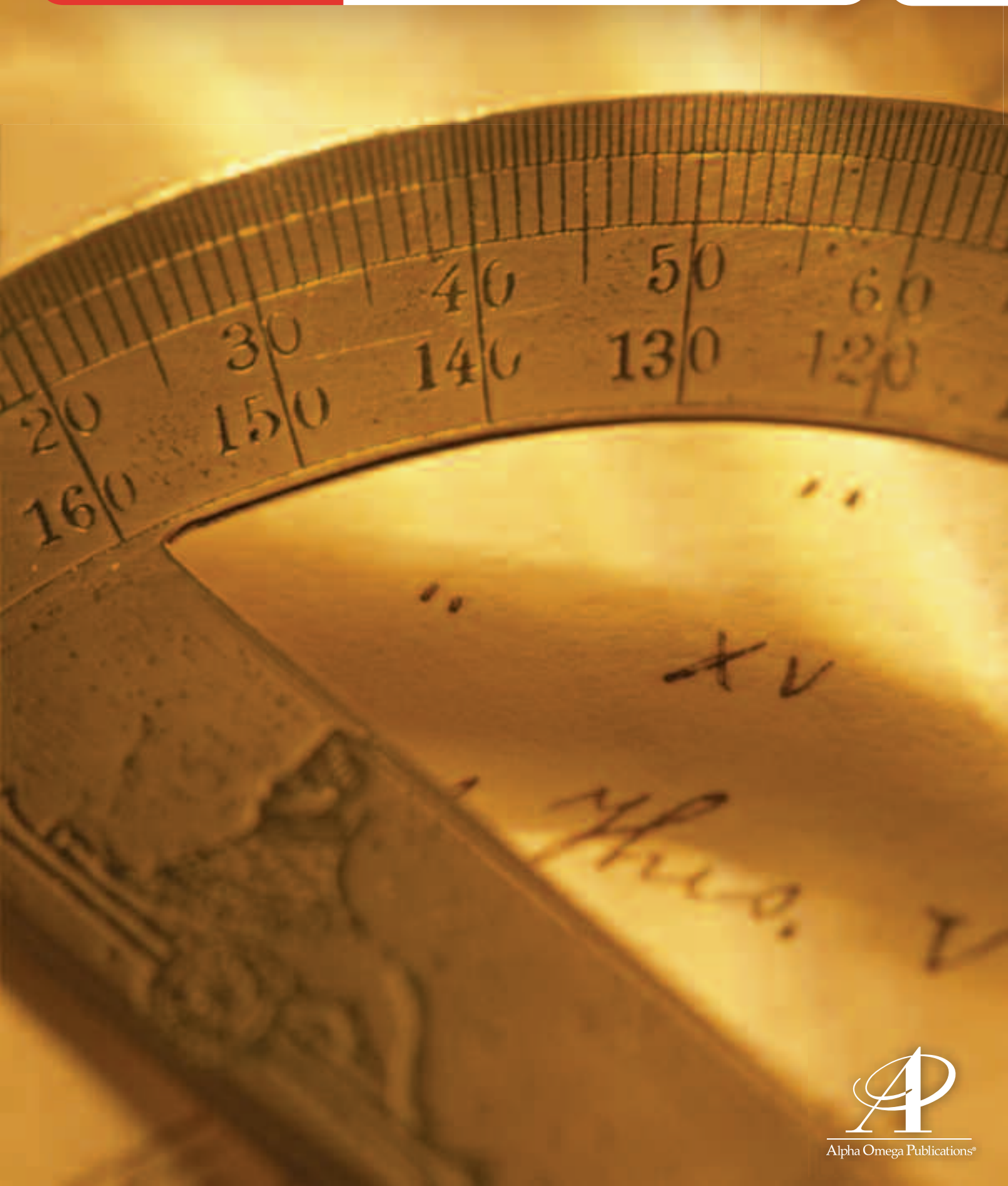
LIFEPAC®

# Math

Student Book

Grade **12**
Unit **1**



Alpha Omega Publications®

# MATHEMATICS 1201
# RELATIONS AND FUNCTIONS

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# RELATIONS AND FUNCTIONS

Relations and functions are sets of ordered-pair numbers. These sets of numbers represent lines (the linear function), simple curves (the quadratic function), and curves of higher degree functions. You will soon study the exponential function, the logarithmic function, and the circular function (trigonometry). The study of functions becomes more necessary and important as you prepare for the study of calculus.

## OBJECTIVES

**Read these objectives.** The objectives tell you what you should be able to do when you have successfully completed this LIFEPAC®.

When you have completed this LIFEPAC, you should be able to:

1. Specify the domain and range of a given relation.
2. Determine whether a given relation is also a function.
3. Identify and work with rules of correspondence for given relations.
4. Read and work with function notation.
5. Find values of a given function for specified elements of the domain.
6. Perform the arithmetic of functions.
7. Find the composition of two or more functions.
8. Graph the constant and identify functions.
9. Find the inverse of a function.

## I. ORDERED-PAIR NUMBERS

An ordered-pair number is a pair of numbers that go together. The numbers are written within a set of parentheses and separated by a comma. (6, 2) is an ordered-pair number; the order is designated by the first element 6 and the second element 2. The pair (2, 6) is not the same as (6, 2) because of the different ordering. Sets of ordered-pair numbers can represent relations or functions. Some relations and functions are defined by rules of correspondence.

### RELATIONS

This section discusses the concept of relations and gives its definition.

**DEFINITION**

**A *relation* is any set of ordered-pair numbers.**

Suppose that the weights of any number of students are recorded by the school nurse.

| Student | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Weight | 150 | 130 | 100 | 160 |

The pairing of the student number and his corresponding weight is a relation and is written:

$A = \{(1, 150), (2, 130), (3, 100), (4, 160)\}$

These data are written as a set of ordered-pair numbers. Each element of the set is an ordered-pair.

The first element of each pair is the student number, and the set of all first elements is called the *domain* of the relation or data set.

The domain of $A = \{1, 2, 3, 4\}$

The second element of each pair is the weight, and the set of second elements is called the *range* of the relation or data set.

The range of $A = \{150, 130, 100, 160\}$

*WRITE THE DOMAIN AND RANGE OF THE FOLLOWING RELATIONS IN LIST FORM.*

1.1 $\{(5, 0), (6, 1), (7, 2)\}$

a. Domain _____

b. Range _____

1.2 $F = \{(6, \sqrt{2}), (7, \sqrt{3}), (8, \sqrt{4}), (9, \sqrt{5})\}$

a. Domain of $F$ _____

b. Range of $F$ _____

1.3 $G = \{(\frac{1}{2}, \frac{\pi}{6}), (\frac{1}{2}, \frac{\pi}{4}), (\frac{1}{2}, \frac{\pi}{3}), (\frac{1}{2}, \frac{\pi}{2})\}$

a. Domain of $G$ _____

b. Range of $G$ _____

1.4 $J = \{(6.2, 0.3), (7.3, 0.3), (8.4, 0.3), (9.5, 0.3)\}$

a. Domain of $J$ _____

b. Range of $J$ _____

*WRITE THE REQUIRED RELATIONS.*

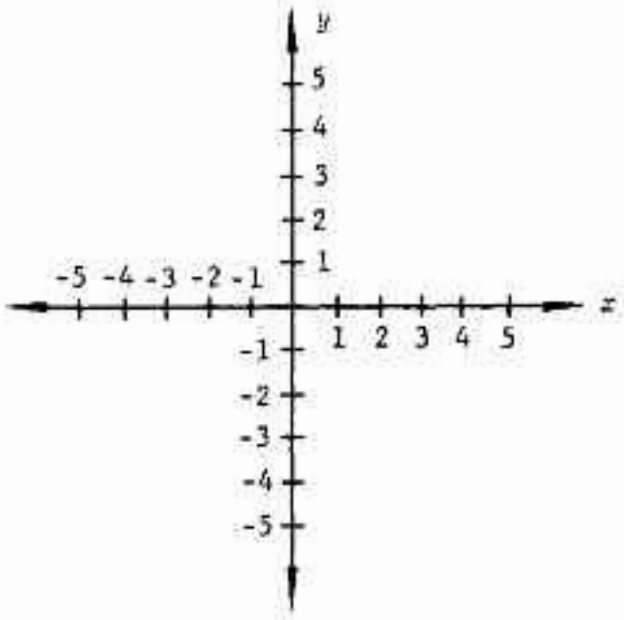
1.5 Write a relation in ordered-pair form for six different packages of fruit whose weights are the domain of the relation and are 10, 15, 20, 30, 60, and 90; and whose costs are the range of the relation and are \$2, \$3, \$4, \$6, \$8, and \$10.

$Q =$ _____

1.6 A physics student, conducting an experiment of a falling object, drops a baseball from the top of a building and records the distance traveled for each second of elapsed time. If the distances recorded are 16 ft., 64 ft., 144 ft., 256 ft., and 400 ft. respectively for each second of time, write this relation in ordered-pair form.

$F =$ _____

A relation that consists of ordered pairs of real numbers is a subset of $R \times R$. This set, read "$R$ cross $R$," is made up of all the possible combinations of real numbers denoting ordered pairs. To visualize $R \times R$, think of a horizontal number line extending infinitely in both directions, crossed by a vertical number line extending infinitely in both directions.



Each positive and negative real number on the horizontal number line can be paired with each positive and negative real number on the vertical number line to give all possible combinations of two real numbers. Any relation that consists of ordered pairs of real numbers, therefore, is a subset of $R \times R$.

Some relations can be specified or defined by a rule enabling you to determine the element or elements of the range paired with each element of the domain. For example the solution set over $R \times R$ of an open sentence such as $x - y = 4$ is a relation $L$ that you can specify:

$$L = \{(x, y): (x, y) \in R \times R \text{ and } x - y = 4\}$$

or

$$L = \{(x, y): y = x - 4, x \in R\}$$

The domain of the relation $L$ is the set of real numbers.

domain of data = {Real numbers}

The range of the relation $L$ is the set of numbers $x - 4$, where $x$ is any real number.

range of data = $\{x - 4; x \in R\}$

Whenever an open sentence specifies a relation whose domain and range are not explicitly stated, we agree to include in the domain and range those real numbers and *only* those real numbers for which the open sentence is true.

*STUDY THESE EXAMPLES:*

Determine (a) the domain and (b) the range of

$$H = \{(x, y): y = \sqrt{1 - x^2}\}.$$

Solution: a. Since each value of $y$ is to be a real number, each value of $1 - x^2$ must be a nonnegative number. Since $x^2 = |x|^2$, $1 - x^2 \geq 0$ means $1 - |x|^2 \geq 0$ or $|x|^2 \leq 1$. Then $|x| \leq 1$.

Therefore, the domain of $H$ is $\{x: -1 \leq x \leq 1\}$.

b. $|x| \leq 1$ implies $0 \leq x^2 \leq 1$.
Then $0 \leq 1 - x^2 \leq 1$.

Therefore, the range of $H$ is $\{y: 0 \leq y \leq 1\}$.

State the domain of $F = \{(x, y): y = \frac{3x}{(x - 2)(x + 5)}\}$.

Since each value of $y$ is to be a real number, $(x - 2)(x + 5) \neq 0$; since if $(x - 2)(x + 5) = 0$, then $y$ would be undefined. $(x - 2)(x + 5) = 0$ when $x = 2$ or $x = -5$. Therefore, the domain of $F$ consists of all the real numbers except 2 and -5.

*STATE THE DOMAIN OF EACH OF THE FOLLOWING RELATIONS.*

1.7 $\{(x, y): x = \sqrt{y}\}$

Domain _____

1.8 $\{(r, s): s = \frac{2r - 5}{13r}\}$

Domain _____

1.9 $\{(a, b): ab = 12\}$

Domain _____

*STATE THE DOMAIN AND RANGE OF EACH OF THE FOLLOWING RELATIONS.*

1.10 $\{(x, y): y = x + 1, x \text{ is some even integer}\}$

a. Domain _____

b. Range _____

1.11 $\{(x, y): y = |x|\}$

a. Domain _____

b. Range _____

**FUNCTIONS**

**DEFINITION**

A *function* is a relation in which no two ordered pairs have the same first element.

A function, therefore, associates each element in its domain with one and only one element in its range.

*STUDY THESE EXAMPLES:*

$F = \{(1, 2), (2, 3), (3, 4), (4, 5)\}$

$F$ is a function because each element of the domain is associated with a unique element of the range.

$G = \{(-1, 3), (0, 3), (2, 3), (-5, 3)\}$

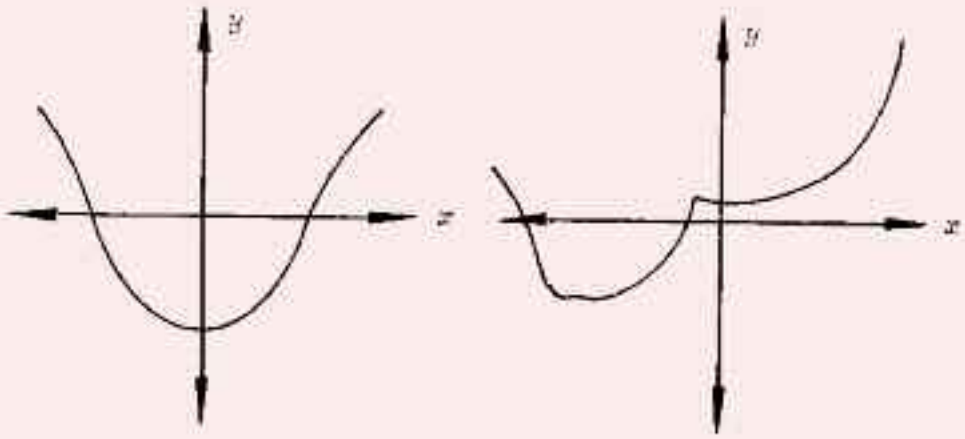
$G$ is a function because each first element is not paired with more than one second element.

$H = \{(1, 6), (2, 7), (1, 8), (3, 9)\}$

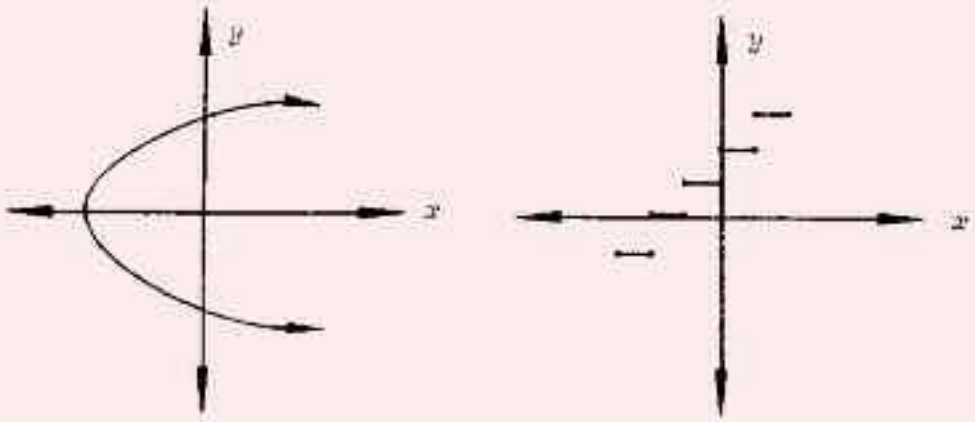
$H$ is *not a function* because the first element, 1, does not have a unique second element. In other words, the element 1 is double valued; hence, $H$ is not a function.

A function can be identified from a graph. If both of two points lie on the same vertical line, then $x$ is double valued and hence the relation is *not* a function. if any vertical line drawn through the graph cuts the graph at more than one point, then the relation is *not* a function.

These graphs are relations, $\{(x, y)\}$, which are also functions.




These graphs are relations, $\{(x, y)\}$, which are not functions.




*ANSWER THESE QUESTIONS.*

1.12 $F = \{(6, 3), (5, 4), (4, 5), (3, 6), (2, 7)\}$

a. Is $F$ a function? _____

b. If not, why? _____

1.13 $G = \{(1, 1), (1, 2), (1, 3), (1, 4)\}$

a. Is $G$ a function? _____

b. If not, why? _____

1.14 $H = \{(2, 5), (3, 5), (4, 5), (5, 5), (6, 5)\}$

a. Is $H$ a function? _____

b. If not, why? _____

1.15 $K = \{(-3, -3), (-4, -4), (-5, -5), (-6, -6)\}$

a. Is $K$ a function? _____

b. If not, why? _____

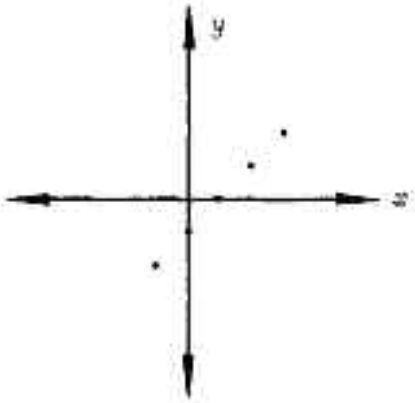
1.16 $J = \{\quad\}$

a. Is $J$ a function? _____

b. If not, why? _____

*WHICH OF THE FOLLOWING GRAPHS ARE FUNCTIONS? FOR a., WRITE yes OR no; FOR b., EXPLAIN WHY OR WHY NOT.*

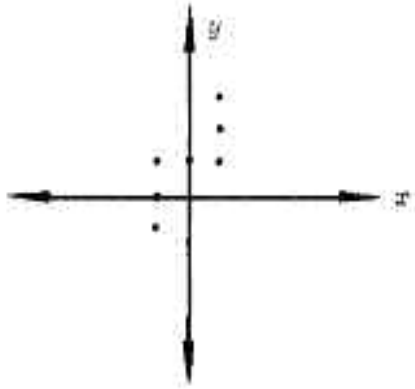
1.17




a. _____

b. _____

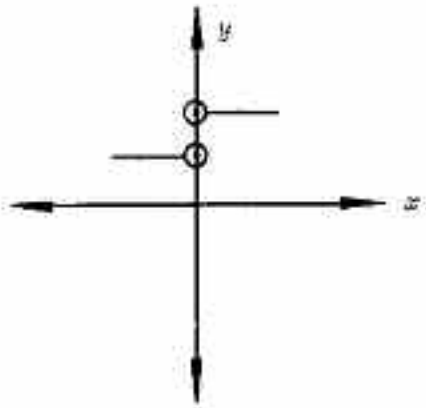
1.18




a. _____

b. _____

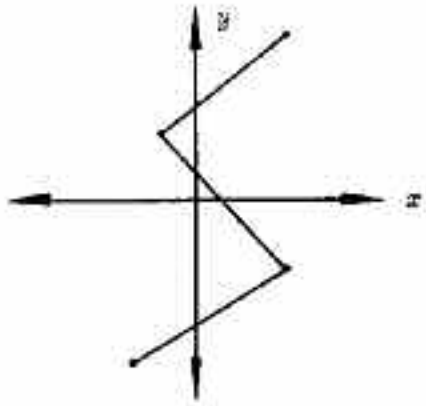
1.19




a. _____

b. _____

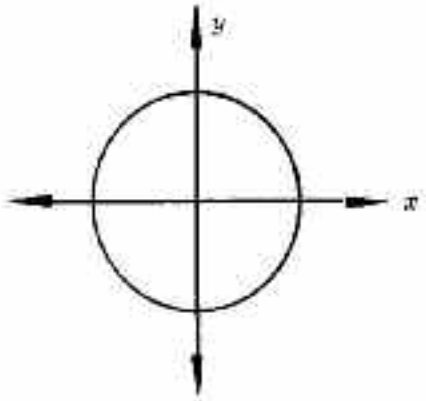
1.21




a. _____

b. _____

1.20




a. _____

b. _____

**RULES OF CORRESPONDENCE**

Many relations and functions are defined by an algebraic rule whereby any ordered pair within the relation or function may be calculated.

*STUDY THESE EXAMPLES:*

$$A = \{(1, 2), (2, 4), (3, 6), \ldots\}$$

In this relation the second element is always double the first element. If we call the first element $x$ and the second element $y$, then the rule of correspondence will be

$$y = 2x.$$

$B = \{(1, 4), (2, 6), (3, 8), \ldots\}$

The rule in this relation is a little more difficult to find; however, the rule is

$$y = 2(x + 1).$$

Verify the Set $B$ and add a few solutions of your own. Would (-4, -6) be a solution to Set $B$? The answer is yes.

$C = \{(-2, 3), (-1, 0), (1, 0), (2, 3), (3, 8), \ldots\}$

The rule in this relation is also a little more difficult to find; however, the rule is

$$y = x^2 - 1.$$

$D = \{(1, 3), (2, 1), (3, 2.5)\}$

Don't try too long on this one, because it has no rule that will compute the ordered-pair numbers.

*WRITE A RULE OF CORRESPONDENCE FOR EACH OF THESE RELATIONS.*

1.22 $A = \{(x, y): (1, 2), (2, 5), (3, 8), \ldots\}$ _____

1.23 $B = \{(x, y): (2, 3), (4, 4), (6, 5), \ldots\}$ _____

1.24 $C = \{(x, y): (6, 15), (8, 21), (10, 27), \ldots\}$ _____

1.25 $D = \{(x, y): (3, 9), (5, 25), (-2, 4), \ldots\}$ _____

1.26 $E = \{(x, y): (2, 15), (5, 30), (8, 45), \ldots\}$ _____

*COMPLETE THESE ACTIVITIES.*

1.27 The diagonals of a polygon are lines that join any two nonadjacent vertices. The square has two diagonals, the pentagon has five, and the hexagon has nine. Write a rule of correspondence for the number of diagonals of any $n$-sided polygon.

$A = \{(n, D): (4, 2), (5, 5), (6, 9), \ldots\}$ _____

1.28 If the first element of the following pairs of numbers is a measure of hours and the second element is a measure of distance in miles, write a rule of correspondence for $D$.

$B = \{(t, D): (1, 50), (2, 100), (3, 150), \ldots\}$ _____

1.29 Write a rule of correspondence for $S$.

$C = \{(t, S): (0, 0), (1, 16), (2, 64), (3, 144), \ldots\}$ _____

1.30 Given: $S = \frac{n}{2}(n + 1)$. Finish this table of values.

$D = \{(n, S)$: (0, a. ), (1, b. ), 2, c. ), (d. , 6), (100, e. )$\}$

1.31 A rectangular area of 2,000 sq. ft. is to be fenced on three sides with fence costing 30¢ per foot and on the fourth side with fence costing 50c per foot. If $x$ denotes the length of the fourth side and $c = f(x)$ denotes the corresponding cost of the fence in cents, (a) express $c$ as a function of $x$. (b) What is the domain of the function?

a. _____

b. _____

Review the material in this section in preparation for the Self Test. The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific areas where restudy is needed for mastery.

## SELF TEST 1

*WRITE THE REQUIRED INFORMATION (each answer, 3 points).*

1.01 If $x = \{(2, 3), (2, 4), (2, 5), (2, 6)\}$, is $x$ a function? _____

1.02 If $x = \{(2, 4), (3, 4), (4, 4), (5, 4)\}$, is $x$ a function? _____

1.03 If $x = \{3, 2, 4, 1, 6\}$, the range of $x$ = _____

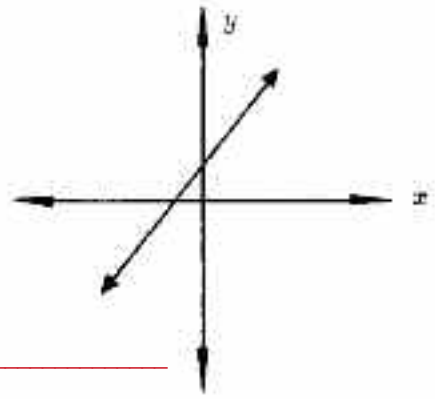
1.04 If $y = \{(2, 6), (3, 1, 4), (1), (5, 6)\}$, is $y$ a function? _____

1.05 The domain of 1.01 is _____

1.06 The range of 1.02 is _____

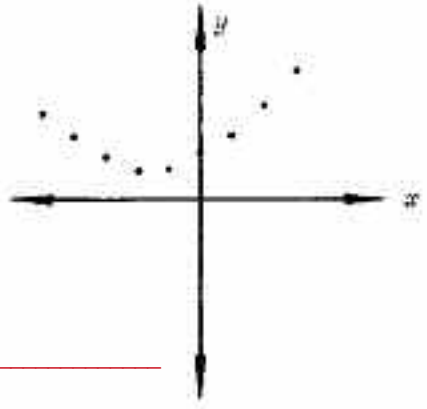
*TELL WHICH OF THE FOLLOWING GRAPHS REPRESENT FUNCTIONS BY WRITING* yes *OR* no *IN THE SPACE PROVIDED (each answer, 3 points).*

1.07



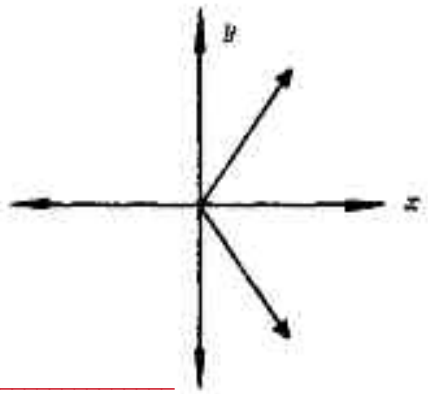

_____

1.08

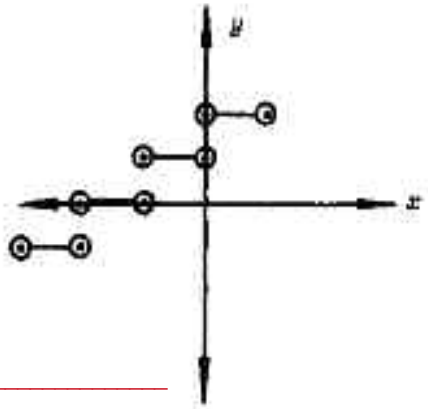



_____

1.09




1.010




*WRITE A RULE FOR EACH OF THE FOLLOWING RELATIONS (each answer, 5 points).*

1.011 $P = \{(x, y): (1, 0), (2, 4), (3, 8), (4, 12), \ldots\}$

1.012 $Q = \{(x, y): (2, 8), (3, 27), (4, 64), (5, 125), \ldots\}$

1.013 $R = \{(x, y): (2, 3), (4, 4), (6, 5), (8, 6), \ldots\}$

1.014 $S = \{(x, y): (4, 5), (8, 7), (12, 9), (16, 11), \ldots\}$

40/50

**Score**
**Teacher Check**

**Initial** **Date**