Welcome to the Learn Math Fast System, Volume IV. This book will cover basic Geometry. For the best results, you should read Volumes I - III of the Learn Math Fast System, first. But if you are already proficient in basic math and pre algebra, you are ready for this book.

Be sure to read each lesson and then complete the worksheet or test at the end of the lesson. Compare your answers with the ones in the back of the book. If you get stuck on a problem, use the answer key to help you solve it.

This book comes with a Math Kit. As you read through the book, you will learn about each item in the kit.

Once you finish this book, you are ready for Learn Algebra Fast, the next book in the Learn Math Fastseries.

## CHAPTER 1

## LINES AND ANGLES

## LESSON 1: LINES

Geometry is the study of shapes, angles, and lines. In geometry, a line goes on forever with no end. Since we can't draw a line that goes on forever, we put arrows on the end to show that it does. The line below is called line AD. In geometry, it is written as $\overleftrightarrow{A D}$ for short. Notice the symbol on top of AD is a miniature picture of a line.


Since lines are so long, we usually only talk about one little chunk of a line at a time. In geometry a chunk of a line is called a line segment. Below is a line with some points on it, to show just a segment of the line.


This is called line segment $B C$. To make it shorter, we write $\overline{B C}$ to say line segment $B C$. Again, the symbol above $B C$ is just a miniature line segment.
On the next page is another type of line; it is called a ray. A ray is different from a line because it does have an end, so it only gets one arrow. The arrow goes on forever, but the end has a fixed point called an endpoint.

Think of a ray as a "sun ray." The endpoint is attached to the sun and the arrow is shooting out to the earth.

The point that is attached to the sun is called the endpoint. The endpoint in ray $A B$ below is called endpoint $A$.


Ray $A B$ is written as $\overrightarrow{A B}$ for short. Notice the symbol above $A B$; it is a miniature picture of a ray. When two rays join together at their endpoints, they can't help but create an angle.


This is written as $\angle A B C$. That is the long name. You can call it angle $B$ for short because that is the letter at the endpoint. When endpoints come together and make an angle, that new combined endpoint is called a vertex. In that last picture, $B$ is the vertex of the angle.

Take a quick test to make sure you understand everything so far. Be sure to include arrows in your drawings whenever necessary.

## WORKSHEET 4-1

1. Draw a line with two points creating a line segment $A B$.
2. Draw an angle and label the points as $C D E$ with point $D$ being the vertex.
3. Draw ray $K L$ with $K$ as the endpoint.
4. Name each picture as a line, a line segment, a ray, or an angle.

5. Using the little symbols we just learned: $\longleftrightarrow, \rightarrow$, , and $\angle$, write the short name for each of the following. For example, Line EF would be written as $\overrightarrow{E F}$.
a. Line $A B$
b. Line segment $C D$
c. Ray BC
d. Angle EFG

## LESSON 2: ANGLES

If you correctly answered all the problems on the last worksheet, then you are ready to move on. If you got a wrong answer, go back and find out why and then move on.

You will need your protractor for this next lesson. Look at your protractor. The numbers go around the edge from 0 to 180. There is a second set of numbers going the opposite direction, from 0 to 180 . But no matter which direction you go, 90 is always on top in the center.


A protractor is the tool we use to measure an angle. Angles are measured in degrees. To help explain what a degree is, I have drawn an angle inside a circle below. A protractor measures how much of a circle's curve is taken up by the angle. The entire circle is 360 degrees. Our angle below is only 75 of the 360 degrees. A protractor is used to find out exactly how many degrees this angle measures.


Look at the pictures to see how a protractor measures this angle.


Take a closer look, so you can read the measurement on the protractor. This angle is 75 degrees.


Our protractor is only half of a circle. It measures from 1 to 180 degrees. Below is a picture of an angle that is so big, it is almost the entire half circle. It measures about $179^{\circ}$. It is nearly a straight line.


If this angle gets 1 degree bigger, it will be a straight line; also called a straight angle. It is hard to tell the difference between a straight line and a straight angle, but technically a straight angle would be 180 degrees and a line... is just a line.

You may have heard someone say, "He made a one-eighty and came right back." Look at your protractor. A completely straight angle, which looks just like a straight line, is 180 degrees. So, if someone were walking with their arm pointing straight out in front of them and then turned around to face the opposite direction, their arm would have made a 180 degree arc. That's why making a "180" is making a complete turn-around.


Instead of writing out the word degrees every time, you can use a symbol. The symbol for degrees is a little circle drawn towards the top, right side of a number like this, $180^{\circ}$.

Often times, during a sports competition you'll hear, "He did a three-sixty!" When someone does a "360" on a snowboard, it means he jumped up and made a complete circle before landing. That's because turning in a complete circle is a $360^{\circ}$ turn. Look at your protractor again. If you start at zero and spin a half circle, it is $180^{\circ}$. If you keep spinning to make a full circle, you will have gone twice that much or 360 degrees. That's why the symbol for degrees is a little circle above the number. At least I think that's why - it makes sense.

## $360^{\circ}$

We use a protractor to measure angles in degrees. On the next page, I have drawn a perfect $90^{\circ}$ angle. Find the vertex of this angle.


Now look at your protractor again. Along the flat edge in the very center you will see a tiny hole.


Put that over top of the vertex and line up ray $B C$ with the $0^{\circ}$ line drawn on the protractor.

Once you have the vertex lined up with the center hole and the "zero" lines are lined up together, you will see that ray $B A$ is pointing to exactly $90^{\circ}$. That's how you know it is a $90^{\circ}$ angle. A perfect $90^{\circ}$ angle is called a right angle.

A lot of math books draw a little box in the corner of a right angle to prove that the angle is exactly $90^{\circ}$.


This little box tells us that this angle is not $89^{\circ}$, and it is not $91^{\circ}$, it is exactly $90^{\circ}$; also called a right angle. Knowing it is exactly $90^{\circ}$ will be helpful later in geometry when we have to try to solve mystery numbers. Below are two $90^{\circ}$ angles, back to back.


Now let's squish them together.


Now they look like this...


Now we have our first mystery. It is a super easy mystery to solve, but it is still a mystery. What is the measurement of the angle without a box?


That's simple; you know the other angle is $90^{\circ}$ because we just squished two $90^{\circ}$ angles together. But you could have figured it out mathematically. Earlier we learned that a straight line is $180^{\circ}$, so whatever angles are on top have to equal $180^{\circ}$. If one side is $90^{\circ}$, then the other side must also be $90^{\circ}$ because $90+90=$ 180.

The majority of the geometry questions you will answer will be "find the missing number" type of questions. You will be given a few clues and you use those clues to figure out the missing number.

Now what do you suppose would happen, if I moved this line over by $1^{\circ}$ ?


First of all, we would have to remove the little box in the corner because it would no longer be $90^{\circ}$. If I made that angle $1^{\circ}$ bigger, it would be a $91^{\circ}$ angle.


Now how big is this angle?
When I moved the line over by 1 degree, it made the angle bigger. But the other angle got 1 degree smaller, so it must be $89^{\circ}$ now. You could also figure this out mathematically, instead of logically. You know the straight line measures $180^{\circ}$. You know one angle measures $91^{\circ}$. Do the math 180-91=89. That proves the angle is $89^{\circ}$.

Now I will move the other line even further. What is the missing angle?


Since a straight line is $180^{\circ}$, we know that the two angles must equal 180. We know one of the angles is $45^{\circ}$, so the other angle must be $135^{\circ}$.

If this seems super easy, you're right, it is. If this sounds complicated, you are over thinking it. Go back and read the last few pages again. This should be as easy as adding numbers together to get 180.

Earlier I told you that a $90^{\circ}$ angle is called a right angle. Any angle that is smaller than $90^{\circ}$ is called an acute angle. You can remember this by thinking, "Oh look at that cute little angle."

Any angle that is bigger than $90^{\circ}$ is called an obtuse angle. The $135^{\circ}$ angle from the last problem is an obtuse angle. The $45^{\circ}$ angle is an acute angle.

When two angles share one side and share a vertex, they are called adjacent angles. Look at the drawing below.


Angles $A B C$ and $C B D$ are adjacent angles because they share a side $B C$ and they share vertex $B$.

Let's review those last three new words: acute, obtuse, and adjacent angles. An acute angle is smaller than $90^{\circ}$. It's "a cute" little angle. An obtuse angle is bigger than $90^{\circ}$ - it's obese. And adjacent angles are right next to each other, sharing one side and a vertex.

## WORKSHEET 4-2

Look at the drawing below and then answer the questions.


1. Is angle ABD obtuse or acute?
2. Is angle $D B C$ obtuse or acute?
3. What is the measurement of angle $A B D$ ?
4. What is the measurement of angle $D B C$ ?
5. Look at angle $A B D$. Which letter is the vertex?
6. Name the angle that is adjacent to angle DBC.

If that last worksheet was easy and you got all the answers correct, keep reading. If you had any problems, go back and learn what you missed.
Look at this next drawing. The only clue you are given is that one angle is $45^{\circ}$.
That is all we need, to figure out the rest of the angles. We know this angle must be $135^{\circ}$ because $180-45=135$.


But wait! This is a straight line too, so the angles on top of it must also equal $180^{\circ}$. If one side of this line is $135^{\circ}$, then the other side must be $45^{\circ}$.


And if we know that this angle is $45^{\circ}$, then we know that this angle is $135^{\circ}$. In fact, opposite angles are always the same. The two $45^{\circ}$ angles above are called opposite angles. The two $135^{\circ}$ angles above are also opposite angles. Opposite angles share a vertex, but not a side. Look at these next two lines; they form a perfect cross.


Can you guess what would happen, if I moved this line one way or the other?
I will move it over a little. Watch how the angles change, when I push the line over $20^{\circ}$.


Notice how two angles on a straight line always equal 180 and how opposite angles are always the same. Those are two big clues to figuring out missing numbers. Fill in the correct size for angles $A, B$, and $C$ below.


Our first clue is that opposite angles are equal. Angle B is opposite a $30^{\circ}$ angle. Since opposite angles are always equal, angle $B$ must be $30^{\circ}$ too.

And we know that when two angles form a straight line they total $180^{\circ}$. To figure out angle $A$, we subtract $180-30=150$. Angle $A=150^{\circ}$. Opposite angles are always equal, so angle $C$ is also $150^{\circ}$. Here are the answers.

Angle $A=150^{\circ}$
Angle $B=30^{\circ}$
Angle $C=150^{\circ}$
Look at this next picture. What size is each angle?


Do you remember what the little box means? It means this angle is exactly $90^{\circ}$. With that one little clue you should be able to figure out all 4 angles. Remember our clues: Opposite angles are always equal and angles that form a straight line always equal 180. Have you figured out the answers? That's right; all 4 angles are $90^{\circ}$ each. Together they total $360^{\circ}$, just like a circle.


Take the Chapter Review Test, to see if you are ready to continue. You must get all the answers correct before you can continue.

## CHAPTER 1 REVIEW TEST

All questions will be about this drawing:


1. Is angle $B$ an obtuse angle or an acute angle?
2. Is angle $C$ an obtuse angle or an acute angle?
3. If angle $A$ is $50^{\circ}$, what size are angles $B, C$, and $D$ ?
4. Are angles $A$ and $D$ opposite or adjacent angles?
5. Are angles $A$ and $B$ opposite or adjacent angles?
6. If angle $D$ is $129^{\circ}$, what size is angle $B$ ?
7. If angle $D$ is $129^{\circ}$, what size is angle $C$ ?
8. Why are there arrows on the ends of the lines?
9. Draw a picture to portray each of the following geometric terms.

A Line segment
A Ray
An angle with a vertex $Q$
Any Angle
A Right angle
An Obtuse angle
An Acute angle
Two opposite angles
Two adjacent angles

