

From the above class activity, we observe that $3(x + y) = 3x + 3y$. In general, we have the **distributive law** of multiplication over addition:

$$a(x + y) = ax + ay$$

We say that $a(x + y)$ is expanded to $ax + ay$. The expressions $a(x + y)$ and $ax + ay$ are equivalent expressions.

This law can be generalized and applied as follows:

$$\begin{aligned} 1. \quad (x + y)a &= a(x + y) \\ &= ax + ay \\ &= xa + ya \end{aligned}$$

Multiplication can be distributed over addition from the right.

$$\begin{aligned} 2. \quad a(x - y) &= a[x + (-y)] \\ &= ax + a(-y) \\ &= ax - ay \end{aligned}$$

Multiplication can be distributed over subtraction.

$$3. \quad a(x + y + z) = ax + ay + az$$

Multiplication can be distributed over several terms.

Example 3

Expand each expression by removing the parentheses.

$$(a) \quad a(3b + c) \qquad (b) \quad -x(2y - z)$$

Solution

$$(a) \quad a(3b + c) = 3ab + ac$$

$$\begin{aligned} (b) \quad -x(2y - z) &= -x[2y + (-z)] \\ &= (-x)(2y) + (-x)(-z) \\ &= -2xy + xz \end{aligned}$$

Try It! 3

Expand each expression by removing the parentheses.

$$(a) \quad a(2b - 3c) \qquad (b) \quad -x(-5y + z)$$

RECALL

$$\begin{aligned} (a)(b) &= ab \\ (-a)(-b) &= ab \\ (a)(-b) &= -ab \\ (-a)(b) &= -ab \end{aligned}$$

Note: The distributive law is applicable when removing parentheses in algebraic expressions such as $x - (a - b)$. This expression can be interpreted as $x + (-1)(a - b)$.

$$\begin{aligned} x - (a - b) &= x + (-1)(a - b) \\ &= x + (-1)[a + (-b)] \\ &= x + (-1)(a) + (-1)(-b) \\ &= x - a + b \end{aligned}$$

From Class Activity 1, we can summarize the steps involved in problem solving with linear equations as follows:

- STEP 1** Read the question carefully and identify the unknown quantity.
- STEP 2** Use a letter to represent the unknown quantity (e.g. x).
- STEP 3** Express other quantities in terms of x .
- STEP 4** Form an equation based on the given information.
- STEP 5** Solve the equation.
- STEP 6** Write down the answer statement.

Note: It is a good practice to check whether the solution you have obtained satisfies the conditions in the original problem. For instance, some problems may require the solution to be a positive integer. If we get a solution $x = -\frac{2}{3}$, it should be rejected.

Example 13

The sum of three consecutive integers is 111. Find the integers.

Solution

STEP 1 We are going to find the three integers.

STEP 2 Let x be the smallest integer.

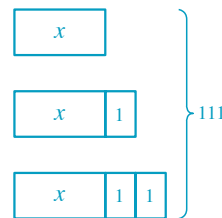
STEP 3 Middle integer = $x + 1$
Largest integer = $x + 2$

STEP 4 Sum of three integers = 111
 $\therefore x + (x + 1) + (x + 2) = 111$
 $3x + 3 = 111$

STEP 5 $3x = 111 - 3$
 $3x = 108$
 $x = \frac{108}{3}$
 $\therefore x = 36$

STEP 6 The three integers are 36, 37, and 38.

Model Method



\therefore the equation is $3x + 3 = 111$.

REMARKS



It is important to set just one variable x and express the other two integers in terms of x . Otherwise, there would be too many variables to be solved.

Check:

When $x = 36$,
 $x + 1 = 37$
and $x + 2 = 38$.

$$36 + 37 + 38 = 111$$

The solution $x = 36$ is correct.

Try It! 13

The sum of three consecutive integers is 144. Find the integers.

REVIEW EXERCISE 5



1. Solve the following equations.

(a) $13x - 22 = 30$

(b) $2(5x - 8) + 6 = 11$

(c) $\frac{2x}{3} + \frac{x}{5} = 13$

(d) $1 - \frac{4}{7}x = 23 + x$

(e) $\frac{4x - 5}{2} = \frac{7x - 3}{9}$

(f) $\frac{x - 4}{3} - \frac{2x + 1}{6} = \frac{5x - 1}{2}$

(g) $\frac{2}{x - 7} = 6$

(h) $\frac{4x - 1}{5x + 1} = \frac{5}{7}$

2. Given the formula $D = b^2 - 4ac$,

(a) find the value of D when $a = 1$, $b = -5$, and $c = 3$,

(b) find the value of c when $a = 2$, $b = 3$, and $D = 49$.

3. Given the formula $S = \frac{n(a + b)}{2}$,

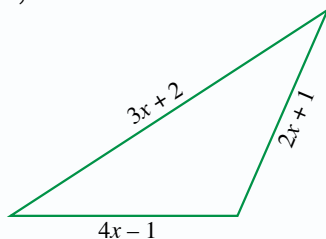
(a) find the value of S when $a = 1$, $b = 25$, and $n = 12$,

(b) find the value of a when $b = 41$, $n = 15$, and $S = 330$.

4. The lengths of the sides of a triangle are $(2x + 1)$ cm, $(3x + 2)$ cm, and $(4x - 1)$ cm.

(a) Find the perimeter of the triangle in terms of x .

(b) If the perimeter of the triangle is 47 cm, find the value of x .



5. Peter has 96 stamps and Sam has 63. How many stamps should Sam give Peter so that Peter will have twice as many stamps as Sam?

6. A boy is 26 years younger than his father.

In 3 years' time, his age will be $\frac{1}{3}$ his father's age. Find the boy's present age.

7. The price of a skirt is \$25 more than the price of a T-shirt. The total price of 3 skirts and 8 T-shirts is \$339. Find the price of a skirt.



8. In a certain week, the amount of time Lisa spent on watching television was 3 hours more than twice the time she spent on doing her mathematics homework. If the total time she spent on these two activities was 30 hours in that week, how many hours did Lisa spend on doing her mathematics homework?

9. The number of books in a class library is 17 more than 3 times the number of students in the class. If 5 students are absent, each student can borrow exactly 4 books from the library. Find the number of students in the class.

10. A number is 4 times greater than another number. By subtracting 3 from each number, the first number becomes 5 times greater than the second. What are the two numbers?



EXTEND YOUR LEARNING CURVE

Matchstick Triangle Patterns

Johnny uses matchsticks to form a pattern of triangles as shown below.



Suppose m matchsticks are required to form n triangles.

(a) Copy and complete the following table.

n	1	2	3	4	5	6
m	3					

- (b) Find a formula connecting m and n .
- (c) How many matchsticks are required to form 100 triangles?
- (d) How many triangles can be formed with 2,005 matchsticks?
- (e) Suppose the area of a triangle is $\sqrt{3}$ cm². Find the total area of the triangles formed in (d). Give your answer correct to the closest whole numbers.

WRITE IN YOUR JOURNAL

Did you find the general strategy used to solve word problems on linear equations helpful? Why or why not?

6

RATIO, RATE AND SPEED



LET'S LEARN TO...

- 1 understand the concept of ratio
- 2 state the relationship between ratio and fraction
- 3 solve problems involving ratio
- 4 understand the concepts of rate and average rate
- 5 solve problems involving rate
- 6 understand the concepts of uniform speed and average speed
- 5 solve problems involving speed

The density of a material is defined as its mass per unit volume, that is, $\text{density} = \frac{\text{mass}}{\text{volume}}$. Because the ratio of the density of pure ice to that of sea water is 9 : 10, typically, only a small part of a giant iceberg is above water. The shape of the underwater portion can be difficult to judge by looking at the portion above the surface. Do you know what fraction of the volume of the iceberg is exposed?

6.1 Ratios Involving Rational Numbers

A Meaning of Ratio

We have learned the idea of a ratio in the previous grade. Let us recall its meaning.

Given any two similar quantities, a and b , the ratio of a to b (denoted by $a : b$) is defined as

$$a : b = \frac{a}{b}, \text{ where } b \neq 0.$$

A ratio is a comparison of two similar quantities. In this section, we will limit our discussion on ratios to those involving rational numbers.

Example 1

There are 16 boys and 20 girls in a class. Find the ratio of

- (a) the number of boys to the number of girls,
- (b) the number of girls to the total number of students in the class.

Solution

(a) Ratio of the number of boys to the number of girls
 $= 16 : 20$ $16 : 20 = \frac{16}{20} = \frac{4 \times 4}{4 \times 5} = \frac{4}{5}$
 $= 4 : 5$

(b) Total number of students $= 16 + 20$
 $= 36$

Ratio of the number of girls to the
total number of students
 $= 20 : 36$
 $= 5 : 9$

Note: Since a ratio can be expressed as a fraction, we will reduce the ratio to its simplest form $a : b$, where a and b have no common factors except for 1.

Try It!

A bag consists of 25 green balls and 15 red balls. Find the ratio of

- (a) the number of green balls to the number of red balls,
- (b) the number of red balls to the total number of balls in the bag.

EXERCISE 7.2



BASIC PRACTICE

- Find the unknown quantity in each case.
 - 30% of a is 18.
 - 37.5% of $\$b$ is $\$108$.
 - $22\frac{2}{9}\%$ of c kg is 44 kg.
 - 150% of d cm² is 126 cm².
 - 0.5% of e °C is 7 °C.
 - $\frac{1}{3}\%$ of f hours is 12 hours.



FURTHER PRACTICE

- Adam attempts 65% of the questions in a test. If he attempts 52 questions, find the total number of questions in the test.
- 45% of the members in a council are women. There are 72 female council members. Find
 - the total number of council members,
 - the number of male council members.
- 85% of the customers of a supermarket were residents of the neighborhood. Given that 2,380 of the customers on a particular day were residents, find
 - the total number of customers,
 - the number of customers who were not residents of the neighborhood on that day.
- After cycling 18 km at an average speed of 12 km/hr, Lucy finds that she still has to cycle 55% of the total distance. She then completes the rest of her journey at an average speed of 16.5 km/hr. Find
 - the total distance of her journey,
 - the remaining distance she needs to cycle to complete the journey,
 - the time taken for the whole journey,
 - the average speed for the whole journey.



MATH@WORK

- A drink stall in the mall sells bottled water, juice and soda. On a particular day, it sold 175 out of its 200 bottled water, 85% of its 220 bottled juice and 180 bottles of soda which is 80% of its original stock of bottled soda.
 - Which drink has the highest number of bottles sold? How many bottles of this drink were sold?
 - Which drink has sold the highest percentage of its stock? What was the percent figure?
 - Which drink has the greatest number of bottles in its original stock? What was the figure?
- 38% of Sumiko's music CDs are Chinese music CDs, 44% of them are English music CDs and the rest are Japanese music CDs. There are 45 Japanese music CDs.
 - Find the total number of CDs in the collection.
 - Find the number of Chinese music CDs.
 - If 60% of the English music CDs are CDs with pop songs, find the number of CDs with English pop songs.



BRAINWORKS

- Is 60% of 30 equal to 30% of 60? Explain your answer. Generalize your findings using variables x and y .
 - If 60% of x is 30 and 30% of y is 60, is x equal to y ? Explain your answer.
 - Write a real-life problem that involves the mathematics in (a).



Meaning of Percentage

$$n\% = \frac{n}{100}$$

$$1\% = \frac{1}{100}$$

$$100\% = 1$$

Expressing One Quantity as a Percentage of Another and Reverse Percentage

If a is $n\%$ of b ,

then $a = \frac{n}{100} \times b$

and $b = \frac{100a}{n}$.

Percentage Increase

$$\text{Increase} = \text{Increased value} - \text{Original value}$$

$$\text{Percentage increase} = \frac{\text{Increase}}{\text{Original value}} \times 100\%$$

$$\text{Increased value} = (100\% + \text{Increase } \%) \times \text{Original value}$$

Percentage Decrease

$$\text{Decrease} = \text{Original value} - \text{Decreased value}$$

$$\text{Percentage decrease} = \frac{\text{Decrease}}{\text{Original value}} \times 100\%$$

$$\text{Decreased value} = (100\% - \text{Decrease } \%) \times \text{Original value}$$

Discount

$$\text{Discount} = \text{Marked price} - \text{Selling price}$$

$$\text{Percentage discount} = \frac{\text{Discount}}{\text{Marked price}} \times 100\%$$

$$\text{Selling price} = (100\% - \text{Discount } \%) \times \text{Marked price}$$

Tax

$$\text{Tax} = \text{Tax rate} \times \text{Cost}$$