LESSON 29: FACTORING A QUADRATIC

Look at the equation below.

$$(5x + 3)(2x + 3) = (10x^2 + 15x + 6x + 9)$$

Factor Factor Product

The binomials (5x + 3) and (2x + 3) are the factors of $10x^2 + 15x + 6x + 9$. What are the factors of this problem?

$$(7x + 2)(2x + 4)$$

The factors are the "numbers" being multiplied. In this case, (7x + 2) is being multiplied by (2x + 4), so those are the factors. I will use the FOIL method to multiply these two factors together. Try to come up with the answer yourself, before you look at my answer.

$$(7x + 2)(2x + 4) = 14x^2 + 28x + 4x + 8$$

Combine the like terms.

$$14x^2 + 32x + 8$$

That is the product of those two factors. Guess what? When you multiply a binomial expression by another binomial expression, the answer is a *quadratic* expression. Get it? Quad means 4, and bi means 2, so 2 x 2 = 4, and a binomial times a binomial = a quadratic. Whenever you multiply two binomials, the answer will always fit into this mold. Do you see how the answer above looks a lot like this quadratic form?

$$ax^2 + bx + c \leftarrow \text{a constant}$$

"a" is the coefficient of x^2 "b" is the coefficient of x

The letter "a" is the coefficient of x^2. Sometimes the coefficient will be the number 1, which doesn't need to be written, so "a" may be invisible at times.

$$ax^2 + bx + c$$

The letter "b" represents the coefficient of "x" or whatever variable is being used. And the letter "c" is any number by itself, no variable; it's called a constant.

Whenever you have a polynomial that fits into this exact format, it is called a q*uadratic.* And remember, a **QUAD**ratic is the product of two **BI**nomials.

Now, it is time for the next big challenge in algebra. It is called *factoring a quadratic expression.*

To factor a quadratic means to do the reverse of what we just did. We just took two binomials (7y + 2) and (2y + 4), multiplied them together, and ended up with the quadratic, $14y^2 + 32y + 8$. This time, we will start with the quadratic and then we have to figure out what two binomials were multiplied together, to get that product.

Let's start with a simple one. I'll give you the quadratic and then we'll figure out what two binomials (factors) were multiplied together to get that product. We start with two empty sets of parentheses. We will fill in the blanks as we figure them out. Let's factor this quadratic expression.

Quadratic: $x^2 + 6x + 8$

Start with two empty sets of parentheses. ( )( )

Take a look at the first term, x^2. What two factors were multiplied together, to get x^2 as a product? The two factors of x^2 are x and x. So that must be the two "first" terms.

Quadratic: $x^2 + 6x + 8$

Put the two factors of x^2 in the parentheses. $(x \quad)(x \quad)$

Now take a look at the last term in the quadratic, 8. When we use the FOIL method, the last step we do is multiply the LAST numbers together. This "8" is a product of the two LAST numbers in the parentheses. What two numbers do you suppose were multiplied together to get 8? Two factors of 8 are 4 and 2. Let's put those two numbers in the parentheses and see if it makes sense.

Quadratic: $x^2 + 6x + 8$

Put the two factors of 8 in the parentheses. $(x \quad 2)(x \quad 4)$

Since we have all positive number in our quadratic, it is safe to put "+" signs inside the parentheses too.

$$x^2 + 6x + 8$$
$$(x + 2)(x + 4)$$

We can find out if we are right, by using the FOIL method. Multiply those two binomials to see if the product (answer) is the quadratic we started with. Here is the math to check our work.

$$(x + 2)(x + 4)$$
$$x^2 + 4x + 2x + 8$$
Combine like terms.
$$x^2 + 6x + 8$$

Yep! When we multiply (x + 2) by (x + 4), we get a product of $x^2 + 6x + 8$. Now let's say we didn't use 4 and 2 for factors of 8. Let's say we chose 8 and 1, to be the factors of 8. Our binomials would have been (x + 8)(x + 1). Use the FOIL method to get the product of these two binomials instead.

$$(x + 8)(x + 1)$$

$$x^2 + 1x + 8x + 8$$

Combine like terms.

$$x^2 + 9x + 8$$

That answer is not the same as the quadratic we started with, so it is not the right answer. That's how I know that 4 and 2 are the right choice, not 8 and 1. Let's try another one. We will start with a quadratic and an empty set of parentheses.

$$x^2 + 8x + 15$$
$$(\quad)(\quad)$$

Look at the first term in the quadratic, x^2. What are the factors of x^2? Put those in the parentheses.

$$x^2 + 8x + 15$$
$$(x \quad)(x \quad)$$

Look at the last term in the quadratic, 15. What are the factors of 15? 5 and 3 are factors of 15, so are 1 and 15. Look at the term in the center of the quadratic, 8x. Which of those two sets of factors will total 8 when combined?
5 and 3, or 15 and 1? Definitely 5 and 3. Put those in the parentheses.

$$x^2 + 8x + 15$$
$$(x \quad 5)(x \quad 3)$$

All the numbers are positive, so it is safe to put "+" signs in the parentheses.

$$x^2 + 8x + 15$$
$$(x \ + 5)(x + 3)$$

We can find out if we are right by multiplying the two binomials, to see if we come up with the same quadratic.

$$(x + 5)(x + 3)$$
$$x^2 + 3x + 5x + 15$$
$$x^2 + 8x + 15$$

Yep, we got the right answer, so our factors are correct. Let's try another one. This time, there will be a negative number involved. Here is the quadratic expression.

$$x^2 - 7x + 12$$

Start with the first term in the quadratic, x^2. What two factors can produce x^2? It must be x and x again. Put those in the empty parentheses.

$$x^2 - 7x + 12$$
$$(x \quad)(x \quad)$$

The last term in the quadratic is 12. What are the possible factors of 12? I can think of two possible sets of factors, 6 and 2, and 3 and 4. Look at the center term and decide which set would work best. The center term is -7x, so I think 3 and 4 would be the best choice. Put those two numbers in the parentheses.

$$x^2 - 7x + 12$$
$$(x \quad 3)(x \quad 4)$$

But wait! The center term is *negative* 7, can you figure out what signs you will need to use for the 3 and 4?

Keep in mind that this amount is a *sum* and this amount is a product.

$$x^2 - 7x + 12$$

A *sum* is the answer to an addition problem. A product is the answer to a multiplication problem. Adding negative numbers, gives a negative sum. Multiplying negative numbers, gives a positive product.

In the problem above, we want the sum of 3 and 4 to be -7 and we want the product of 3 and 4, to be positive. That's easy, make them both negative!

$$-3 + -4 = -7$$
$$-3 \times -4 = +12$$

Fill in the parentheses and then use the FOIL method to see if we have the right factors.

$$(x - 3)(x - 4)$$
$$x^2 - 4x - 3x + 12$$
Combine like terms.
$$x^2 - 7x + 12$$

Yep, that's the right answer. Do you see how you can look at the factors of the last term and then decide which ones would be best by looking at the center term? Let's try another one. Here is the quadratic and an empty set of parentheses.

$$y^2 - 10y + 21$$
$$(\quad)(\quad)$$

What are the factors of y^2? Put those two factors in the parentheses.

$$y^2 - 10y + 21$$
$$(y \quad)(y \quad)$$

Look at the last term. What are the possible factors of 21? Keep in mind that the center term is -10y, so whatever factors you choose, they better equal -10 when combined. The possible factors of 21 are 7 and 3, or 1 and 21. Which two will equal 10? Definitely 7 and 3, just make them negative, so when they are multiplied together they will be positive and when they are added together they will be negative.

$$y^2 - 10y + 21$$
$$(y - 7)(y - 3)$$

Use the FOIL method, to see if we got the right answer.

$$(y - 7)(y - 3)$$
$$y^2 - 3y - 7y + 21$$
Combine like terms
$$y^2 - 10y + 21$$

That's right! Let's try one more. This time our quadratic will have all negative numbers. If the last term, which is a product, is a negative number, then the two factors MUST have opposite signs.

Here is the next quadratic and an empty set of parentheses to start with.

$$m^2 - 3m - 18$$
$$(\quad)(\quad)$$

What are the factors of m^2?
What are the possible factors of 18?
Choose factors of 18 that total -3 when combined.
The possible factors of 18 are, 9 and 2, 6 and 3, or 18 and 1.
Which set of factors will total -3 when combined?
Remember the last term is negative, so you will have two different signs.
If we use -6 and +3, we will have a total of -3, so let's use those.

$$m^2 - 3m - 18$$
$$(m - 6)(m + 3)$$

Check our work with the FOIL method.

$$(m - 6)(m + 3)$$
$$m^2 + 3m - 6m - 18$$
$$m^2 - 3m - 18$$

That's it! Now try some on your own on the next worksheet. You will be given a quadratic and then you have to figure out what two binomials were multiplied together, to get that quadratic. In other words, factor the quadratic.

Name_____ Date _____

Worksheet 29 Factoring Quadratics

Factor these quadratic expressions.

1. $x^2 + 8x + 16$

2. $m^2 + 5m + 6$

3. $x^2 - 8x + 7$

4. $y^2 + 4y - 32$

5. $x^2 + 13x + 42$

6. $n^2 + 5n - 36$

7. $b^2 + 7b - 30$

8. $c^2 - 10c + 16$

9. $a^2 + 8a - 33$

10. $x^2 + 16x + 48$