollars. $40, $10, $50

(ii) Using the results in **(i)**, find the sum of the approximate costs of the three items. $100

(b) Find the sum of the exact costs of the three items. $103.60

(c) Compare the actual sum in **(b)** with the estimated sum in **(a)**.
What can you conclude?

The estimate sum is slightly lower than the actual sum.

2. The manager of an events management company sold 1,324 concert tickets at $18.95 each. He used a calculator to work out the revenue. As the calculator displayed a sum of over $250,000, he jumped up in excitement.

(a) **(i)** Write down the price of each ticket rounded to 1 significant figure. $20

(ii) Write down the number of tickets sold rounded to 1 significant figure. $1,000

(iii) Find the approximate revenue using the results in **(i)** and **(ii)**. $20,000

(b) Based on your answer obtained in **(a)**, do you expect the manager to remain excited for long? Why?

No. The sum on the calculator is too high compared to the approximate, so there was a gross miscalculation of the revenue.

(c) Find the actual revenue. $25,089.80

(d) What do you think went wrong in the manager's calculation?

He may have mistakenly added a zero to the end of the number of concert tickets sold.

(b) Do the calculators display the expected answers?

The four-function calculator did not but the scientific calculator did.

(c) Can you give reasons for the differences in answers in **(a)**, **(b)**, or **(c)**?

The four-function calculator rounds off the result of the operations.

4. **(a)** In Operation 3, do the calculators display the expected answers in **(a)(i)** and **(b)(i)**?

The scientific calculator did not but the four-function calculator did.

(b) Do the calculators display the expected answers in **(a)(ii)** and **(b)(ii)**?

Yes.

5. What do you think is the rule of rounding in your calculators?

The scientific calculator stores the digits it cannot display while the four-function calculator rounds down to the last displayed digit.

Extend Your Learning Curve

Nanometer Technology

At times, even small things can affect our lives to a large extent. In recent years, a new branch of science, called *nanometer technology* or *nano-technology*, has allowed scientists to make molecular changes to the structure of certain substances. This technology can be applied, for instance, to the production of computer chips, the development of nanoscale drug delivery systems and implantation medicine.

Research and write a brief report on this technology and its possible applications.

Suggested Brief Report

Nano particles refer to those particles of size 1 to 100 nm. At the nanoscale, the physical, chemical and biological properties differ in fundamental ways. Nanotechnology has allowed us to create new materials which are lighter and stronger, and new drugs which have specified penetration characteristics.

The following websites provide more information about nanotechnology.

1. National Nanotechnology Initiative
http://www.nano.gov

2. Nano Science Technology Institute
http://www.nsti.org

Try It!

Section 1.1

1. Simplify the following.

(a) $a^{10} \times a^{8}$ **(b)** $(5x^3y^4) \times (6xy^7)$

Solution

(a) $a^{10} \times a^{8} = a^{10+8}$
$= a^{18}$

(b) $(5x^3y^4) \times (6xy^7) = (5 \times 6)x^{3+1}y^{4+7}$
$= 30x^4y^{11}$

2. Simplifythe following.

(a) $q^{17} \div q^{11}$ **(b)** $(30r^9s^{10}) \div (6r^8s^3)$

Solution

(a) $q^{17} \div q^{11} = q^{17-11}$
$= q^{6}$

(b) $(30r^9s^{10}) \div (6r^8s^3) = \left(\frac{30}{6}\right)(r^{9-8})(s^{10-3})$
$= 5rs^7$

3. Simplify the following.

(a) $(b^9)^5$ **(b)** $(c^6)^8 \div (c^3)^{10}$

Solution

(a) $(b^9)^5 = b^{9\times 5}$
$= b^{45}$

(b) $(c^6)^8 \div (c^3)^{10} = (c^{6\times 8}) \div (c^{3\times 10})$
$= (c^{48})(c^{30})$
$= c^{48-30}$
$= c^{18}$

4. Simplify the following.

(a) $(p^2q^5)^4 \times (p^3q)^6$ **(b)** $\left(\frac{3a^5}{b^3}\right)^4 \div \left(\frac{-9a^3}{b^6}\right)^2$

Solution

(a) $(p^2q^5)^4 \times (p^3q)^6 = (p^{2\times 4}q^{5\times 4}) \times (p^{3\times 6}q^{1\times 6})$
$= (p^8q^{20}) \times (p^{18}q^6)$
$= (p^{18+8}q^{20+6})$
$= p^{26}q^{26}$
$= (pq)^{26}$

(b) $\left(\frac{3a^5}{b^3}\right)^4 \div \left(\frac{-9a^3}{b^6}\right)^2 = \left(\frac{3^4a^{5\times 4}}{b^{3\times 4}}\right) \div \left(\frac{(-9)^2a^{3\times 2}}{b^{6\times 2}}\right)$
$= \left(\frac{81a^{20}}{b^{12}}\right) \div \left(\frac{81a^6}{b^{12}}\right)$
$= \left(\frac{81a^{20}}{b^{12}}\right) \times \left(\frac{b^{12}}{81a^6}\right)$
$= \frac{81}{81}(a^{20-6})(b^{12-12})$
$= a^{14}$

Section 1.2

5. Evaluate the following.

(a) $3^0 - 3^{-2}$ **(b)** $5^{-9} \times 5^6$

(c) $(2^{-9})^5 \div (2^6)^{-7}$ **(d)** $\left(\frac{7}{6}\right)^{-1}$

Solution

(a) $3^0 - 3^{-2} = 1 - \frac{1}{3^2}$
$= 1 - \frac{1}{9}$
$= \frac{8}{9}$

(b) $5^{-9} \times 5^6 = 5^{-9+6}$
$= 5^{-3}$
$= \frac{1}{5^3}$
$= \frac{1}{125}$

(c) $(2^{-9})^5 \div (2^6)^{-7} = 2^{-9\times 5} \div 2^{6\times(-7)}$
$= 2^{-45} \div 2^{-42}$
$= 2^{-45-(-42)}$
$= 2^{-3}$
$= \frac{1}{2^3}$
$= \frac{1}{8}$

(d) $\left(\frac{7}{6}\right)^{-1} = \frac{6}{7}$

6. Simplify the following and express your answers in positive exponential notation.

(a) $(m^{-4}n^3)^{-5}$
(b) $(p^6q^{-2})(p^{-6}q^{-3})$
(c) $(5x^{-4}y^8) \div (15x^{-3}y^6)$

Solution

(a) $(m^{-4}n^3)^{-5} = m^{(-4)\times(-5)}n^{3\times(-5)}$
$= m^{20}n^{-15}$
$= \frac{m^{20}}{n^{15}}$

(b) $(p^6q^{-2})(p^{-6}q^{-3}) = p^{6+(-6)}q^{-2+(-3)}$
$= p^0q^{-5}$
$= \frac{1}{q^5}$ $(p^0 = 1)$

(c) $(5x^{-4}y^8) \div (15x^{-3}y^6) = \left(\frac{5}{15}\right)(x^{-4-(-3)})(y^{8-6})$
$= \frac{1}{3}(x^{-1})(y^2)$
$= \frac{y^2}{3x}$

Brainworks

5. The solution of the equation $4^x = 2^n$ is an integer x. Find two possible values of the constant n.

Solution

$$4^x = 2^n$$
$$2^{2x} = 2^n$$
$$\therefore\ 2x = n$$
$$n = 2 \text{ or } 4$$

Exercise 1.5

Basic Practice

1. Express the following numbers in scientific notation.

(a) 83,700 **(b)** 720,000
(c) 96,200,000 **(d)** 1,450,000,000
(e) 0.00016 **(f)** 0.000028
(g) 0.0000095 **(h)** 0.000000030

Solution

(a) $8\underbrace{3{,}700}_{\text{4 places}} = 8.37 \times 10^4$

(b) $7\underbrace{20{,}000}_{\text{5 places}} = 7.2 \times 10^5$

(c) $9\underbrace{6{,}200{,}000}_{\text{7 places}} = 9.62 \times 10^7$

(d) $1{,}\underbrace{450{,}000{,}000}_{\text{9 places}} = 1.45 \times 10^9$

(e) $0.\underbrace{0001}_{\text{4 places}}6 = 1.6 \times 10^{-4}$

(f) $0.\underbrace{00002}_{\text{5 places}}8 = 2.8 \times 10^{-5}$

(g) $0.\underbrace{000009}_{\text{6 places}}5 = 9.5 \times 10^{-6}$

(h) $0.\underbrace{00000003}_{\text{8 places}}0 = 3.0 \times 10^{-8}$

2. Express the following numbers as integers.

(a) 9.8×10^3 **(b)** 5×10^4
(c) 7.23×10^6 **(d)** 1.06×10^8

Solution

(a) $9.8 \times 10^3 = 9{,}800$
(b) $5 \times 10^4 = 50{,}000$
(c) $7.23 \times 10^6 = 7{,}230{,}000$
(d) $1.06 \times 10^8 = 106{,}000{,}000$

3. Express the following numbers as decimals.

(a) 4×10^{-3} **(b)** 3.6×10^{-5}
(c) 1.58×10^{-6} **(d)** 2.07×10^{-10}

Solution

(a) $4 \times 10^{-3} = 0.004$
(b) $3.6 \times 10^{-5} = 0.000036$
(c) $1.58 \times 10^{-6} = 0.00000158$
(d) $2.07 \times 10^{-10} = 0.000000000207$

4. Evaluate the following without using a calculator and express your answers in scientific notation.

(a) $3.6 \times 10^4 + 4.7 \times 10^5$
(b) $6.8 \times 10^{-9} + 9 \times 10^{-7}$
(c) $4 \times 10^6 - 9.8 \times 10^5$
(d) $5.4 \times 10^{-11} - 6.6 \times 10^{-12}$
(e) $(5 \times 10^6) \times (3 \times 10^8)$
(f) $(4 \times 10^{-5}) \times (1.7 \times 10^9)$
(g) $(2 \times 10^{-3}) \div (8 \times 10^{-7})$
(h) $(3.4 \times 10^{-4}) \div (1.7 \times 10^5)$

Solution

(a) $3.6 \times 10^4 + 4.7 \times 10^5 = 0.36 \times 10^5 + 4.7 \times 10^5$
$= 5.06 \times 10^5$

(b) $6.8 \times 10^{-9} + 9 \times 10^{-7} = 0.068 \times 10^{-7} + 9 \times 10^{-7}$
$= 9.068 \times 10^{-7}$

(c) $4 \times 10^6 - 9.8 \times 10^5 = 4 \times 10^6 - 0.98 \times 10^6$
$= 3.02 \times 10^6$

(d) $5.4 \times 10^{-11} - 6.6 \times 10^{-12} = 5.4 \times 10^{-11} - 0.66 \times 10^{-11}$
$= 4.74 \times 10^{-11}$

(e) $(5 \times 10^6) \times (3 \times 10^8) = 5 \times 3 \times 10^6 \times 10^8$
$= 15 \times 10^{6+8}$
$= 1.5 \times 10^{15}$

(f) $(4 \times 10^{-5}) \times (1.7 \times 10^9) = (4 \times 1.7) \times 10^{-5} \times 10^9$
$= 6.8 \times 10^{-5+9}$
$= 6.8 \times 10^4$

(g) $(2 \times 10^{-3}) \div (8 \times 10^{-7}) = (2 \div 8) \times (10^{-3} \div 10^{-7})$
$= 0.25 \times 10^{-3-(-7)}$
$= 0.25 \times 10^4$
$= 2.5 \times 10^3$

(h) $(3.4 \times 10^{-4}) \div (1.7 \times 10^5)$
$= (3.4 \div 1.7) \times (10^{-4} \div 10^5)$
$= 2 \times 10^{-4-5}$
$= 2 \times 10^{-9}$

Further Practice

5. Evaluate the following without using a calculator and express your answers in scientific notation.

(a) $(1.3 \times 10^7)^2$ **(b)** $(5 \times 10^{-6})^3$
(c) $\sqrt{1.96 \times 10^8}$ **(d)** $\sqrt[3]{2.16 \times 10^{-10}}$
(e) $\dfrac{(2 \times 10^{-5}) \times (6 \times 10^7)}{4 \times 10^{-8}}$ **(f)** $\dfrac{8 \times 10^{-4} + 1 \times 10^{-5}}{5 \times 10^6 - 5 \times 10^5}$

(ii) Approximate area of the field
$= 64 \times 26$
$= 1{,}664$
$= 1{,}700 \text{ m}^2$ (rounded to 2 sig. fig.)

(c) No.

Brainworks

9. Determine if $25{,}735 \times 58$ is greater than $25{,}736 \times 57$ without using a calculator.

Solution

$25{,}736 \times 57 = (25{,}735 + 1) \times (58 - 1)$
$= 25{,}735 \times 58 + 58 - 25{,}735 - 1$
$< 25{,}735 \times 58$

$\therefore$ $25{,}735 \times 58$ is greater than $25{,}736 \times 57$

10. Give two examples of estimation used in real-world situations.

Solution

To have a rough idea of the amount to pay for groceries.
To have a rough idea of how many people the lift can take.

11. Give an example of a computation in which the answer obtained using a calculator or computer software is not exact.

Solution

The value of π in the calculator.

Review Exercise 1

1. Simplify the following, expressing your answers with positive exponents.

(a) $(x^3y)(x^4y^2)$
(b) $(a^5b^6) \div (a^3b^8)$
(c) $(2a^3b^4)^3$
(d) $\dfrac{(-2p^3q^2)^2}{(4p^2q^5)^3}$

Solution

(a) $(x^3y)(x^4y^2) = x^{3+4}y^{1+2}$
$= x^7y^3$

(b) $(a^5b^6) \div (a^3b^8) = a^{5-3}b^{6-8}$
$= a^2b^{-2}$
$= \dfrac{a^2}{b^2}$

(c) $(2a^3b^4)^3 = 2^3a^{3\times3}b^{4\times3}$
$= 8a^9b^{12}$

(d) $\dfrac{(-2p^3q^2)^2}{(4p^2q^5)^3} = \dfrac{4p^{3\times2}q^{2\times2}}{4^3p^{2\times3}q^{5\times3}}$
$= 4^{1-3}p^{6-6}q^{4-15}$
$= 4^{-2}p^0q^{-11}$
$= \dfrac{1}{16q^{11}}$

2. Evaluate the following.

(a) $10^{-2} + 10^0 + 10^2$
(b) $3^{-1} - 4^{-2}$
(c) $\left(\dfrac{4}{5}\right)^{-3}$
(d) $\left(\dfrac{2}{3}\right)^2 \div \left(\dfrac{9}{4}\right)^{-2}$

Solution

(a) $10^{-2} + 10^0 + 10^2 = \dfrac{1}{100} + 1 + 100$
$= 101\dfrac{1}{100}$

(b) $3^{-1} - 4^{-2} = \dfrac{1}{3} - \dfrac{1}{4^2}$
$= \dfrac{1}{3} - \dfrac{1}{16}$
$= \dfrac{16}{48} - \dfrac{3}{48}$
$= \dfrac{13}{48}$

(c) $\left(\dfrac{4}{5}\right)^{-3} = \left(\dfrac{5}{4}\right)^3$
$= \dfrac{5^3}{4^3}$
$= \dfrac{125}{64}$
$= 1\dfrac{61}{64}$

(d) $\left(\dfrac{2}{3}\right)^2 \div \left(\dfrac{9}{4}\right)^{-2} = \dfrac{2^3}{3^3} \times \left(\dfrac{4}{9}\right)^{-2}$
$= \dfrac{8}{27} \times \dfrac{9^2}{4^2}$
$= \dfrac{3}{2}$

3. Simplify the following, expressing your answers with positive exponents.

(a) $(x^3y^{-1})(x^{-4}y^5)$
(b) $(-2x^4y^3)^0(-3x^{-4}y^5)^{-2}$
(c) $\dfrac{(6p^2q^{-3})^5}{(-3pq^{-5})^3}$
(d) $\left(\dfrac{x^4}{y^{-2}}\right)^3 \div \left(\dfrac{x^{-2}}{y^3}\right)^4$

Solution

(a) $(x^3y^{-1})(x^{-4}y^5) = x^{-1}y^4$
$= \dfrac{y^4}{x}$