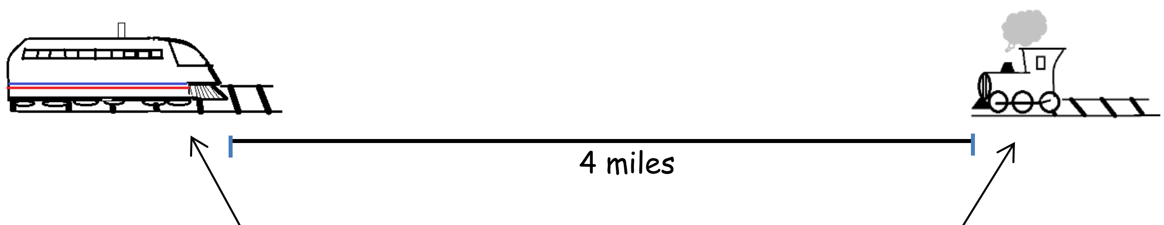
LESSON 3: DISTANCE FORMULA IN STORY PROBLEMS

Now that you understand how to use the Distance Formula, let's bump it up a bit, so it's a little more Algebra "Two-ish."

Most upper level math tests, like the SAT or college placement tests, will give you a story problem that involves two trains going opposite directions. They sound nearly impossible to solve, but once you realize how easy it is to build an equation, you will find these types of problems simple. Read the example below.

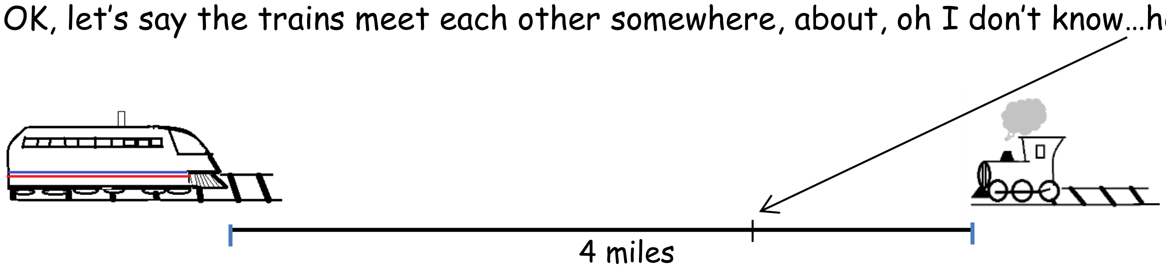


A commuter train is heading east at 50 miles per hour. A little train is heading west at 30 miles per hour. The trains are 4 miles apart. How long until they meet?

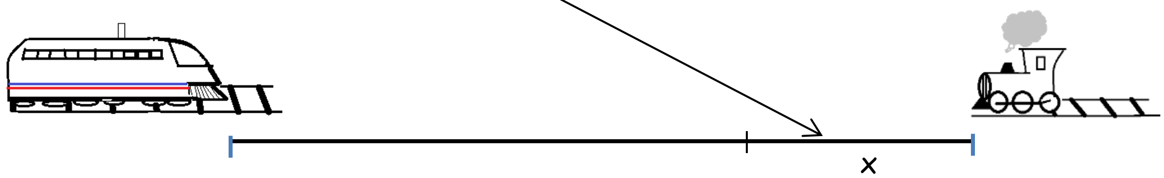
OK, you know this is a "Distance Formula" question, but there are two different rates, no time, and their distances are combined! Yikes!

We have to look at this problem logically. First, we will focus our attention on the little train. It is going to travel some UNKNOWN distance at 30 miles per hour. And we would like to know how long it will take this little train to get there.

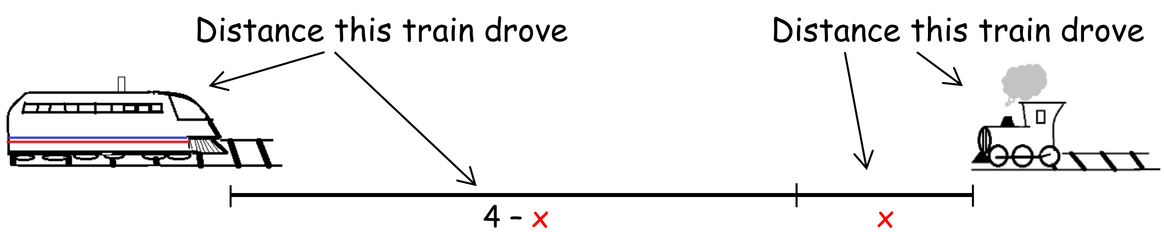
OK, let's say the trains meet each other somewhere, about, oh I don't know…here.



Then this distance is one of our unknown amounts. We will call it x.



So if the little train traveled "x" miles and the entire distance is 4 miles, then the commuter train will have to travel "4 miles – x miles." Do you understand that? Let's pretend the little train went 1 mile. Then the commuter train would go 4 – 1 miles. But we don't know how far the little train went, so we call it "x."



OK, now we have two distances and two rates. Let's make an equation for each train using the Distance Formula.

Commuter train at 50 mph

Little train at 30 mph

$$\frac{d}{r} = t \qquad \frac{4 - x}{50} = t \qquad \frac{x}{30} = t$$

Now comes the fancy part! Look at this equation here. Solve for x by multiplying both sides by 30. You get this, right?

$$x = 30t$$

Now we know how far the little train traveled. It traveled $30t$ miles. So that means that the big commuter train traveled $4 - 30t\ miles$.

Replace the "x" in the commuter train's equation with $30t$.

Commuter train

$$\frac{4-x}{50} = t$$

Little train

$$x = 30t$$

Solve for "t." Start by multiplying both sides by 50.

$$\frac{4-30t}{50} = t$$

$$4 - 30t = 50t$$

Add 30t to both sides and combine like terms.

$$4 = 80t$$

Divide both sides by 80 to get "t" by itself.

$$\frac{4}{80} = t \quad Reduce\ down\ to\ \frac{1}{20} = t$$

OK, so $t = \frac{1}{20}$, what does that mean? That is the time it will take for the two trains to meet each other, $\frac{1}{20}$ of an hour. But what is that? Well let's see, an hour is 60 minutes long, so we want to know how much is $\frac{1}{20}$ of 60 minutes. I'll change those words into math.

$$x = \frac{1}{20} \times \frac{60}{1}$$

$$x = \frac{60}{20}\ or\ \frac{6}{2}\ or\ 3\ minutes$$

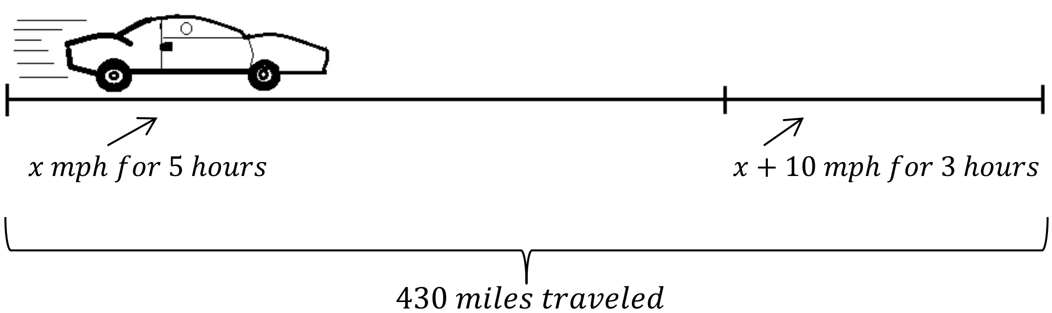
Do you see how changing a story problem into an algebraic equation can help you solve for the answer? Let's try another one together. Read the following story problem.

Anita drives for 5 hours at some unknown speed and then she speeds up by 10 mph and drives for 3 more hours. She traveled a total of 430 miles. What two rates of speed did she travel?

The first step is to figure out WHAT we are looking for; the unknown. This question asks, "What two rates of speed…" so that is our "x." Anita drove x mph.

$$x = first\ speed$$

And how long did she drive that fast? She drove x mph for 5 hours. Then she sped up and drove 10 miles per hours faster for 3 more hours. That would be written as "x + 10 mph," wouldn't it? I'll draw a little picture, so this is easier to visualize.



Let's pull out our distance formula and fill in some of the variables with our info.

$$\frac{d}{r} = t$$

Mmm…since we know the distance is 430 miles, let's rearrange this formula, so it says, "Distance equals something."

$$d = rt$$

That's more like it, now I can fill in the distance.

$$430\, miles = rt$$

OK, we've got our distance, now we need to fill in the "rate" and "time." The first portion of Anita's trip was driven at x mph for 5 hours. That is the rate and time.

$$430 = 5x$$

But don't forget about the second portion of her trip. She also drove for 3 hours at x + 10 mph. But think about that for a moment…how will you write that? If you wrote $3x + 10$, that would be, "3 hours at the original speed, plus 10 miles." That's not right. We want to write 3 hours TIMES $x + 10$.

$$430 = 5x + 3(x + 10)$$

Rub your palms together briskly, and let's do some algebra!

$$430 = 5x + 3(x + 10)$$

$$430 = 5x + 3x + 30$$

Use the Distributive Property of Multiplication

$$430 = 8x + 30$$

Add the like terms.

$$430 - 30 = 8x$$

Swing the 30 over to the other side and change the sign.

$$400 = 8x$$

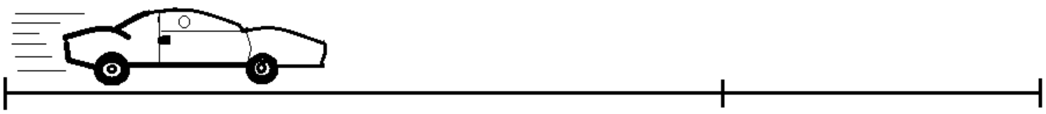
$$\frac{400}{8} = x$$

$$\mathbf{50 = x}$$

Divide both sides by 8.

$400 \div 8 = 50$

Tada! We have an answer for x. So now what? Let's go back to our picture and fill in the "x" with 50.



$x\ mph\ for\ 5\ hours$ $\qquad$ $x + 10\ mph\ for\ 3\ hours$

Anita traveled at 50 miles per hours for 5 hours and 60 mph for 3 hours.

There is one other way to use the distance formula. It isn't just for solving "traveling" types of problems; it can be used to solve other kinds of problems too. For example, the "rate" doesn't have to be "miles per hour," it could be "number of lawns mowed per hour" or "number of books read per day." Here is an example of a "rate" story problem. Get it? Instead of a "distance" problem, I said "rate."

Esther can peel 4 potatoes per minute. How long will it take her to peel 68 potatoes?

Let's pull out the distance formula…ahem…I mean the RATE formula. (In case you don't get my humor, they are the same thing.)

$$\frac{d}{r} = t$$

In this situation the "Rate" is how fast Esther can peel potatoes. She can peel 4 per minute, $\frac{4}{1\ minute}$. That amount goes here.

Remember: This line means "per."

$$\frac{d}{\frac{4}{1}} = t$$

Don't be nervous about having a fraction in the denominator, it will be really easy to solve. We are looking for the "time" it will take to peel "68" potatoes. So you could say that peeling 68 potatoes would be "going the distance." And "time" is our unknown variable.

$$\frac{68}{\frac{4}{1}} = t$$

Do you know how to solve this equation? Since $\frac{4}{1}$ is the same thing as 4, just divide, $68 \div 4$.

$$\frac{68}{4} = t$$

$$17 = t$$

It will take Esther 17 minutes to peel 68 potatoes at the rate of 4 potatoes per minute.

Now let's say Esther gets some help. Her mom, Keisha, has a fancy automatic potato peeler that can peel 13 potatoes per minute. Now how fast can the two of them peel 68 potatoes together?

That's no big deal. I'll just add Keisha's rate and Esther's rate together.

$$\frac{68}{\frac{4}{1} + \frac{13}{1}} = t$$

Before I do anything else, I'm going to add these two whole numbers together.

$$\frac{68}{17} = t$$

$$4 = t$$

It will take only 4 minutes for the two of them to peel 68 potatoes.

Complete the next worksheet. If you get stuck, try really hard to figure it out yourself. But if you just can't figure it out, take a peek at the answer to give yourself a little hint.

WORKSHEET 3

Name ____ Date _____________

1. Trinity walked to town at a rate of 4 mph. She rode the bus back at 40 mph. The town is 2 miles away. How long did it take Trinity to travel to town and back?

2. A dog is resting against a pole to which he is attached with a 24 meter chain. He watches a cat approach to within 8 meters of the pole and promptly gives chase. The dog runs at 12 meters per second and the cat runs at 10 meters per second. Which does the dog reach first, the cat or the end of the chain?

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3. Gunnar flew north against the wind, which was blowing at 40 mph, for 5 hours. Returning south at the same speed, with the wind pushing him 40 mph, he made the trip in 3 hours. What was the speed of the plane? (Hint: the distance is the same in both directions).

4. Two workers plan to mow 14 lawns. The older worker can mow 2 ½ lawns per hour. The younger worker can mow 1 lawn per hour. How long is it going to take the two workers to mow all 14 lawns.