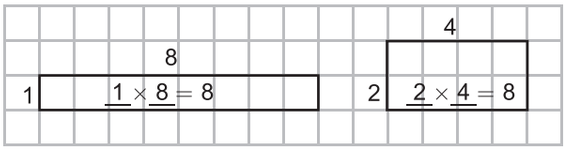
NS8-58 Perfect Squares

1. Find the factors of each number below by drawing all the different rectangles (with whole number side lengths) that have an area equal to the number.

Example:



So the factors of 8 are: 1, 2, 4, and 8.

a) 4

b) 5

c) 6

d) 7

e) 8

f) 9

2. For which numbers in Question 1 could you draw a square? _____

A number larger than 0 is called a **perfect square** if you can draw a square with whole number side lengths having that area.

3. a) Draw squares with side lengths 1, 2, 3, 4, and 5 on the grid.

b) Write the first five perfect squares larger than 0.

_____ _____ _____ _____ _____

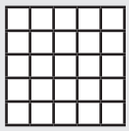
4. Explain why a square with an area of 20 cm^2 does not have a whole number side length.

5. Can a prime number be a perfect square? Explain.

6. Show that 36 is a perfect square by drawing a square with area 36.

7. Show that 10 is not a perfect square by drawing all non-congruent rectangles with area 10.

Any perfect square can be written as a product of a whole number with itself.

Example: $25 = 5 \times 5$.  Area $= 5 \times 5 = 25$ squares

NOTE: Since $0 = 0 \times 0$, we say that 0 is a perfect square even though you cannot draw a square with area 0.

8. Write the first ten perfect squares larger than 0.

$1 \times 1 =$ _____ $2 \times 2 =$ _____ $3 \times 3 =$ _____ $4 \times 4 =$ _____ $5 \times 5 =$ _____

$6 \times 6 =$ _____ $7 \times 7 =$ _____ $8 \times 8 =$ _____ $9 \times 9 =$ _____ $10 \times 10 =$ _____

When we multiply a number by itself, we get a perfect square. This process is called **squaring the number**. Example: 6 squared is $6 \times 6 = 36$. We write $6^2 = 36$. (The 2 is because we multiplied two 6s.)

9. Write each perfect square as a product and evaluate it.

a) $5^2 = 5 \times 5 = 25$ b) $3^2 =$ c) $8^2 =$ d) $0^2 =$ e) $7^2 =$

10. Write the numbers from smallest to largest.

a) 3^2 5^2 4^2

9 *25* *16*

9 *16* *25*

b) 10^2 8^2 9^2

_____ _____ _____

_____ _____ _____

c) 5^2 12^2 7^2

_____ _____ _____

_____ _____ _____

d) 3^2 5 10 4^2 2^2

_____ _____ _____ _____ _____

_____ _____ _____ _____ _____

e) 50 7^2 9^2 8^2 85

_____ _____ _____ _____ _____

_____ _____ _____ _____ _____

NS8-59 Factors of Perfect Squares

To list all the factors of a given number (the pairs of numbers that multiply to give that number), stop when you get a number that is already part of a pair.

1. Make a chart to find all the pairs of numbers that multiply to give each number.

a) 20

1st	2nd
1	20
2	10
3	
4	5
5	Done!

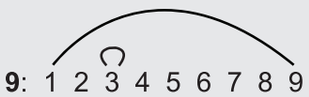
b) 12

1st	2nd

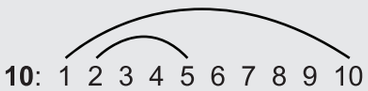
c) 15 d) 14 e) 25
f) 5 g) 26 h) 30
i) 42 j) 72 k) 63
l) 100 m) 64 n) 91

A **factor rainbow** for a number pairs the factors that multiply to give that number.

Factor rainbow for 9



Factor rainbow for 10

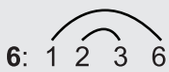


2. Finish the factor rainbow for each number.

6: 1 2 3 4 5 6 **8**: 1 2 3 4 5 6 7 8 **12**: 1 2 3 4 5 6 7 8 9 10 11 12

As a shortcut to making a factor rainbow, we can leave out all numbers that are not factors.

Example:



3. Using the shortcut, make a factor rainbow for each number from 1 to 20.
For the numbers from 11 to 20, you will need to list the factors first.

1: 1 **2**: 1 2 **3**: 1 3 **4**: 1 2 4 **5**: 1 5

6: 1 2 3 6 **7**: 1 7 **8**: 1 2 4 8 **9**: 1 3 9 **10**: 1 2 5 10

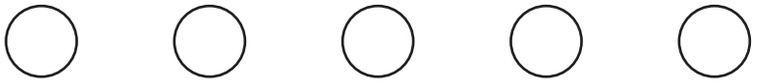
11: **12**: **13**: **14**: **15**:

16: **17**: **18**: **19**: **20**:

4. a) Look at your answers to Question 3. Which numbers have an odd number of factors?

_____, _____, _____, and _____.

b) Extend the sequence of numbers you found in part a) by using the gaps between the numbers.



_____ , _____ , _____ , _____ , _____ , _____

Do you recognize the numbers in the sequence? What are they called? _____

c) All perfect squares have an odd number of factors. Why?

Hint: Look at the factor rainbows from Question 3. When is there a factor that is paired up with itself?

d) Write the reverse (see p. 116) of the statement from part c). Is it also true?

INVESTIGATION ▶ Which numbers have exactly 3 factors?

A. Explain why any number with exactly 3 factors is a perfect square.

B. List all the factors of the first 10 perfect squares greater than 0.

Perfect Square	Factors
$1=1^2$	1
$4=2^2$	1, 2, 4
$9=3^2$	1, 3, 9
$16=4^2$	1, 2, 4, 8, 16
$25=5^2$	
$36=6^2$	
$49=7^2$	
$64=8^2$	
$81=9^2$	
$100=10^2$	

C. Which perfect squares between 1 and 100 have exactly 3 factors?

_____^{2} , _____^{2} , _____^{2} , and _____^{2}

D. What are the prime numbers between 1 and 10?

_____ , _____ , _____ , and _____

E. Compare your answers to parts C. and D. What do you notice?

F. Make a conjecture about which numbers have exactly 3 factors.

G. Use your conjecture to find the first 3 numbers greater than 100 that have exactly 3 factors.

NS8-60 Square Roots of Perfect Squares

The number 5 is called the **square root** of 25 because 25 is the **square** of 5.

We write $\sqrt{25} = 5$ because $25 = 5^2 = 5 \times 5$.

Square roots are numbers, so you can add, subtract, multiply, and divide them.

1. Find the square root by writing the same number in each box.

a) $9 = \square \times \square$ b) $49 = \square \times \square$ c) $0 = \square \times \square$ d) $25 = \square \times \square$

2. Evaluate.

a) $\sqrt{49} =$ 7 b) $\sqrt{16} =$ _____ c) $\sqrt{9} =$ _____ d) $\sqrt{36} =$ _____

e) $\sqrt{1} =$ _____ f) $\sqrt{100} =$ _____ g) $\sqrt{81} =$ _____ h) $\sqrt{64} =$ _____

3. Evaluate.

a) $\sqrt{25} + \sqrt{4}$ b) $\sqrt{36} \times \sqrt{25}$ c) $\sqrt{64} - \sqrt{9}$ d) $\sqrt{100} \div \sqrt{4}$ e) $\sqrt{49} + \sqrt{64}$

$= 5 + 2 = 7$

f) $\sqrt{36} - \sqrt{25}$ g) $\sqrt{36} \div \sqrt{4}$ h) $\sqrt{36} + \sqrt{25} - \sqrt{1}$ **BONUS ▶** $\sqrt{25} + \sqrt{16} \times \sqrt{9}$

4. Order these numbers from smallest to largest.

a) $\sqrt{49}$ $\sqrt{64}$ $\sqrt{25}$ $\sqrt{9}$ $\sqrt{16}$ b) $\sqrt{100}$ 3^2 5 4^2 $\sqrt{4}$ $\sqrt{8^2}$

5. Evaluate the two expressions. Then write $=$ (equal) or $\neq$ (not equal) in the box.

a) $\sqrt{4 \times 9}$ $\boxed{=}$ $\sqrt{4} \times \sqrt{9}$

$= \sqrt{36}$ $= 2 \times 3$

$= 6$ $= 6$

b) $\sqrt{9+16}$ $\square$ $\sqrt{9} + \sqrt{16}$

$= \sqrt{}$ $= \_\_ + \_\_$

$= \_\_\_\_$ $= \_\_$

c) $\sqrt{169 - 25}$ $\square$ $\sqrt{169} - \sqrt{25}$

$= \sqrt{}$ $= \_\_ - \_\_$

$= \_\_\_\_$ $= \_\_$

d) $\sqrt{100 \div 4}$ $\square$ $\sqrt{100} \div \sqrt{4}$

$= \sqrt{}$ $= \_\_ \div \_\_$

$= \_\_\_\_$ $= \_\_$

6. The factor rainbow for each perfect square is shown. Find the square root.

a)

144: 1 2 3 4 6 8 9 12 16 18 24 36 48 72 144

So $\sqrt{144} =$

b)

196: 1 2 4 7 14 28 49 98 196

So $\sqrt{196} =$

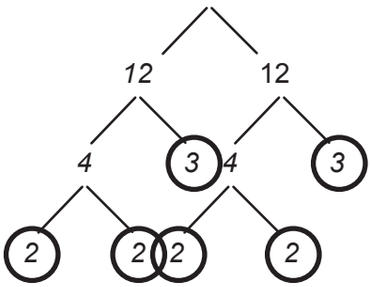
7. a) How can you find the square root of a perfect square by looking at its factor rainbow?

b) Draw a factor rainbow for 225 and find $\sqrt{225}$.

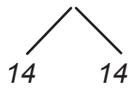
NS8-61 Prime Factorizations of Perfect Squares

1. Find the prime factorization of each perfect square by first finding the prime factorization of its square root. Circle the prime numbers.

a) 144



b) 196



c) 64

d) 225

e) 256

f) 400

2. How many times does the prime number 2 occur in the prime factorization of each number and its square root in Question 1?

a) $144 = 2 \times 2 \times 3 \times 2 \times 2 \times 3$ and $12 = 2 \times 2 \times 3$ So 2 occurs *4* times in 144 and *2* times in 12.

b) $196 =$ and $14 =$ So 2 occurs _____ times in 196 and _____ times in 14.

c) $64 =$ and _____ $=$ So 2 occurs _____ times in 64 and _____ times in _____.

d) $256 =$ and _____ $=$ So 2 occurs _____ times in 256 and _____ times in _____.

e) $225 =$ and _____ $=$ So 2 occurs _____ times in 225 and _____ times in _____.

f) $400 =$ and _____ $=$ So 2 occurs _____ times in 400 and _____ times in _____.

3. The prime number 2 occurs three times in the prime factorization of 56.

How many times will 2 occur in the prime factorization of $56 \times 56 = 56^2$? _____

How do you know? _____

_____

4. Can the prime number 2 occur an **odd** number of times in the prime factorization of a perfect square? Explain. _____

_____

INVESTIGATION ▶ Can any prime number occur an odd number of times in the prime factorization of a perfect square?

A. $18 = 2 \times 3 \times 3$ so $18^2 = 18 \times 18 =$ _____

The prime number 3 occurs two times in the prime factorization of 18.

How many times does it occur in the prime factorization of $18^2 = 18 \times 18$? _____

B. $250 = 2 \times 5 \times 5 \times 5$ so $250^2 = 250 \times 250 =$ _____

The prime number 5 occurs three times in the prime factorization of 250.

How many times does it occur in the prime factorization of $250^2 = 250 \times 250$? _____

C. a) **Double** the number of times each prime factor occurs. Then use a calculator to find the **square root** of the result. In parts iii)–vii) you have to find the factors first.

i) $45 = 3 \times 3 \times 5$ $3 \times 3 \times 3 \times 3 \times 5 \times 5 = 2025$ and $\sqrt{2025} = 45$

ii) $28 = 2 \times 2 \times 7$ _____ = _____ and $\sqrt{} =$ _____

iii) 48 iv) 35 v) 91 vi) 27 vii) 63

b) What do you notice? _____

D. a) **Halve** the number of times each prime factor occurs, then find the **square** of the result. In parts iii)–vii) you have to find the factors first.

i) $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$ $2 \times 2 \times 3 = 12$ and $12^2 = 144$

ii) $324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$ _____ = _____ and _____^{2} = _____

iii) 5625 iv) 576 v) 1936 vi) 11 025 vii) 27 225

b) What do you notice? _____

E. Explain why a number is a perfect square if all its prime factors occur an even number of times in its prime factorization.

---

5. Which numbers are perfect squares? Find their prime factorizations to decide.

a) 6 300 b) 6 400 c) 2 268 d) 243 e) 729 f) 1 296

6. a) Extend the pattern.

b) Find the prime factorization of all 10 terms in the pattern.

c) Circle the perfect squares in the pattern in part a).

d) Will the 100th term be a perfect square? How do you know?

NS8-62 Square Roots of Non-Perfect Squares

We can find the square of non-whole numbers, too. Example: $1.3^2 = 1.3 \times 1.3 = 1.69$

1. Evaluate each square.

a) $1.4^2 = \underline{1.4 \times 1.4}$
$= \underline{\qquad\qquad}$

b) $0.8^2 = \underline{0.8 \times 0.8}$
$= \underline{\qquad\qquad}$

c) $2.5^2 = \underline{\quad} \times \underline{\quad}$
$= \underline{\quad}$

The number 19 is not a perfect square because there is no whole number whose square is 19.

$4^2 = 16$ is **less** than 19 and $5^2 = 25$ is **more** than 19.

But we can still try to find its square root! The number $\sqrt{19}$ is the decimal number that, multiplied by itself, gives 19.

2. a) Explain why $\sqrt{19}$ is more than 4 and less than 5.

b) Calculate $4.5 \times 4.5 =$ ____ ____.____ ____. Is $\sqrt{19}$ more or less than 4.5? _____

c) Guess $\sqrt{19}$ to one decimal place. $\sqrt{19} \approx 4.$____

d) Check your guess by multiplying. 4.____ $\times$ 4.____ = ____

e) Was your guess too low or too high? _____

f) Increase your estimate by one tenth if your estimate was too low and decrease it by one tenth if your estimate was too high. Square your new estimate. 4.____ $\times$ 4.____ = ____

g) Is your new answer closer to 19 or farther away?

h) Continue guessing and revising until your answer is as close to 19 as you can make it.

i) Estimate $\sqrt{19}$ to one decimal place. $\sqrt{19} \approx 4.$____

To calculate $\sqrt{19}$ on a calculator, one of these three sequences will work:

Step 1: Key in 19.
Step 2: Press the [2nd] or [INV] key.
Step 3: Press the [x^2] key.

OR **Step 1:** Key in 19.
Step 2: Press the [$\sqrt{\ }$] key.

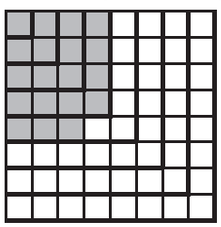
OR **Step 1:** Press the [$\sqrt{\ }$] key.
Step 2: Key in 19.

3. Calculate $\sqrt{19}$ on a calculator. Round your answer to two decimal places, then one decimal place.

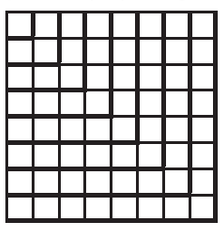
$\sqrt{19} \approx 4.$____ ____ $\approx 4.$____ Does your answer agree with your answer to Question 2 i)? Explain.

4. Shade as many full layers as you can until you have shaded the given number of squares.

Which two perfect squares is the number between? Do parts c)–i) on grid paper.

a) 

19 is between ____^{2} and ____^{2}

b) 

11 is between ____^{2} and ____^{2}

c) 44 d) 21 e) 35 f) 50 g) 72 h) 65 i) 42

5. Which perfect squares is each whole number between? Which consecutive whole numbers is each square root between?

a) 7 is between ___4___ and ___9___
So 7 is between ___2^2___ and ___3^2___
So $\sqrt{7}$ is between ___2___ and ___3___

b) 15 is between _____ and _____
So 15 is between _____^{2} and _____^{2}
So $\sqrt{15}$ is between _____ and _____

c) 85 is between _____ and _____
So 85 is between _____^{2} and _____^{2}
So $\sqrt{85}$ is between _____ and _____

d) 52 is between _____ and _____
So 52 is between _____^{2} and _____^{2}
So $\sqrt{52}$ is between _____ and _____

e) $\sqrt{45}$ f) $\sqrt{91}$ g) $\sqrt{13}$ h) $\sqrt{55}$ i) $\sqrt{6}$ j) $\sqrt{72}$

6. Estimate each square root to one decimal place by guessing, checking, and revising. Show your work.

a) $\sqrt{12}$ b) $\sqrt{22}$ c) $\sqrt{15}$ d) $\sqrt{30}$

7. Calculate each square root on a calculator and round your answer to one decimal place.

a) $\sqrt{12} \approx$ _____ b) $\sqrt{22} \approx$ _____ c) $\sqrt{15} \approx$ _____ d) $\sqrt{30} \approx$ _____

8. Do your answers to Questions 6 and 7 agree? Explain.

9. Guled took the square root of a number and his calculator showed 6.324 555 3.

a) Multiply this number by itself. What number did Guled take the square root of?
b) Was the calculator's answer an approximation? How do you know?

10. Find the closest perfect square to each number and the closest whole number to its square root.

a) closest perfect square to **19** is ___16___ = ___4___^{2}
closest whole number to $\sqrt{19} \approx$ ___4.36___ is ___4___.

b) closest perfect square to **27** is ____ = ____^{2}
closest whole number to $\sqrt{27} \approx$ ____ is ____

c) closest perfect square to **21** is ____ = ____^{2}
closest whole number to $\sqrt{21} \approx$ ____ is ____

d) closest perfect square to **44** is ____ = ____^{2}
closest whole number to $\sqrt{44} \approx$ ____ is ____

11. Look at your answers to Question 10 and complete this statement.

If n^2 is the closest perfect square to x, then ____ is the closest whole number to $\sqrt{x}$.

12. Estimate each square root to its nearest whole number by finding the nearest perfect square to the number you are taking the square root of.

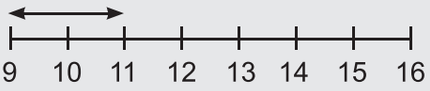
a) $\sqrt{24}$ b) $\sqrt{32}$ c) $\sqrt{20}$ d) $\sqrt{75}$ e) $\sqrt{68}$

NS8-63 Estimating Square Roots

Estimate $\sqrt{11}$ as follows.

11 is between $9 = 3^2$ and $16 = 4^2$.

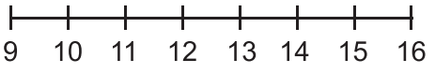
11 is $\frac{2}{7}$ of the way from 9 to 16.



So $\sqrt{11}$ is approximately $\frac{2}{7}$ of the way from 3 to 4.

So $\sqrt{11} \approx 3\frac{2}{7} = 3 + (2 \div 7) \approx 3.285\,7 \approx 3.3$. On a calculator, $\sqrt{11} \approx 3.316\,6 \approx 3.3$.

1. Use a number line to estimate each square root.
Write your answer rounded to one decimal place.



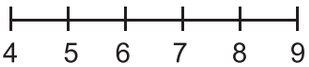
a) 12 is _____ of the way from 9 to 16.

So $\sqrt{12}$ is approximately _____ of the way from 3 to 4.

So $\sqrt{12}$ is approximately _____ $= 3 +$ _____ $\div$ _____ $\approx$ _____

b) $\sqrt{15} \approx$ _____ c) $\sqrt{10} \approx$ _____ d) $\sqrt{14} \approx$ _____ e) $\sqrt{13} \approx$ _____

2. Estimate each square root using the number line.



Write your answer as a mixed number.

a) $\sqrt{5} \approx$ _____ b) $\sqrt{6} \approx$ _____ c) $\sqrt{7} \approx$ _____ d) $\sqrt{8} \approx$ _____

3. Calculate each square root from Question 2 to one decimal place using a calculator.
Were your estimates correct?

a) $\sqrt{5} \approx$ _____ b) $\sqrt{6} \approx$ _____ c) $\sqrt{7} \approx$ _____ d) $\sqrt{8} \approx$ _____

4. Estimate each square root to two decimal places and then to one decimal place.
Use a number line.

a) $\sqrt{32} \approx 5.$___ ___ $\approx 5.$___ b) $\sqrt{50} \approx 7.$___ ___ $\approx 7.$___ c) $\sqrt{85} \approx$___.___ ___ $\approx$ ___.___

5. Calculate each square root from Question 4 on your calculator.

a) $\sqrt{32} \approx 5.$___ ___ $\approx 5.$___ b) $\sqrt{50} \approx 7.$___ ___ $\approx 7.$___ c) $\sqrt{85} \approx$___.___ ___ $\approx$ ___.___

6. For how many decimal places did your estimates and calculations in Questions 4 and 5 agree? _____

7. Estimate and then calculate each square root. To how many decimal places is your estimate accurate?

a) $\sqrt{21}$ b) $\sqrt{13}$ c) $\sqrt{39}$ d) $\sqrt{69}$ e) $\sqrt{45}$ f) $\sqrt{2}$