

PDM7-1 Stem and Leaf Plots

The **leaf** of a number is its right-most digit. The **stem** is all digits except the right-most digit. The stem of a one-digit number is 0, since there are no digits except the right-most one.

stem → 4 87 ← leaf

1. In each number, underline the leaf and circle the stem. The first one is done for you.
- a) 1 23 b) 5 c) 7 4 d) 9 e) 2 3 f) 8 7 1 2 g) 3 4 5
2. Write a number with...
- a) leaf 0. _____ b) stem 0. _____
3. Underline the numbers in each group that have the same stem. Hint: Circle each stem first, as you did in Question 1.
- a) 7 8 7 9 5 9 b) 3 4 5 3 4 3 4 9 c) 5 7 8 2 5 7 8 5 7 4
- d) 7 8 7 8 e) 4 5 6 7 4 5 6 4 5 6 6 f) 1 2 2 3 4 1 2 3 4 1 2 3 3

Build a **stem and leaf plot** for this data set: 45, 9, 23, 35, 29, 32.

Step 1: The stems are 4, 0, 2, and 3. Write the stems in a column, ordered from smallest to largest.

Stem	Leaf
0	
2	
3	
4	

Step 2: Write each leaf in the same row as its stem.

Stem	Leaf
0	9
2	3 9
3	5 2
4	5

Step 3: Put the leaves in each row in order.

Stem	Leaf
0	9
2	3 9
3	2 5
4	5

4. Put the leaves in the correct order. Then list the data from least to greatest.
- a)

Stem	Leaf
2	1 4
3	8 5 6
5	3 2

rough work

→

Stem	Leaf
2	1 4
3	5 6 8
5	2 3

final answer

21 24 35 36 38 52 53
- b)

Stem	Leaf
0	5
8	7 3
2 3	4 9 6

rough work

→

Stem	Leaf

final answer

5. Use the data sets to create stem and leaf plots.
- a) 13 14 19 23 31 b) 5 19 23 39 217 c) 99 98 102 99 101
- d) 2 37 88 2 104 e) 23 34 50 29 23 f) 3 4 17 4 3 12 3 2 10 3 2 19
- 39 87 3 43 100 35 47 46 33 24 3 4 12 4 3 12 4 3 14 3 8 21

What you can learn from a stem and leaf plot:

The **smallest number** is 981 and the **largest number** is 1 006. The range is:

$$\begin{array}{r} \text{the largest number} \\ 1006 \end{array} - \begin{array}{r} \text{the smallest number} \\ 981 \end{array} = 25$$

Stem	Leaf
98	1 4 9
99	8 8 8
100	2 3 3 6

The number that occurs most often is called the **mode**. The mode here is 998, and it occurs three times.

Stem	Leaf
98	1 4 9
99	8 8 8
100	2 3 3 6

6. In each case, find the smallest value, the largest value, the range, and the mode.

a)

Stem	Leaf	Smallest: _____
2	3 6 7	Largest: _____
3	0 1 1 2	Range: _____
5	2 3 3 3 8	Mode: _____

b)

Stem	Leaf	Smallest: _____
31	0 5 5	Largest: _____
33	1 6 9	Range: _____
37	8 8 8	Mode: _____

7. Find the largest value, the smallest value, the range, and the mode of the sets in Question 5.

8. A gym class collected the results from their long-jump trials and ordered them in a stem and leaf plot.

Stem	Leaf
17	2 9
18	5 7 8 8
19	0 2 2 4
20	1 2 5 5 5
21	2 7 8
22	4 9

- a) Circle the units of measurement that you think the class used.

mm cm m km

- b) Riana's jump was recorded as 202. Underline the leaf for Riana's result in the plot.

- c) What was the distance of the longest jump? (Include the units.) _____

- d) What was the shortest jump? _____ What was the mode? _____

- e) If the class were divided into two equal groups according to how they jumped, which group would Riana be in? Explain.

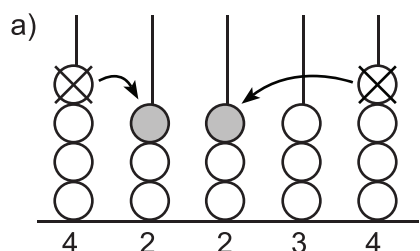
9. a) Write two numbers where the smaller number has the greater leaf.
b) Can you find two numbers where the smaller number has the greater stem? Explain.

10. Either explain why each statement is true or give a counter-example to show that it is false.

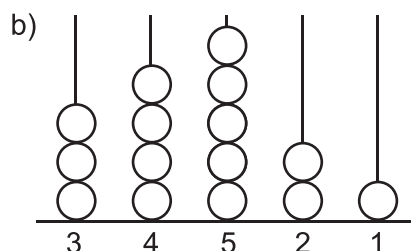
- a) A one-digit number always has stem 0.
b) A one-digit number always has leaf 0.
c) Two numbers with the same stem always have the same number of digits.
d) Two numbers with the same leaf always have the same number of digits.
e) The stem is always a single digit.
f) The leaf is always a single digit.

PDM7-2 The Mean

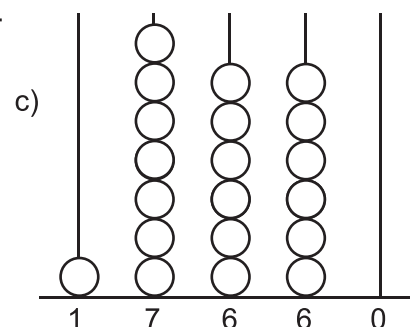
1. Move enough beads so that all rods have the same number of beads.
The **mean** is the number of beads on each rod.



Mean: 3



Mean:



Mean:

$$\begin{array}{ccccccccc} 4 & 2 & 2 & 3 & 4 & & 3 & 3 & 3 & 3 & 3 \\ \hline \end{array}$$

Total number of beads = $4 + 2 + 2 + 3 + 4 = 15$

Mean = number of beads on each rod
= total number of beads \div number of rods
= $15 \div 5$

So **mean** = **sum of data values** \div **number of data values**.

2. Find the mean without using beads.

a) 0 3 4 6 7

$$\begin{array}{r} \square \text{ sum of} \\ \div \square \text{ data values} \\ \hline \square \text{ number of} \\ \square \text{ data values} \\ \hline \square \text{ mean} \end{array}$$

b) 1 4 5 7 8

$$\begin{array}{r} \square \\ \div \square \\ \hline \square \end{array}$$

c) 2 5 6 8 9

$$\begin{array}{r} \square \\ \div \square \\ \hline \square \end{array}$$

d) 3 6 7 9 10

$$\begin{array}{r} \square \\ \div \square \\ \hline \square \end{array}$$

Compare the data values in the sets above — each one is 1 greater than the same value in the previous set. In your notebook, explain how the mean changes when you add 1 to each data value.

3. Find the mean.

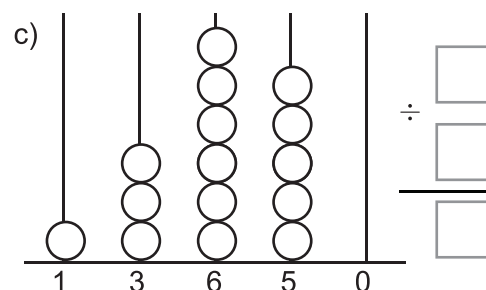
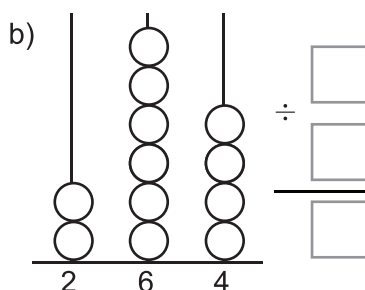
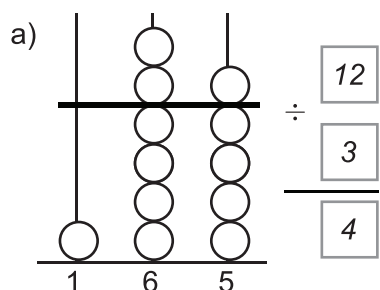
a) 0 1 2 3 4 5

b) 24 25 27 29 21

c) 0 25 10 12

Explain why the beads model does not help to find the mean in a).

4. Find the mean and draw a horizontal line to show it.



INVESTIGATION ► How does adding new data values to a set affect the mean?

- A.** The data set 1, 4, 10 has mean $15 \div 3 = 5$. Add the following data values to this set and decide if the mean increased, decreased, or stayed the same.

- | | | |
|-----------------------------|-----------------------|------------------------|
| i) New data value: 3 | ii) New data value: 4 | iii) New data value: 5 |
| New mean: $18 \div 4 = 4.5$ | New mean: _____ | New mean: _____ |
| The mean <u>decreased</u> . | The mean _____. | The mean _____. |
| iv) New data value: 6 | v) New data value: 7 | vi) New data value: 8 |

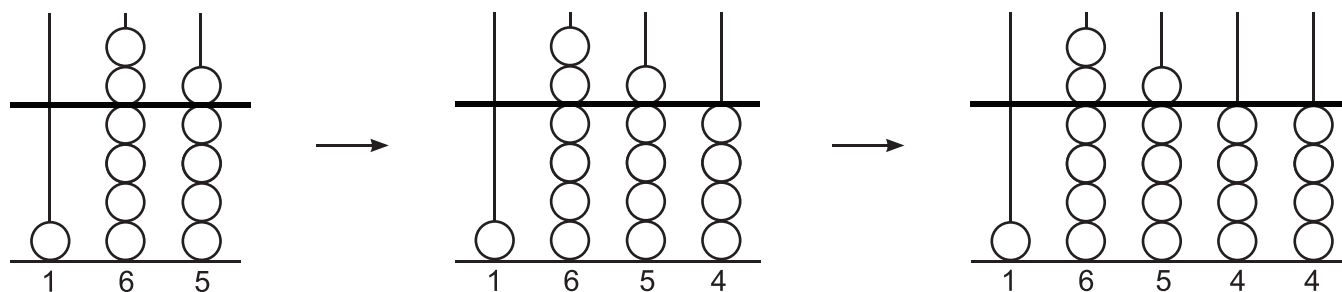
- B.** Make a conjecture:

When the new data value is smaller than the mean, the mean _____.

When the new data value is larger than the mean, the mean _____.

- C.** Test your conjecture with a different set of data: 2, 3, 7. First find the mean, then add different new values above and below the mean and find the new mean. Did the mean increase, decrease, or stay the same?

- D.** If you add a value that is the same as the mean, would the mean change? Use this model to explain your thinking.



- 5.** The data set is 3, 4, 7, 8, 8.

- Find the mean.
- Is the data value 7 above or below the mean?
- Will removing 7 from the set of data increase or decrease the mean?
Find the mean of the set 3, 4, 8, 8 to check your prediction.

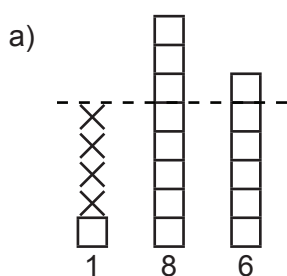
- 6.** Ten people work in an office. They get paid different salaries depending on their job.

Salesperson: \$50 000 per year Secretary: \$35 000 per year Clerk: \$25 000 per year

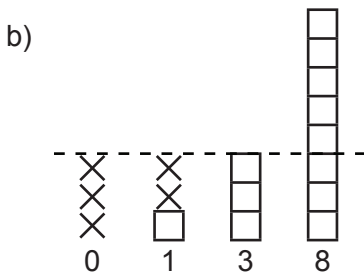
There are 5 salespeople, 2 secretaries, and 3 clerks.

- Find the mean salary in the office.
- What is the mode (the most common value) of the salaries?
- Will the mean salary increase or decrease if a secretary retires?
- Will the mean salary increase or decrease if two new salespeople are hired?

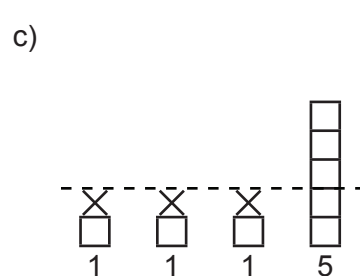
7. Count the spaces below the mean and the blocks above the mean.



4 spaces *below* mean
4 blocks *above* mean



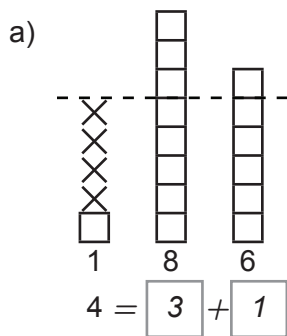
 spaces *below* mean
 blocks *above* mean



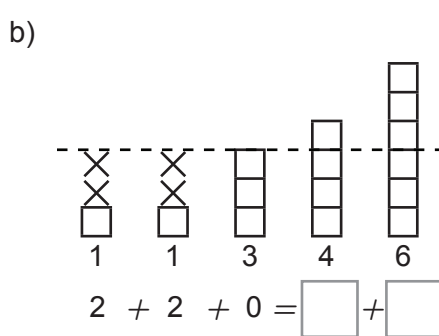
 spaces *below* mean
 blocks *above* mean

8. Look at your answers to Question 7. What do you notice? Explain.

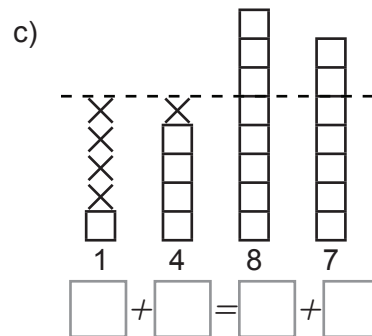
9. The number of spaces below the mean is the same as the number of blocks above the mean. Write a number sentence to show this.



$$4 = 3 + 1$$



$$2 + 2 + 0 = 4 + 6$$



$$1 + 4 = 8 + 7$$

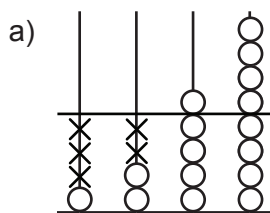
Do the remaining parts on grid paper. Draw the blocks and find the mean first.

d) 2 6 7

e) 3 4 8 6 9

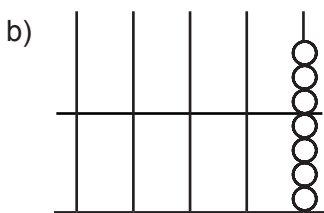
f) 2 4 5 5

10. Find data sets with mean 4 using the number sentences. Draw beads to help you.



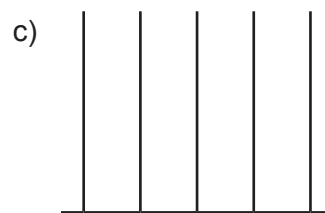
$$3 + 2 = 1 + 4$$

Data: 1 2 5 8



$$2 + 2 + 1 = 2 + 3$$

Data:



$$1 + 1 + 1 + 1 = 4$$

Data:

11. Create three different sets of data with mean 6.

12. **PROBLEM** ► The mean of a set of data is 10. The data values are 2, 19, 7, 4, 15, and one other number. What is the other number? Solve the problem in two ways:

- Use the beads model.
- Let x represent the missing number, write the expression for the mean, and solve the equation.

PDM7-3 Mean, Median, Mode, and Range

To find the **median** of a data set, put the data in order. Count from either end until you reach the middle.

2 3 **6** 7 11

The median is 6.

2 3 **7 9** 11 15

The median is halfway between 7 and 9.
The median is 8.

1. Find the mean, the median, and the range.

a) 9 20 22

mean: $(9 + 20 + 22) \div 3 =$

$51 \div 3 = 17$

median: 20

range: $22 - 9 = 13$

b) 38 40 42

mean: _____

median: _____

range: _____

c) 10 15 20 25 30

mean: _____

median: _____

range: _____

Now order the numbers from smallest to largest first. Show your work in your notebook.

d) 15 18 40 32 25

e) 29 21 27 16 22 17 15

f) 40 25 10 15 20

2. When finding the median, does it matter whether you write the data list from lowest to highest, or from highest to lowest?

INVESTIGATION 1 ► When is the median the same as one of the data values?

A. Circle the middle number or numbers. Find the median.

a) 2 4 6 7 8

b) 2 3 3 8

c) 7 9 13 14 26

d) 3 4 6 10 11 17

B. In which of the sets is the median the same as one of the data values? _____

C. If there is an odd number of data values in the set, is the median always one of the numbers in the set? How do you know?

D. Create a set with an even number of data values and no mode. Find the median. Is the median a number in the set? Can you create a set that changes your answer?

E. The set 2, 3, 5, 5 has an even number of values and a mode. Find the median. Is the median a number in the set?

F. Create a set with 6 data values, a mode, and a median that is a number in the set. Are the median and the mode the same?

G. Is it possible to create a set with 6 data values so that the median is a number in the set, but the median is different from the mode? Explain.

BONUS ► Create a set with 8 data values so that the median is a number in the set, but the median is different from the mode.

3. Find the mean and the median of each set of two data values. Choose your own numbers in d).

a) 8 14 mean: _____ median: _____ b) 9 12 mean: _____ median: _____
c) 3 15 mean: _____ median: _____ d) _____ mean: _____ median: _____

What do you notice about the mean and the median when there are only two data values?

INVESTIGATION 2 ► How does adding new data values affect the mean, median, mode, and range?

A. Find the range, mean, median, and mode of this set: 7, 7.

B. Add the data value 10 to the set. The new set is 7, 7, 10. Will the range, the mean, the median, and the mode increase, decrease, or stay the same as for the set 7, 7? Check your predictions.

Add 4 to the set instead of 10. Predict how the mean, the median, the mode, and the range change. Check the predictions again.

C. Look at the set 7, 7, x (where x represents any number). Can you find the mean, median, mode, and range for this set? Cross out the measures you cannot find.

mean _____ median _____ mode _____ range _____

D. What is the smallest number of values you have to add to the set 7, 7 so that 7 will not be the mode anymore? Give an example of such values.

E. What data value should you add to the set 7, 7 to make the mean 5? To make the mean 9?

F. How many data values should you add to the set 7, 7 to change the median? What could these values be?

4. A set of 4 data values has modes 7 and 5. What are the data values in the set?

5. a) Create two different sets of 4 values with mode 7 and median 6.

b) Is it possible to create a set of 4 data values with mode 7, median 6, and the largest number greater than 7? Explain why or why not.

c) Create a set of 4 data values with mode 7, median 6, and mean 5.

6. a) Use blocks to find a set of 5 test scores ranging from 2 to 7 with mean 5, median 5, and mode 7.

b) Add a test score of 12 to your data set from part a). Find the new mean, median, mode, and range and compare them to those in part a). How have they changed?

mean: _____ median: _____ mode: _____ range: _____

PDM7-4 Using the Mean, Median, and Mode

1. You are going on a trip during the March break. What kind of clothes should you pack? This chart shows the highest daily temperatures at your destination during March break last year:

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
°C	23	22	20	21	22	23	23	10	2	2	5	7	8	12	17

- a) Find the range, mean, median, and mode.
- b) What does the range of temperatures tell you about what you should pack?
- c) If you looked only at the mode of the temperatures, what mistake might you make in your packing?
2. In a company there are 20 employees. One person has a salary of \$ 200 000, two people have salaries of \$75 000, and all the rest have salaries of \$17 500.
- a) What is the mean salary?
- b) What is the median salary?
- c) What is the mode of the salaries?
- d) Which better reflects the salaries in the company, the mean or the mode? Explain.
3. In a company there are 20 employees. 1 person has a salary of \$300 000, two people earn \$70 000, another one earns \$45 000, and the rest have different salaries from \$17 000 to \$20 000, with no two salaries the same.
- a) Is there enough information to find the mean salary? The median salary? The mode of the salaries? The range of salaries?
- b) Which of the mean, the median, and the mode best reflects the salaries in the company? Explain.
4. You work at a clothing store, and your manager says that every week you need an average daily sales of at least \$500. What kind of average do you think your manager is talking about, a mean, a mode, or a median? Why?

5. This chart shows the class marks on a test.

76	78	69	76	73	76	74	66	69	85
74	66	71	76	87	96	66	98	91	73

- a) Create a stem and leaf plot for the data.
- b) Find the range, mode, median, and mean of the data. Which value is hardest to read or calculate from the stem and leaf plot? Explain.
- c) Tom's mark was 76. Which of the following statements that he told his parents were true? Explain using the mean, mode, median, or range.
- i) I did better than half of the class! ii) My grade is higher than the average!
- iii) A lot of students had the same grade as me. iv) Only 6 students got a better mark than me!
- v) 76 was the most common mark.

6. This table shows the price for the same pair of shoes at seven different stores.

Store	A	B	C	D	E	F	G
Price (\$)	83	85	84	86	86	82	81

- a) Find the mean, median, and mode of the prices.

mean: _____ median: _____ mode: _____

- b) Store B claims that its prices are lower than average. Which “average” could they use to make this statement true: the mean, mode, or median? Do you think the claim is misleading? Why?

INVESTIGATION ►

- A. Consider the set 5, 5, 5, 5, 5. What are the mean, the median, and the mode of this set?

mean: _____ median: _____ mode: _____

Why are the mean, the median, and the mode the same? _____

- B. Consider the set 4, 5, 5, 5, 6. Find the mean, the median, and the mode. Compare your answers to the answers in part A.

mean: _____ median: _____ mode: _____

- C. Use the bead or the block model to explain why the answers in A and B are the same.

- D. Create another set of 5 numbers where the mean, the median, and the mode are 5.

- E. Will the mean and median be the same or different in this set: 3, 4, 5, 6, 7?
Is there a mode?

- F. Use the bead or the block model to explain why the mean and the median are the same in the set 5, 5, 7, 8, 10.

-
7. Find a data set, not all numbers equal, where the mean, mode, and median are all the same.

8. a) Find the mean and the median of these sets.

Set A: 2, 3, 4

Set B: 2, 3, 4, 5, 6

- b) Move one data value from set A to set B. Find the mean and the median of the new sets. Did the mean and the median of each set increase, decrease, or stay the same?

- c) Explain why the mean and the median of set B cannot increase if a data value from set A is added.

9. When a soccer player moved from Team A to Team B, the mean age of **both** teams increased. Give an example of data to show how this could happen.

BONUS ► You have a set of 6 whole numbers (some of them can be 0). Find the lowest possible sum of all the numbers in the set if...

- a) the median is 50. b) the mean is 500. c) the mode is 5 000.

PDM7-5 Outliers

INVESTIGATION 1 ► Which value in a set affects the range the most?

A. Find the range of the set 2, 3, 5, 202. Range: _____

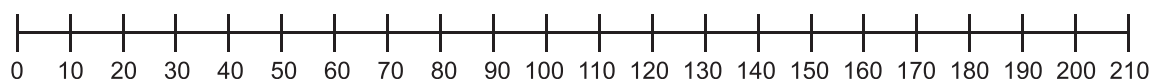
B. Remove one value from the set and find the range of the new set. Repeat with all other values.

New set: 3, 5, 202 New set: 2, 5, 202 New set: _____, _____, _____ New set: _____, _____, _____

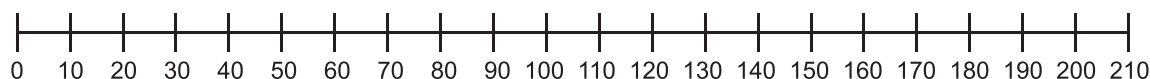
Range: $202 - 3 = 199$ Range: _____ Range: _____ Range: _____

Which value affects the range the most?

C. Mark the data values on the number line below. Circle the value that affects the range the most.



D. Repeat parts B and C with this set: 2, 52, 102, 152, 202.



E. How does removing the value that most affects the range change the range?

An **outlier** is a data value that is far from the rest of the data values in the set. Removing an outlier changes the range of the set a lot. Examples:

In the set 2, 3, 4, 5, 90, the number 90 is an outlier.



In the set 2, 400, 406, 435, 470, 475, the number 2 is an outlier.



1. For each set below, create a stem and leaf plot of the data. Identify the outlier in each set.

a) 23, 25, 65, 25, 37, 761, 41

b) 789, 792, 770, 764, 200

c) 75, 77, 71, 90, 13, 86, 80

How can stem and leaf plot help you to identify the outlier?

2. This set does not have an outlier: 2, 3, 100, 201, 202. Explain why none of the values is an outlier.

INVESTIGATION 2 ► Does adding an outlier to a set affect the mode?

A. What is the mode of the set 110, 115, 110, 113, 114? ____ Does this set have an outlier? ____

B. Add the outlier 200. The new set is ____, ____, ____, ____, ____, 200. What is the mode? ____

C. Add the outlier 1. The new set is ____, ____, ____, ____, ____, ____, 200. What is the mode? ____

D. Does an outlier change the mode of the set? ____

INVESTIGATION 3 ► Consider the set 2, 3, 4, 5, 76.

A. Circle the outlier.

B. Find the mean and the median of the set.

C. Write the set without the outlier: ____, ____, ____, ____.

Find the mean and the median of the new set.

D. Which value was changed more by the removal of the outlier, the mean or the median?

E. Choose any other value in the set. Write the set without it: ____, ____, ____, 76.

Find the mean and the median of the new set.

F. Repeat the investigation with these sets.

i) 78, 77, 12, 69, 74

ii) 4, 4, 4, 4, 5, 6, 35.

G. What affects the mean and the median more, the removal of the outlier or the removal of another value?

3. The set 2, 3, 4, 5 does not have an outlier. If you add 100 as a new data value, what will be affected more, the mean or the median? Check your prediction.

4. a) Find the outlier in each set of data.

A: Ages of the members of the Bridge club in Golden Age Retirement Residence:
68, 76, 78, 89, 94, 69, 102, 69, 75, 7, 77

B: Yearly salaries of permanent employees in a company: \$25 000, \$35 000,
\$300, \$45 000

C: Hourly rate of contract workers in a company: \$25, \$17.50, \$300, \$45

b) In which of the above situations is the outlier likely a mistake in the data? Explain.

5. Katie's math test scores (out of 20) are 16, 17, 17, 5, 19, 18, 17, 20, 19.

a) What is the outlier in this set?

b) When Katie's teacher finds the average for the report, should he include the outlier? Discuss with a partner.

6. One day a scene from a movie is filmed outside 10 Daffodil Street. At 8 Daffodil Street, there is a small coffee shop, and the daily sales at the coffee shop are 1 000 times larger than ever.

a) Does the set of the daily sales for the year contain an outlier?

b) To keep track of sales, the coffee shop finds the average daily sales for the year. Should the owner of the shop include the outlier in the calculation?

c) The owner decides to sell the store. She includes the average daily sales in an advertisement. Should she include the outlier in the calculation of the average?