CHAPTER

1

PLACE VALUE AND POWERS OF TEN

In this chapter, you will learn to:

- Follow the order of operations
- Use powers of ten and expanded form to write numbers
- Use scientific notation

LESSON 1: PLACE VALUE REVIEW

Place Value Detective

You Will Need:

- Place Value Activity Sheets (in the back of the answer key)
- Masking tape
- Scissors
- A prize (see note in the answer key)

You Will Do:

1. Have your parent set up this activity according to the instructions in the answer key.
2. Start on the paper marked "start" and read the clues. Use your knowledge of place value and numbers to guess the correct number.
3. Find the next activity sheet that has the same number as your answer on the top. Unfold it to read a new set of clues and guess the next number.
4. Continue working like this until you have guessed all the numbers. The last answer will lead you to a prize.

Place value is the value of each digit in a number. It is the system that we use to write down numbers, and the location of each digit affects its value. Each group of 3 digits is called a **period,** is separated by a comma, and follows a pattern of ones, tens, and hundreds.

PLACE VALUE:
A system of writing numbers where the location of a digit affects its value

PERIOD:
Each group of 3 digits separated by a comma

Review the place value chart below.

PLACE VALUE CHART

Millions			Thousands			Ones		
Hundred Million	Ten Million	One Million	Hundred Thousand	Ten Thousand	One Thousand	Hundreds	Tens	Ones
1	2	3,	4	5	6,	7	8	9

STANDARD FORM:

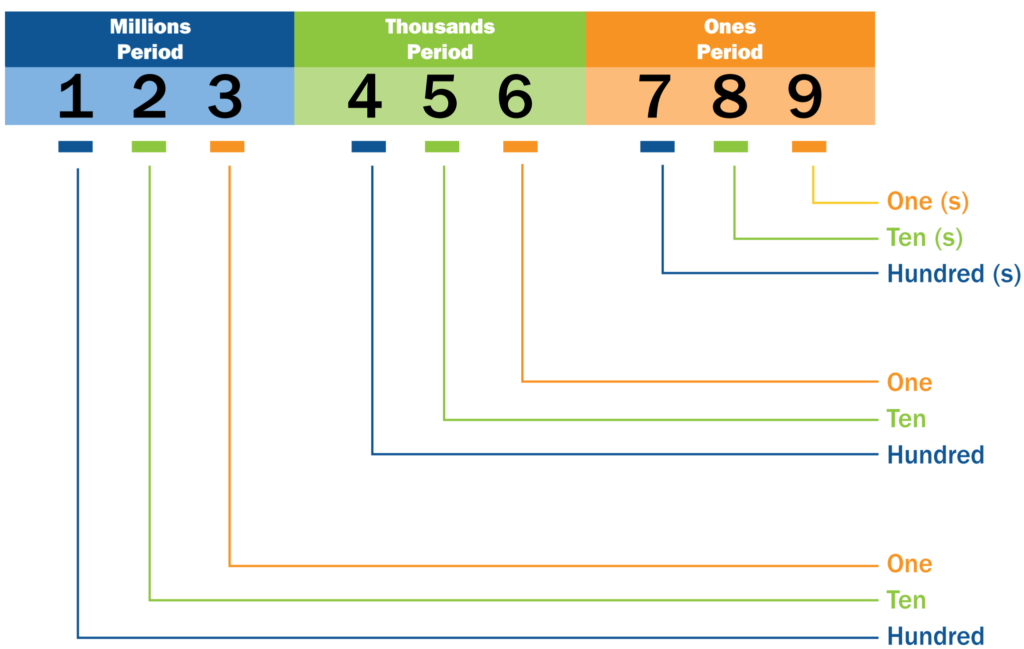
123,456,789

EXPANDED FORM:

100,000,000 + 20,000,000 + 3,000,000 + 400,000 + 50,000 + 6,000 + 700 + 80 + 9

WORD FORM:

one hundred twenty-three million, four hundred fifty-six thousand, seven hundred eighty-nine



We can write numbers in standard form using place value. We can also use expanded form as another way of representing the value of the number.

EXAMPLE 1: **Rewrite the number 87,045 in expanded form.**

80,000 + 7,000 + 40 + 5

The expanded form of the number shows exactly how many ten thousands, thousands, tens, and ones are in the number.

1. Write each number in expanded form.

a. 543,788

________________ + ________________ + ________________

+ ___________ + ___________ + ______

b. 23, 451

_____________ + _____________ + _____________

+ ___________ + ______

c. 5,699

_____________ + _____________ + ___________ + ______

2. Write each number in standard form.

a. 300,000 + 70,000 + 1,000 + 300 + 30 + 1 = ____________

b. 20, 000 + 4,000 + 500 + 4 = ____________

3. Write the value of the underlined digit.

a. 2<u>3</u>,800 ____________

b. <u>8</u>99,107 ____________

c. 17,8<u>4</u>5 ____________

d. 239,<u>9</u>80 ____________

3. It is hard to know exactly, but some scientists believe that there are at least eight million, seven hundred thousand different species of animals on Earth. Circle the number below that correctly shows that amount.

a. 870,000

b. 8,700,000

c. 8,070,000

Life is abundant on Earth.

LESSON 2: ORDER OF OPERATIONS

Order Matters

You Will Do:

1. Take a trip in the car with your parents. Ask them about the rules at the intersections you encounter. Who gets to go first? What is the order for traffic flow?

2. Discuss with your parents if they have ever driven somewhere with different rules. Some countries drive on the opposite side of the street than where people drive in the United States.
3. Why do you think we have to have certain rules for the order of driving? Talk it over with your parents. Do you think it is important to follow the rules of the road?

Just like in driving, mathematicians have rules for the order in which we do math. Otherwise, there could be a lot of confusion and possibly a catastrophe! It is important that we all agree on how to do a certain problem or else we will get different answers, and we won't be able to work together.

Look at the two problems below as an example.

Micah's Work	Isaiah's Work
$4 + 2 \times 3$	$4 + 2 \times 3$
6×3	$4 + 6$
18	10

Micah and Isaiah both worked on the same problem. They didn't make any mistakes when they added or multiplied, but they got two completely different answers. This is because they did the problem in a different order. Micah added 4 and 2 before multiplying by 3. Isaiah multiplied 2 and 3 before adding 4. Who has the correct answer?

Long ago, mathematicians noticed this problem and decided that we need to all agree on an order. They call it the **order of operations**. This order of operations is what all mathematicians follow so that we know we will all get the same answer when performing mathematical operations. In this book, we will learn 3 steps in the order of operations.

The Order of Operations

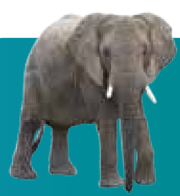
1. Perform operations that are in grouping symbols such as parentheses
2. Multiply and Divide (from left to right)
3. Add and Subtract (from left to right)

You will learn about the first step (grouping symbols) in the next lesson. The last 2 steps show that if there are 2 or more operations in a problem, you should do all the multiplication and division before you go back and do all of the addition and subtraction. It is also important that you always work from left to right. This isn't hard to remember because you read from left to right. Math is the same. Multiply and divide from left to right. Then go through and add and subtract from left to right.

Look back at the work Micah and Isaiah did. Who got the correct answer? I hope you see and understand that Isaiah's answer is the correct one.

Going left to right, Isaiah did the multiplication first. Then going left to right, Isaiah did the addition second. Now you know that following the rules is very important in math if we all want to find the same value.

When you find the value of a numerical expression, it is called **evaluating** a numerical expression. It is just a more mathematical way of saying "find the value by doing all the operations in the right order."



THE ORDER OF OPERATIONS: an agreed upon order for doing math problems

EVALUATE: To find the value of an expression

EXAMPLE 1: **Evaluate the expression.**

$3 \times 5 + 2 \times 4$

First you need to do the multiplication from left to right. To help us remember this, we can put parentheses around the multiplication operations. After we multiply we will add.

$(3 \times 5) + (2 \times 4)$

$15 + (2 \times 4)$

$15 + 8$

23

EXAMPLE 2: **Evaluate the expression.**

$6 + 10 \times 8 - 2$

$6 + (10 \times 8) - 2$	Start with the multiplication. Draw in parentheses if that helps you.
$6 + 80 - 2$	Now do the addition because it is on the left.
$86 - 2$	Finally, subtract.
84	

EXAMPLE 3: **Evaluate the expression.**

$6 - 12 \div 4 + 9$

$6 - (12 \div 4) + 9$	Write parentheses around the division so that you remember to do that operation first.
$6 - 3 + 9$	Next subtract because we move from left to right.
$3 + 9$	Finally, add.
12	

Notice that if you tried to add $3 + 9$ first it would have been a different answer. Students often forget to add and subtract moving from left to right.

"Do multiplication and division from left to right. Then do addition and subtraction from left to right."

1. Follow the order of operations to evaluate each expression. Show each step of your work.

a. $3 \times 4 + 5$

b. $7 - 2 \times 3$

c. $10 - 8 + 6$

d. $4 + 16 \div 4$

e. $3 \times 8 + 7 \times 2$

f. $10 + 72 \div 9$

g. $24 \div 6 + 4 \times 2$

h. $8 + 5 \times 4 - 5$

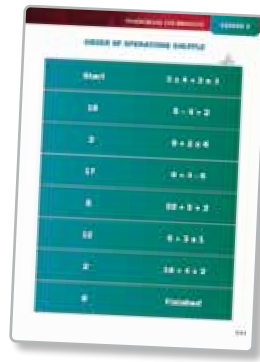
i. $6 + 10 \times 5 - 3$

j. $5 + 32 \div 4 + 3$

2. Write your own order of operations problem below. Then write the correct answer. Cover the answer with a piece of paper and see if someone can solve it correctly.

LESSON 3: PARENTHESES AND BRACKETS

Order of Operations Shuffle



You Will Need:

- Lesson 3: Activity Sheet
- Scissors
- Tape

You Will Do:

1. Have your parent tear out the activity sheet from the back of the answer key and cut the strips apart. They should scramble the strips so you do not see the original order.
2. Starting with the problem labeled "Start," evaluate the expression and find the strip with the answer on it. Line that strip up underneath.
3. Once you have all the strips lined up, tape them together. Flip them over to see a picture.

Sometimes mathematicians want to be able to tell you to do a certain step in a problem first. Look at the situation below for an example of this.

Every week Sarah earns $10. She spends $3 and then puts the rest in her bank. How much money will she have after 7 weeks?

To find the answer, you need to multiply by 7 and subtract 3. If we follow the order of operations, we will multiply first and then subtract. But that is not what Sarah did. She spent the $3 before putting the money in the bank. We need a way to show other mathematicians that they need to subtract first. They use **grouping symbols** to do this.

$(10 - 3) \times 7$

The parentheses above are an example of a grouping symbol. Grouping symbols direct us to a certain part of a problem that we need to do first. In this case, we need to subtract 3 before multiplying by 7.

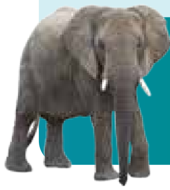
$(10 - 3) \times 7$
7×7
49
Sarah will have $49 after 7 weeks.

There are different kinds of grouping symbols. Whenever you see grouping symbols, you do the math in the symbols first. Then you complete and multiplication and division from left to right. Finally you go back and do any addition and subtraction left to right.

If there is more than one set of grouping symbols, you start in the center and work your way out. Let's do a couple practice problems together.

Grouping Symbols

( )	Parentheses	[ ]	Brackets



GROUPING SYMBOLS:
A set of symbols that indicate you should do certain operations first

EXAMPLE 1: **Evaluate the expression. Show each step.**

2 × [(7 – 3) × (2 + 2)]

I see brackets; we start there. But inside the brackets, I see parentheses. I must start with the expressions in the parentheses that are furthest inside first.

2 × [4 × 4]

Now perform the next operation. The brackets remain and I must do the math inside of them before I move on.

2 × 16

All that is left is multiplication. I can do that easily.

32

EXAMPLE 2: **Evaluate the expression. Show each step.**

25 – [(10 ÷ 2) × (10 – 7)]	Start with the expressions in the parentheses that are furthest inside first.
25 – [5 × 3]	Now do the next operation that is inside the brackets.
25 – 15	Subtract
10	

1. Evaluate each expression. Remember to follow the order of operations and complete steps within grouping symbols first. Show each step of your work.

The Order of Operations

1. Perform operations that are in grouping symbols such as parentheses
2. Multiply and Divide (from left to right)
3. Add and Subtract (from left to right)

a. (5 – 2) × (4 + 5)

b. (25 ÷ 5) + (6 × 4)

c. 28 ÷ [10 – (6 ÷ 2)]

d. 3 × [(9 – 6) + (16 ÷ 2)]

e. 4 × [(3 × 3) – 5]

f. 2 × [(12 – 8) × 2] + 3

ACTIVITY

LESSON 4: POWERS OF TEN

Roll, Write, and Expand

You Will Need:

- 3 dice

You Will Do:

1. Roll the three dice and use the results to write a three-digit number below. Then write out the number in expanded form.
2. Roll the three dice twice and write down a six digit number in the space provided. Write out the number in expanded form.
3. Roll the three dice three times and write down a nine digit number. Write out the number in expanded form.

Number	Expanded form
___ ___ ___	
___ ___ ___ , ___ ___ ___	
___ ___ ___ , ___ ___ ___ , ___ ___ ___	

You've already reviewed the place value chart and learned about what each digit represents. But you may not have noticed the relationship between each spot on the chart. Each spot on the chart is ten times the value of the spot to its right. For instance, the 1,000 place is ten times 100 (10 × 100 = 1,000). Let's look at another example.

Ones		
Hundreds	Tens	Ones
7	0	0

7,000 is ten times as big as 700.

Thousands			Ones		
Hundred Thousand	Ten Thousand	One Thousand	Hundreds	Tens	Ones
		7,	0	0	0

700 × 10 = 7,000

In the same way, each spot on the chart is 1/10 the value of the spot to the left. The 1,000 place is 1/10 of the 10,000 place (10,000 ÷ 10 = 1,000). Let's look at another example.

Thousands			Ones		
Hundred Thousand	Ten Thousand	One Thousand	Hundreds	Tens	Ones
	8	0,	0	0	0

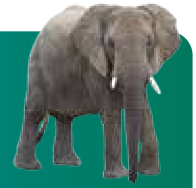
8,000 is 1/10 as big at 80,000.

Thousands			Ones		
Hundred Thousand	Ten Thousand	One Thousand	Hundreds	Tens	Ones
		8,	0	0	0

80,000 ÷ 10 = 8,000

Each of the different values on the place value chart can be represented by multiplying by 10.

10
100 = 10 ×10
1,000 = 10 × 10 × 10
10,000 = 10 × 10 × 10 × 10
100,000 = 10 × 10 × 10 × 10 × 10
1,000, 000 = 10 × 10 × 10 × 10 × 10 × 10



BASE: The number we multiply over and over in an exponent

EXPONENT: The small number written to the right and above the base that tells us how many times to multiply

POWERS OF TEN: A number that is the product of multiplying tens

It is tiring and can be confusing to write out so many tens like that. So, mathematicians came up with a simpler way of showing that you need to multiply by the same number over and over again. The **base** is the number we multiply by over and over and the **exponent** tells us how many times to multiply.

10^3 — Exponent (3), Base (10)

This is the same as

10 × 10 × 10

Common Mistake:

10^3 is not the same as 10 × 3.

10 × 3 = 30

10^3 = 10 × 10 × 10 = 1,000

We can make a chart showing these numbers which we call the **powers of ten**.

The Powers of Ten		
10	10	10^1
100	10 × 10	10^2
1,000	10 × 10 × 10	10^3
10,000	10 × 10 × 10 × 10	10^4
100,000	10 × 10 × 10 × 10 × 10	10^5
1,000,000	10 × 10 × 10 × 10 × 10 × 10	10^6

1. Use your knowledge of place value to answer the questions below.

a. 800 is ten times as much as what number? ______________

b. 90 is 1/10 of what number? ______________

c. 60,000 is ten times as much as what number? ______________

d. 300 is 1/10 of what number? ______________

2. Complete the chart.

Number	Ten times as much	1/10 as much
100		
50		
8,000		
20,000		
300,000		

3. Match each number to the correct power of ten.

100	10^1
10	10^2
10,000	10^3
1,000,000	10^4
1,000	10^5
100,000	10^6

LESSON 5: POWERS OF TEN AND EXPANDED FORM

Powers of Ten Slider

You Will Need:

- Lesson 5: Activity Sheets
- Scissors

You Will Do:

1. Carefully tear out the activity sheets from the back of the answer key. Cut out the blank strips along the dotted lines. Then cut out the slider and cut slits along the dotted lines. The easiest way to do this is to fold the slider in half and cut the slits, and then unfold it.
2. Weave one of the strips into the slider so that it shows through in all of the place value windows. Write in the number 5,122. Move the decimal point in the number one space to the right by sliding the strip to the left. This also gives you the answer to 5,122 × 10. Write the answer below.

   5,122 × 10 = ______________________________

3. Move the decimal point again by sliding the strip to the left one space to find the answer to 5,122 × 100. Write the answer below.

   5, 122 × 100 = ______________________________

4. Slide a new strip into the slider and write the number 98,000. This time we are going to move the decimal point to the left. Slide the strip to the right one space to find the answer to 98,000 ÷ 10. Slide it to the right again to find the answer to 98,000 ÷ 100. Write both answers below.

   98,000 ÷ 10 = ___________________ 98,000 ÷ 100 = ___________________

5. Slide a third strip into the slider and write a number of your choosing. Slide it to the right and to the left to write and answer two of your own problems.

   ____________ × 10 = ____________ ____________ ÷ 10 = ____________

6. Keep the slider in a safe place; you will use it again in Lesson 8.

You've already learned one way to write numbers in expanded form. There are two other ways you can do this that help you see the place value relationship in different ways.

EXAMPLE 1: **Write the number 76,819 in expanded form using multiplication and parentheses.**

$(7 \times 10{,}000) + (6 \times 1{,}000) + (8 \times 100) + (1 \times 10) + (9 \times 1)$

Writing the number out in expanded form like this lets us see each power of ten. We can also write numbers in expanded form using the powers of ten notation.

EXAMPLE 2: **Write the number 76,819 in expanded form using the powers of ten.**

$7 \times 10^4 + 6 \times 10^3 + 8 \times 10^2 + 1 \times 10^1 + 9$

1. Use what you learned in the opening activity to find the answers to these problems. You can make a new strip and act it out if you need to.

a. 3,557 x 10 = ____________________

b. 3,557 x 100 = ____________________

c. 790,000 ÷ 10 = ____________________

d. 790,000 ÷ 100 = ____________________

2. Write each number in expanded form using multiplication and parentheses.

a. 5,671 ______________________________

b. 10,156 ______________________________

c. 99,678 ______________________________

d. 566,123 ______________________________

3. Write the each number in expanded form using the powers of ten.

a. 6,799 ______________________________

b. 11,255 ______________________________

c. 345,115 ______________________________

d. 90,221 ______________________________

4. A researcher estimates that about 291,640 platypuses live in a region of Australia. Write this estimate in expanded form using multiplication and parentheses. Then write it again using powers of ten.

The platypus is an egg-laying, semi-aquatic mammal. They have many unique and surprising characteristics that don't match with what we typically assume about mammals.

LESSON 6: SCIENTIFIC NOTATION

Powers of Ten Matching Activity

You Will Need:

- Lesson 6: Activity Sheet
- Scissors
- Glue stick

You Will Do:

1. Carefully tear out the activity sheet from the back of the answer key.
2. Cut apart the cards and scramble them.
3. Match them to their correct spots. Glue your answers from the opening activity in the spaces below.

100	

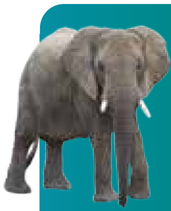
1,000,000	

100,000	

10,000	

1,000	

Powers of ten are used by scientists as well as mathematicians. Scientists use them to write very large numbers more easily. This way of writing numbers is called **scientific notation**. It can also be used to write very small numbers, but you will learn about that in the future.



SCIENTIFIC NOTATION: A way of writing really big or really small numbers using powers of ten

In this lesson, you will practice using powers of ten and scientific notation to write large numbers. You will be practicing a math skill and a science skill at the same time.

To write a number in scientific notation, you rewrite it as a number multiplied by a power of ten. The number you are multiplying must be greater than or equal to 1 and less than 10. In other words, there can only be one digit to the left of the decimal point. Let's look at some examples to get you started.

EXAMPLE 1: **Write the number 8,000 in scientific notation.**

8,000

$8 \times 1{,}000$

8×10^3

8,000 written in scientific notation is 8×10^3.

EXAMPLE 2: **Write the number 2,000,000 in scientific notation.**

2,000,000

$2 \times 1{,}000{,}000$

2×10^6

EXAMPLE 3: **Write the number 4×10^5 in standard form.**

We know that 10^5 is 100,000. You can remember this because it has 5 zeros just like the exponent 5.

$4 \times 100{,}000$

400,000

$4 \times 10^5 = 400{,}000$

1. Write each number in scientific notation.

a. 900

______ × 10 $^{\square}$

b. 8,000

______ × 10 $^{\square}$

c. 4,000

______ × 10 $^{\square}$

d. 70,000

______ × 10 $^{\square}$

e. 8,000,000

______ × 10 $^{\square}$

f. 300,000

______ × 10 $^{\square}$

2. Write the each number in standard form.

a. 3×10^4 ______________________________

b. 6×10^3 ______________________________

3. A researcher estimated that there were 400,000 African elephants. Write this number in scientific notation.

____

African elephant

LESSON 7: PROBLEM SOLVING PRACTICE #1

You have done a great job so far in this chapter learning and reviewing place value. Now we are going to pause our learning of those skills to practice some general problem solving. This is different than order of operations. Perhaps you will remember from previous courses that problem solving is a strategy you develop when you work on problems that can be solved in many different ways. There isn't one certain set of steps you have to follow to get the correct answer, and you might even try several different things before you find the strategy that works for you. Let's review those steps and strategies below.

Problem Solving Process

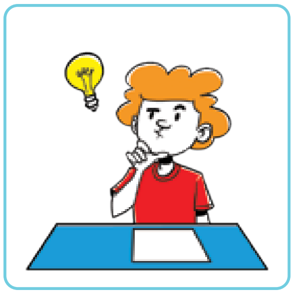
1. Read and understand the problem.
2. Choose your strategy and make a plan.
3. Work through your plan. Change your plan and try something else if necessary.
4. Check your answer. Does it make sense?
5. Present your solution verbally or in writing.

Learning this problem solving process takes time and practice. Every time you work on one of these problems you will not only be focused on solving it, but you will be getting better at this process as well. Before jumping into the problems in the lesson, take some time to review the steps of the problem solving process in more detail.

1. Read and understand the problem.

This step might sound obvious, but it really is important. Never skip this first step. Read the problem slowly and carefully. You may want to read it through twice. I like to read problems out loud to make sure I don't rush and miss something. Underline anything you think is important information. Ask yourself, "What are they asking me to find?" and, "What information was I given?" These questions will help you form a good strategy in step 2.

2. Choose your strategy and make a plan.



You will be learning and practicing many different problem solving strategies throughout this book. You are probably already using many of them now even if you didn't know they had a specific name.

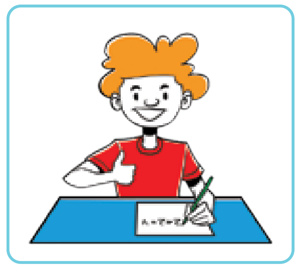
The strategy you choose is up to you, but some work better with certain problems. Here are five suggested strategies.

- Make a list or table
- Act it out
- Guess and check
- Draw a picture
- Work backwards

3. Work through your plan. Change your plan and try something else if necessary.

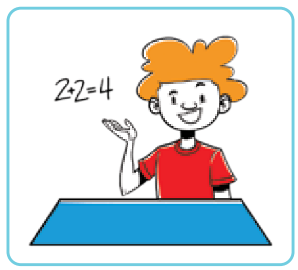
This is a fancy way of saying "solve the problem." Try your plan and see if you can get an answer. If you get stuck, don't get frustrated. That is all just part of the process. Mathematicians often have to try several different strategies before they get the right answer. Don't be afraid to stop what you are doing and start again with a new strategy.

4. Check your answer. Does it make sense?



You did all that hard work, so don't let a little addition error or something like that keep you from getting the correct answer. Double check your work and make sure your answer makes sense in the context of the problem. If you found that a person was 190 years old or that a brand new car cost $5, you probably should look for possible mistakes.

5. Present your solution verbally or in writing.



Communication is a part of being a mathematician. Good mathematicians can clearly explain what they did so that they can work together with other mathematicians. Also, presenting your solution to someone else helps you really understand the process you went through. You may have used a piece of scratch paper to scribble out your different ideas. That is just fine, but take some time to rewrite your work neatly with the steps in order. This clarifies your thinking for yourself and others. You will have a chance to practice this skill in the problem solving lessons and in a poster presentation in the last unit of this book.

One of the coolest things about problem solving is that each mathematician has his or her own style. You might solve a problem in a completely different way than your friend or even your mom or dad. And that isn't just ok, it's great! Creativity and innovative thinking are key components of becoming a great mathematician. You are developing your own style as a problem solver. In this book, you won't be told which type of strategy to use. Instead, it is going to be up to you. If you found the correct answer and can explain what you did to someone else, then you got it right—even if your process doesn't exactly match the answer key. Let's do one together.

EXAMPLE 1: **Wesley is thinking of a number. He gives 3 clues. Can you guess his number?**

1. The number is between 20 and 40.
2. It is an odd number.
3. The digits add up to 10.

Solution #1

One way to solve the problem is to use guess and check. You could use any of the clues to start your guesses. Suppose you started by thinking of a number that has digits that add to 10. You might guess 19.

This guess is too low. It is not between 20 and 40. So from there, you might decide to guess a number in the twenties with digits that add up to 10.

28

The problem with this guess is that it is even. Now try a guess in the thirties.

37

This number fits with all three clues. It is Wesley's number.

Solution #2

Making a list or table can help you organize the information. Make a list using the numbers that satisfy the first clue. The number is between 20 and 40.

21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	

The second clue says the number must be odd. Cross off all the even numbers.

21	~~22~~	23	~~24~~	25	~~26~~	27	~~28~~	29	~~30~~
31	~~32~~	33	~~34~~	35	~~36~~	37	~~38~~	39	

The third clue says that the digits add up to 10. If you look at the remaining numbers there is only one that will work.

37

This is Wesley's number.

Solve each problem below using any strategy you like.
Once you find the answer, explain what you did to your parent.

1. Elijah is thinking of a number. He gives 3 clues. Can you guess his number?

My number is a two-digit number.

Both digits are the same number.

The digits add up to 12.

____

2. Sam is playing with the six numbered cards shown below. He uses the card to make a subtraction problem with a difference of 854. If each card was only used once, where did he place each card in the problem?

1	**3**	**5**
7	**9**	**0**

```
   ___ ___ ___
 – ___ ___ ___
 -------------
    8   5   4
```

SKILLS CHECK

You should have been practicing adding three-digit numbers each day as part of your skills practice. Here are a few more for you to try.

a. $415 + 296$

b. $569 + 188$

c. $205 + 152$

d. $177 + 233$

1. Write each number in expanded form using powers of ten.

a. 8,023 ____________________

b. 501,720 ____________________

2. Write each number in standard form.

a. 5×10^6 ____________________

b. 7×10^3 ____________________

3. Evaluate each expression.

a. $2 \times 3 + 4 \times 8$ __________

b. $10 - 5 + 3$ __________

c. $32 \div [9 - (10 \div 2)]$ ______

d. $4 \times [(7 - 4) + (20 \div 5)]$ ____