

# Principles of Algebra

APPLIED ALGEBRA FROM A BIBLICAL WORLDVIEW



**MASTERBOOKS<sup>®</sup>**  
— CURRICULUM —

**Authors:**

Katherine [Loop] Hannon  
and Dr. Adam Hannon

**Master Books****Creative Team:****Editor:** Craig Froman**Design:** Jennifer Bauer**Cover Design:**

Diana Bogardus

**Copy Editors:**Judy Lewis  
Willow Meek**Curriculum Review:**Kristen Pratt  
Laura Welch  
Diana Bogardus

First printing: March 2021

Copyright © 2021 by Katherine A. Hannon, Dr. Adam F. Hannon, and Master Books®. All rights reserved. No part of this book may be reproduced, copied, broadcast, stored, or shared in any form whatsoever without written permission from the publisher, except in the case of brief quotations in articles and reviews. Permission for any other use of the material must be requested by email from the publisher at [info@nlpg.com](mailto:info@nlpg.com).

For information write:

Master Books®, P.O. Box 726, Green Forest, AR 72638  
Master Books® is a division of the New Leaf Publishing Group, Inc.

ISBN: 978-1-68344-205-9

ISBN: 978-1-61458-761-3 (digital)

Library of Congress Number: 2021934393

Unless otherwise marked, Scripture verses from the King James Version of the Bible.

Where marked ESV, Scripture quotations are from the ESV® Bible (The Holy Bible, English Standard Version®), copyright © 2001 by Crossway, a publishing ministry of Good News Publishers. Used by permission. All rights reserved.

Some review information (including various definitions) was adapted/expanded from *Principles of Mathematics* (Katherine A. Loop, *Principles of Mathematics*, Master Books, Green Forest, AR: 2015-2016); please see *Principles of Mathematics* for more details on math's foundations (on which algebra builds).

*Note:* In putting this material together, many different resources were consulted, many of which are footnoted where appropriate. We do not necessarily recommend these materials; while they were consulted for facts, some do not claim to be from a biblical worldview and should be approached with discernment.

**Printed in the United States of America**

Please visit our website for other great titles:  
[www.masterbooks.com](http://www.masterbooks.com)

Throughout the book, different mathematical concepts are referred to as “tools.” This analogy stems from Walter W. Sawyer’s profound comparison:

---



*“Mathematics is like a chest of tools: Before studying the tools in detail, a good workman should know the object of each, when it is used, how it is used, what it is used for.”<sup>1</sup>*

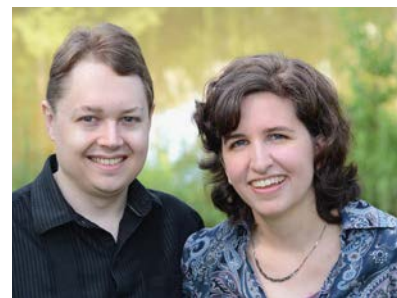
---

It’s our prayer that this book will equip you to use algebra effectively.

## About the Authors

**Katherine [Loop] Hannon** has been writing and speaking about math for more than 15 years. Understanding the biblical worldview in math during her senior year of homeschool made a tremendous difference in her life and started her on a journey of researching and sharing on the topic. Her books on math and a biblical worldview have been used by various Christian colleges, homeschool groups, and individuals.

**Dr. Adam Floyd Hannon**, Katherine’s husband, has long had a passion for math and science. He obtained his doctor of science (ScD) degree in materials science and engineering from the Massachusetts Institute of Technology after obtaining bachelor of science (BS) degrees in both physics and polymer and fiber engineering from the Georgia Institute of Technology. He currently works as a data scientist, applying many mathematical tools and algorithms to help find fraud in the healthcare system.



1 Walter W. Sawyer, *Mathematician’s Delight* (Harmondsworth Middlesex: Penguin, 1943), p. 10, quoted in James D. Nickel, rev. ed., *Mathematics: Is God Silent?* (Vallecito, CA: Ross House Books, 2001), p. 290.

## Preface

Cokie Roberts once said that “as long as there is algebra in school, there will be prayer in school.” While I believe Cokie meant that as a statement about how algebra would make students pray for help in understanding it, it’s our hope that this curriculum will help students pray during their study of algebra out of wonder and awe at God’s handiwork instead.

Rather than simply presenting apparently meaningless facts and problems to solve, our aim in this program is to take students on a journey into discovering how math helps us describe God’s creation and gives us a glimpse into just how faithful and incredible He is. To do this, we’ll be synthesizing information that most students never hear about unless they pursue a degree in a technical field.

It’s our earnest prayer that you will be blessed by this project.

Soli Deo Gloria,  
Katherine and Dr. Adam Hannon

## Acknowledgments

Many people have contributed their time, prayers, and God-given talents to make this work a reality. We would like to specifically acknowledge several people who played a key role in its development:

- Jamin Pratt and Michael Ferreira, for testing the initial draft of the material for us, providing valuable insights from a student’s perspective to help us adjust and refine. Sue Desmarais, a seasoned math teacher at both the high school and college level, for her valuable edits and polishing of the final product; Joy Dubbs, an engineer and friend, for hers; Hank Evans, Adam’s high school Algebra 2 teacher, for his helpful thoughts on the overall outline; Brian Loop, Kate’s engineering brother, for his availability to bounce ideas off from time to time; and Cris Loop, Kate’s mom, for reading through a draft of this course and previewing the optional videos, providing vital feedback from a homeschool mom’s perspective.
- The team at Master Books for making publishing this possible and for doing so with excellence. We especially want to thank Jennifer Bauer, the graphic designer, for making the material come to life, designing illustrations to help make complex concepts simple. We were consistently delighted at how the material brightened after she worked on it. And she patiently persevered through months of edits and adjustments.
- All of our friends, family, and teachers over the years for contributing to our own development and/or encouraging us along during this project. We’d like to specifically thank the Wonderful Wednesday Women for their many prayers and support of Kate during the writing of this curriculum, as well as both of our parents for their support over the years.

Above all, we want to thank God, without whose enabling none of this would be possible.

With gratitude,  
Katherine and Dr. Adam Hannon

# Principles of Algebra About This Curriculum



This complete Algebra 2 program not only teaches algebra, but also shows students why they're learning concepts and how algebra's very existence points us to God. Students will see algebra in action . . . and find their biblical worldview built along the way.

## Are There Any Prerequisites?

At a minimum, students should have completed an Algebra 1 course. Completion of a full course in geometry is recommended, but not required.

Overall, students need to be familiar with the basics of algebra (working with algebraic fractions, exponents, and roots; combining like terms; basic factoring; formulas; basic finding of unknowns; and graphing of linear equations) as well as arithmetic (including converting units of measure, decimals, scientific notation, and using a calculator for exponents, roots, and operations inside of parentheses) and basic geometric ideas (such as perimeter, area, and volume of circles and simple polygons). A basic familiarity with these concepts is assumed.

## How Do I Use This Curriculum?

This curriculum is designed so that it can be self-taught. Students should be able to read the material and complete assignments on their own, with a parent or teacher available for questions. The optional eCourse can limit the amount of reading and provide more guidance through the concepts. If teaching in a classroom, the text can serve as the basis for the teacher's presentations, with the text available as a reference later for students. This *Student Textbook* is divided into chapters and then into lessons. The number system used to label the lessons expresses this order. The first lesson is labeled 1.1 because it is Chapter 1, Lesson 1.



## What Are the Curriculum's Components?

The curriculum consists of this *Student Textbook*, the *Teacher Guide*, and *Solutions Manual*. The *Student Textbook* contains the instructional lessons. The *Teacher Guide* contains an easy-to-follow schedule, as well as the worksheets, quizzes, and tests. The *Solutions Manual* contains a complete answer key (which includes solutions for most problems, as well as notes and explanations of many of the solutions).

### **Optional *Principles of Algebra 2 eCourse* available from Master Books Academy —**

These videos offer presentations of lessons produced by the author and is an addition to the printed material. They're great for students that are more visual/auditory in learning or need more walking through the concepts. Students using the videos should watch the video for that lesson then look over the text, studying it as needed. The eCourse is available through the Master Books Academy at [MasterBooksAcademy.com](http://MasterBooksAcademy.com).



## What Do I Need To Complete This Course?

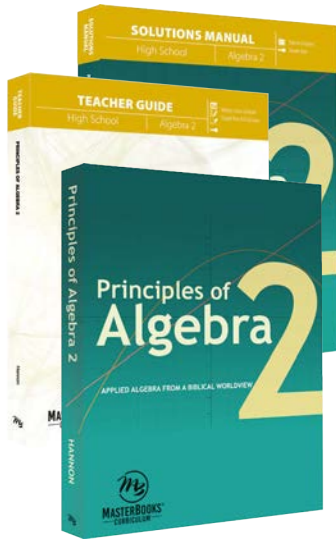
The supplies needed for this course are as follows:

- *Principles of Algebra 2: Applied Algebra from a Biblical Worldview Student Book*
- *Principles of Algebra 2 Teacher Guide*
- *Principles of Algebra 2 Solutions Manual*
- *Principles of Algebra 2 eCourse* (optional)
- Binder with Lined Paper or Other Note-Taking Method
- Calculator (or Online Calculator)
- A College Notebook
- A Second Notebook or Additional Lined Paper
- Index Cards (optional)
- Graph Paper

Please see page 6 of the *Teacher Guide* for a more complete description of each item.

## Where Do I Go Upon Completion?

Upon completion of this course, students should complete a geometry course (if not already completed); if they have already completed geometry, students should be ready to begin a precalculus course or a college algebra program. Advanced students may also want to study for and take the *College Algebra* CLEP test. While this book does *not* cover everything on the test, advanced students may be able to use a CLEP study guide to fill in additional college-level concepts (as well as to gain familiarity with the test format).



# Principles of Algebra

## Table of Contents



<b>Chapter 1: Setting the Foundation</b>	
Lesson 1.1 God and the Laws of Math	11
Lesson 1.2 The Language of Mathematics	17
Lesson 1.3 Understanding, Multiplying, and Dividing Fractions	21
Lesson 1.4 Equivalent Fractions and Simplifying Fractions	26
Lesson 1.5 Understanding Ratios and Proportions	29
Lesson 1.6 Rates	33
Lesson 1.7 Adding and Subtracting Fractions	37
Lesson 1.8 Negative Numbers	41
Lesson 1.9 Chapter Synopsis	45
<b>Chapter 2: Exploring Exponents</b>	
Lesson 2.1 Exponents (and Order of Operations)	49
Lesson 2.2 Negative Exponents (and Scientific Notation)	54
Lesson 2.3 Multiplying and Dividing Exponents	58
Lesson 2.4 Repeated Multiplication of Exponents	62
Lesson 2.5 Exponents and Units	64
Lesson 2.6 Roots and Fractional Exponents	68
Lesson 2.7 Working with Fractional Exponents	72
Lesson 2.8 Chapter Synopsis – And a Glance at Exponential History	77
<b>Chapter 3: Numbers and Sets</b>	
Lesson 3.1 Sets	81
Lesson 3.2 Exploring Numbers: Rational and Irrational Numbers	87
Lesson 3.3 Simplifying Roots	92
Lesson 3.4 Exploring Numbers: Imaginary and Complex Numbers	95
Lesson 3.5 More with Imaginary Numbers	100
Lesson 3.6 Chapter Synopsis – And Different Notations to Describe Sets	103

<b>Chapter 4: Solving for Unknowns and Problem-Solving Skills</b>	
Lesson 4.1 Solving for Unknowns	107
Lesson 4.2 Solving for Unknowns Using Roots	113
Lesson 4.3 Exploring Inequalities	118
Lesson 4.4 Application Problem-Solving Skills	124
Lesson 4.5 Substitution	129
Lesson 4.6 Chapter Synopsis	131
<b>Chapter 5: Solving More In-Depth Problems</b>	
Lesson 5.1 Terms and Combining Like Terms	133
Lesson 5.2 Combining Like Terms in Fractions and More Simplifying Fractions	138
Lesson 5.3 Principles to Help Find Unknowns	142
Lesson 5.4 More Principles to Help Find Unknowns	145
Lesson 5.5 The Distributive Property	149
Lesson 5.6 Factoring Out Common Factors	153
Lesson 5.7 Chapter Synopsis – And Algebra’s Usefulness	158
<b>Chapter 6: Unleashing Algebra: Solving Systems of Equations</b>	
Lesson 6.1 Solving Systems of Equations Using Substitution	161
Lesson 6.2 Solving Systems of Equations by Addition or Subtraction	166
Lesson 6.3 Solving Systems of Equations Using Matrices	170
Lesson 6.4 Chapter Synopsis – And Meaningless Matrices	175
<b>Chapter 7: Introducing Functions and Function Notation</b>	
Lesson 7.1: Understanding Functions	177
Lesson 7.2: Function Notation (and Domains and Ranges)	183
Lesson 7.3: Coordinate Geometry and Graphing Functions	187
Lesson 7.4: Learning About Functions: Intercepts, Asymptotes, and Maximum/Minimum Points	194
Lesson 7.5: Learning About Functions: Concavity and Even or Odd	200
Lesson 7.6: Operations with Functions	207
Lesson 7.7: Composite Functions	212
Lesson 7.8: Inverse Functions	216
Lesson 7.9: Chapter Synopsis	221
<b>Chapter 8: Introduction to Polynomial Functions</b>	
Lesson 8.1: Introducing and Categorizing Polynomials and Polynomial Functions	227
Lesson 8.2: Overview of Linear Functions	233
Lesson 8.3: Beginning to Look at Quadratics (Minimum/Maximum Point and Vertex Form)	240
Lesson 8.4: Working with Polynomial Expressions	246
Lesson 8.5: Chapter Synopsis	250



<b>Chapter 9: Quadratic Functions</b>	
Lesson 9.1 Introducing Quadratic Functions	253
Lesson 9.2 Factoring Quadratic Functions and Applying Roots	262
Lesson 9.3 Understanding and Working with Perfect Squares	268
Lesson 9.4 Completing the Square and the Quadratic Formula	273
Lesson 9.5 Complex Roots of Quadratics	280
Lesson 9.6: Recognizing Quadratics in Certain Forms	283
Lesson 9.7 Chapter Synopsis — And a Geometric Look at Factoring	288
<b>Chapter 10: More with Polynomials</b>	
Lesson 10.1 Expanding to Other Polynomials: The Fundamental Theorem of Algebra	295
Lesson 10.2 Graphically Solving Systems of Equations	301
Lesson 10.3 Graphically Solving Systems of Inequalities	306
Lesson 10.4: Chapter Synopsis	315
<b>Chapter 11: Rational Functions</b>	
Lesson 11.1: Introducing Rational Functions (and Handling Units of Measure)	317
Lesson 11.2: Finding Invalid Inputs and Simplifying Rational Functions	323
Lesson 11.3: Finding Roots of Rational Functions	329
Lesson 11.4: Partial Fractions	336
Lesson 11.5: Polynomial Division	340
Lesson 11.6: Chapter Synopsis	346
<b>Chapter 12: Exponential Functions</b>	
Lesson 12.1: Introducing Exponential Functions	349
Lesson 12.2: More with Exponential Functions	354
Lesson 12.3: Interest Rates and Different Formulas	357
Lesson 12.4: Euler’s Number and Continuous Growth	361
Lesson 12.5: Working with Exponential Functions and Algebraic Explanations	366
Lesson 12.6: More Working with Exponential Functions	371
Lesson 12.7: Chapter Synopsis – And Radioactive Dating	375
<b>Chapter 13: Logarithms</b>	
Lesson 13.1: Introducing Logarithms and Taking the Logarithm of Both Sides of an Equation	379
Lesson 13.2: Shorthand for Common Logarithms and Finding Logarithms on a Calculator	385
Lesson 13.3: Logarithms with Different Bases	389
Lesson 13.4: Adding and Subtracting Logarithms	395
Lesson 13.5: The Power Rule and More Working with Logarithms	399
Lesson 13.6: Introducing Log Scale	405
Lesson 13.7: Introducing Logarithmic Functions	414
Lesson 13.8: Chapter Synopsis	418

<b>Chapter 14: Transforming Functions – Plus More with Functions</b>	
Lesson 14.1: Transforming Functions Vertically	421
Lesson 14.2: Transforming Functions Horizontally	429
Lesson 14.3: Transforming When We Don't Know the Function	440
Lesson 14.4: Transforming Points	447
Lesson 14.5: Connecting Composite Functions and Transformations	455
Lesson 14.6: Introducing Piecewise Functions	462
Lesson 14.7: Intro to Power and Absolute Value Functions	465
Lesson 14.8: Introducing Trigonometric Functions	470
Lesson 14.9: Chapter Synopsis – And Transformations in Action	477
<b>Chapter 15: Functions and Formulas That Help Us Count, Find Probabilities, and Generate Sequences</b>	
Lesson 15.1: Introducing Probability	481
Lesson 15.2: Finding the Probability of <i>Either</i> Event (Adding Probabilities)	485
Lesson 15.3: Factorials	491
Lesson 15.4: Permutations and Combinations	494
Lesson 15.5: Describing Sequences Algebraically	500
Lesson 15.6: More with Sequences	505
Lesson 15.7: Numbering Sequences Differently	508
Lesson 15.8: Series	515
Lesson 15.9: Binomial Theorem	521
Lesson 15.10: Chapter Synopsis – And Warning and Wonder	529
<b>Chapter 16: Review</b>	
Lesson 16.1: Mastering the Mechanics (Reviewing Chapters 1–6 and Mechanics)	535
Lesson 16.2: Functioning with Functions (Reviewing Chapters 7–12)	537
Lesson 16.3: More with Functions – And a Parting Note (Reviewing Chapters 13–15)	542
<b>Appendix A: Math's Message</b>	547
<b>Appendix B: Reference Section</b>	551
<b>Index</b>	559