



# MATH

STUDENT BOOK

▶ **8th Grade** | Unit 1

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# Math 801

## The Real Number System

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# The Real Number System

## Introduction

Pre-algebra is an introductory algebra course designed to prepare junior-high school students for Algebra I. The course focuses on strengthening needed skills in problem solving, integers, equations, and graphing. Students will begin to see the “big picture” of mathematics and learn how numeric, algebraic, and geometric concepts are woven together to build a foundation for higher mathematical thinking.

By the end of the course, students will be expected to do the following:

- Gain an increased awareness of how math is a life skill.
  - Understand how math is like a language, with a set of conventions.
  - Explore concepts taught in previous math courses at higher levels and in real world applications.
  - Practice algebraic thinking in order to model and solve real world problems.
- Utilize new skills and concepts that will help them in future math courses.
  - Introduce variable expressions and equations (single and multiple variable).
  - Introduce linear functions, relationship between dependent and independent variables and coordinate graphing.

In this unit, the student is formally introduced to the subsets of the real number system, including irrationals. Venn diagrams are used to compare and contrast the subsets. The number line is used to discuss distance, midpoint, and absolute value, as well as to compare and order integers.

The properties of the real number system are reviewed. Exponents and order of operations are used to allow the student to apply properties of the real number system. Lastly, scientific notation is explained.

## Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAK. When you have finished this LIFEPAK, you should be able to:

- Classify numbers.
- Evaluate expressions that contain variables.
- Compare and order numbers.
- Determine absolute value.
- Apply the properties of real numbers.
- Use exponents.
- Write numbers in scientific notation.
- Write numbers with square roots.
- Use the order of operations to simplify expressions.

# 1. Relationships

## SUBSETS OF THE REAL NUMBER SYSTEM

Numbers are concepts that are represented by symbols called numerals. Numerals are used to communicate the idea of “how many.” Look at some numerals that have been used in the past to communicate the number ten:

Early Greek	Roman	Babylonian
Δ	X	𐎠

Even though the symbols are different, they each represent the same “idea.” But, what if everyone had their own way to represent numbers? Can you imagine the chaos? Trying to communicate ideas, like what time it is or how much something costs, would be almost impossible!

The sets of numbers we use today exist because there came a time when a universal way to represent numbers was needed.

### Objectives

- Classify numbers.
- Identify irrational numbers.

### Vocabulary

**infinite**—increasing or decreasing without end

**integer**—a number belonging to the set made up of the whole numbers and their opposites

**irrational number**—a number which, when in decimal form, does not terminate or repeat

**natural number**—a number belonging to the set made up of the numbers that are used to count: 1, 2, 3, and so on

**rational number**—a number which can be written as a ratio in fraction form

**real number**—a number which can be written as an infinite decimal

**whole number**—a number belonging to the set made up of zero and the natural numbers

### Real Numbers

All of the numbers that you have worked with so far are called *real numbers*. You might wonder why we call them “real” numbers. Are there “unreal” numbers? Actually, yes, there are! The set of “unreal” numbers are called imaginary or complex numbers, and you will learn about those

later on in math. In pre-algebra, we will focus on the real number system.

Within the system of real numbers, there are several sets or groups of numbers, called subsets. We will use a diagram to help us remember what each group of

numbers is and how it fits within the real number system.

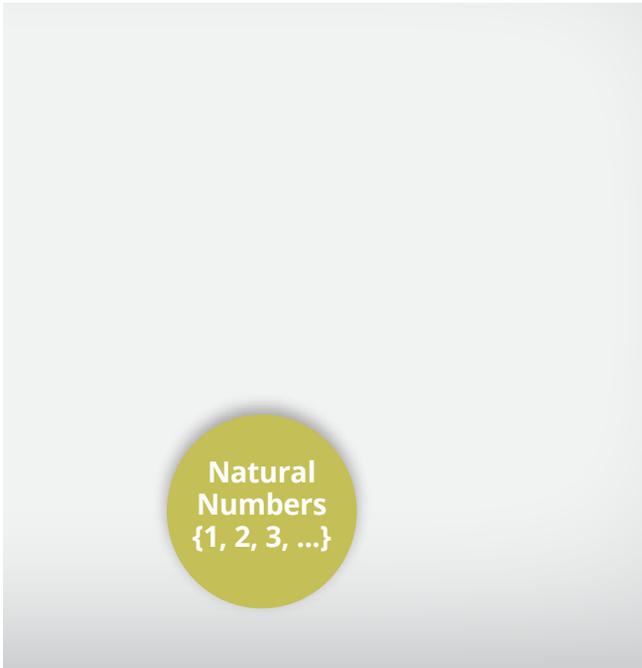
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**Vocabulary!** Remember that a subset is a set in which all its members belong to another set. For example, a set listing several types of dogs is a subset of the set of all animals, because every dog is an animal.

---

People in ancient times began using numbers so they could record or talk about how many of something they had. Zero and negative numbers did not exist as we know them today because people had no need to communicate those concepts. Numbers really were only used for counting, so this set of numbers is called the “counting” numbers or *natural numbers*.

### Natural Numbers



It is difficult to find the history of how zero evolved. The idea of zero was hard for people to accept. Math had always been used to solve “real” problems. People saw

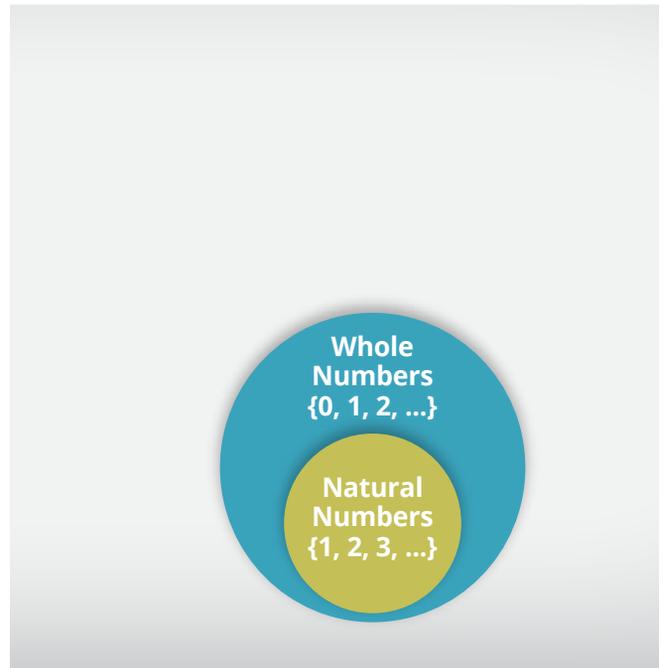
no need to represent zero. It did, however, start to get used as a place holder in numbers, just as we do today. For example, the number 306 would be 36 if we didn’t use the zero to show that there is nothing in the tens place. Eventually, zero was used to represent the result when a number is subtracted from itself. Adding zero to the set of natural numbers gives us the *whole numbers*.

---

**Keep in mind!** The three dots, called an ellipsis, means that the set of numbers is infinite, or continues forever in the same pattern.

---

### Whole Numbers



The negative numbers were ultimately the biggest challenge for mathematicians. As they discovered more about math, mathematicians began to solve problems that had negative answers. At first, people could not agree. Many said that negatives were not real numbers, so there could be no answer to the problems. Over time, however, negative numbers have come to

be accepted. Today, we easily talk about negative temperatures or negative amounts of money (debt).

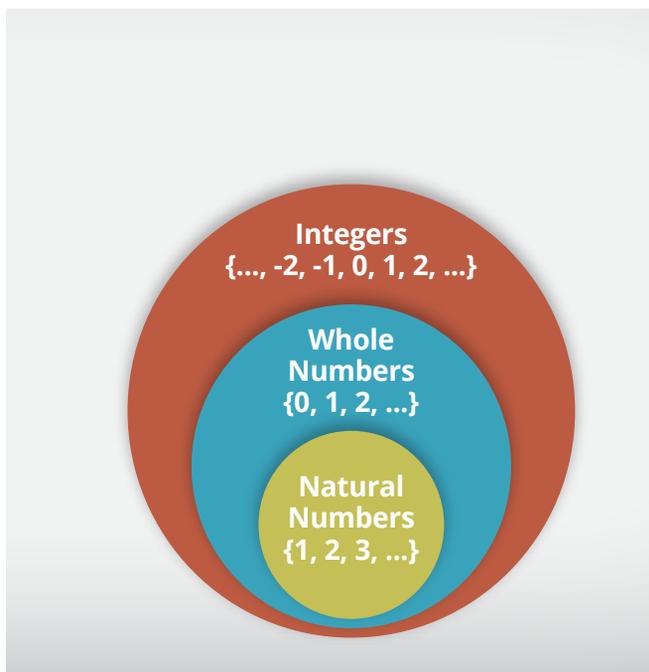
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**This might help!** One way to remember the whole numbers is to think of zero as a “hole.” The words are not spelled the same but sound the same. Try to connect “hole” with the set that starts with zero.

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Take the counting numbers, make them negative, and combine them with the whole numbers. Now you have the *integers*.

### Integers



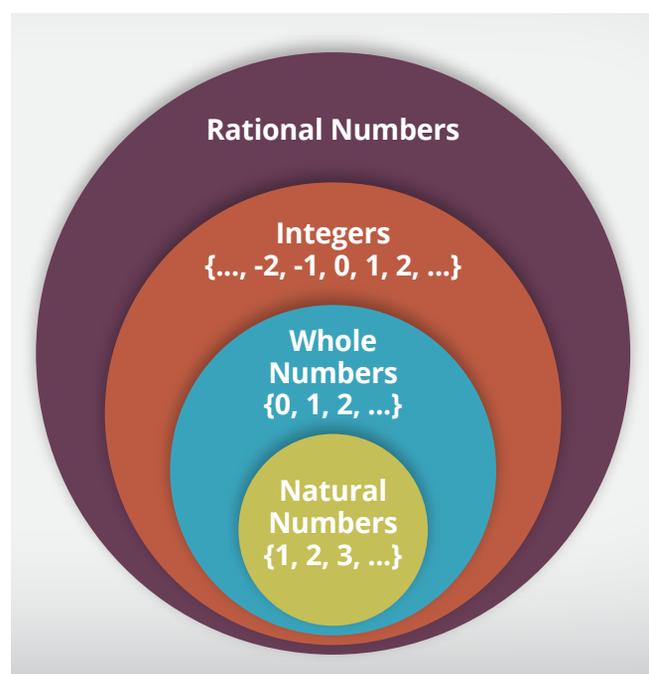
People were actually using fractions before zero or negative numbers. It made sense to people that they could have part of something. Just as with whole numbers, there may have been different symbols, but the concepts were the same. Everyone could “see” what one-half meant. The set of numbers containing all fractions is called the *rational numbers*. Look at the picture relating the sets of numbers:

---

**Key point!** On the diagram, a circle inside another circle shows that the smaller circle is a part of the larger circle. So, all natural numbers are also whole numbers and integers. And all whole numbers are also integers. It doesn't work the other way around though. Not all integers are whole numbers or natural numbers (like  $-1$ ), and not all whole numbers are natural numbers (the number  $0$ ).

---

### Rational Numbers



Rational numbers are numbers that can be written as fractions. The natural numbers, whole numbers, and integers all can be expressed as fractions by placing them over 1. So, they are all rational numbers. Notice in the rational numbers diagram that these three sets are all inside the set of rational numbers. This shows us that they are all rational numbers.

---

**Think about it!** The word rational contains the root word ratio. Ratios are often written in fraction form.

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- Natural numbers can be written as fractions:

$$\frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \dots$$

- The only whole number that is not a natural number is zero, but it can also be written as a fraction:  $\frac{0}{1}$

- Negative integers can be written as fractions:

$$\frac{-1}{1}, \frac{-2}{1}, \dots$$

- Terminating decimals can be written as fractions:

$$0.3 = \frac{3}{10}$$

$$2.07 = 2 \frac{7}{100} = \frac{207}{100}$$

- [Remember... To change a mixed number to a fraction, multiply the whole number by the denominator and add the numerator. This number becomes your new numerator.]

$$-1.9 = -1 \frac{9}{10} = -\frac{19}{10}$$


---

**Vocabulary!** The word terminating means to end. So terminating decimals are decimal numbers that have an end.

---

Most of the numbers that you have used in math so far are rational numbers.

Use the rational numbers diagram to help you classify numbers. We classify numbers by determining which sets they belong to. Remember that numbers may belong

to more than one set. Here are some examples:

- Five is a natural number, whole number, integer, and rational number. We can see this in the diagram because the circle of natural numbers is inside the circles of the whole numbers, integers, and rational numbers.

- $\frac{2}{3}$  is a rational number only.

- $-74$  is an integer and rational number.

- $-\frac{6}{3}$  is an integer and rational number
- 

**Keep in mind!** In the last example, the

fraction  $-\frac{6}{3}$  doesn't look like an integer, but it does simplify to  $-2$ . Make sure to simplify the number, if possible, before determining which sets it belongs to.

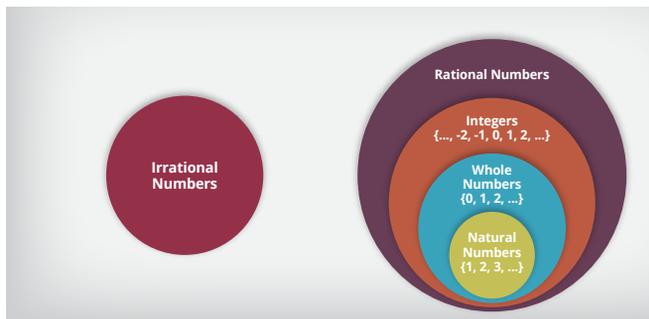
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Do you believe that even with all of these numbers, the set of real numbers is still not complete? The study of geometry introduced even more numbers that had not been necessary before. These numbers were decimal numbers that never terminated or repeated. Remember the number pi? We use the symbol  $\pi$  to represent it. Computers have written it out millions of digits and still there is no pattern to way the digits are written. Here is a little tiny bit of  $\pi$ : 3.1415926535897932384624338327...

Notice that there is no repeating pattern in  $\pi$  and that the number is *infinite* (never ends)! These special decimals that don't repeat or terminate are called *irrational numbers*. Let's look at how the irrational numbers are related to the other sets of numbers.

**Keep in mind!** The prefix “ir-” means “not.” For example, if something is irresistible, you are not able to resist it. Irrational numbers are not rational.

### Real Numbers: Irrational or Rational



Numbers are either rational or irrational. They cannot be both! Notice in the diagram that the circles representing rational and irrational numbers do not overlap or touch each other in any way.

**Make note!** It is a good idea to copy this diagram into your notes, since it is a nice summary of the sets of numbers and how they are related. You'll want to have your notes handy when you are working on the problems and studying for your quiz and test.

There are three types of decimal numbers. Use the following examples to help you

remember whether a decimal number is rational or irrational.

The decimal number is:	Examples	Type
terminating	0.25, 13.3457	rational
repeating	$0.\overline{25}$ , $1.\overline{3}$ , $-74.\overline{125}$	rational
NOT terminating or repeating	713.6925419927...	irrational

The Real Numbers: Irrational or Rational diagram shows the entire real number system as we know it today. The real number system includes any number that can be written as an infinite decimal and represents all of the numbers that you are familiar with. This number system evolved as people needed to express number concepts in consistent ways.

### Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson.

- The real number system can be divided into sets of numbers.
- A number may belong to more than one set of numbers.
- A real number is either rational or irrational, but not both.



**Complete the following.**

1.1 Which of the following is an irrational number?

- $\pi$ 
                         
   $-\frac{16}{3}$ 
                         
   $2.\bar{6}$ 
                         
  18

1.2 The display of a student's calculator shows: 1.2731568 The student is most likely looking at \_\_\_\_.

- a natural number
  a whole number  
 an integer
  an irrational number

1.3 The number  $-5$  is all of the following *except* \_\_\_\_.

- an integer
  a whole number  
 a rational number
  a real number

1.4 If a number is a whole number, then it *cannot* be \_\_\_\_.

- an irrational number
  an integer  
 a natural number
  a rational number

1.5 Luis starts to do a division problem and notices that there is a pattern in the digits to the right of the decimal point. This number is \_\_\_\_.

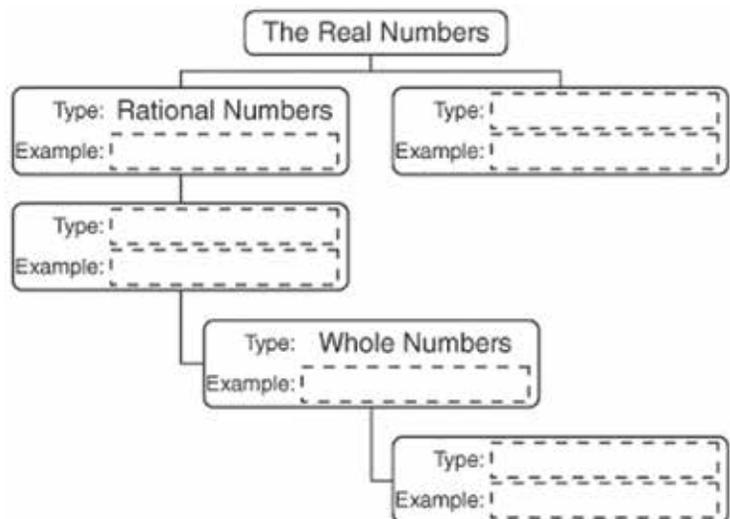
- irrational
  rational
  an integer
  natural

1.6 All of the following are rational *except* \_\_\_\_.

- 0
   $-\frac{2}{5}$ 
  $0.\overline{125}$ 
 3.14159...

1.7 Use the choices to complete the Real Numbers chart.

- $\frac{1}{3}$
- Irrational Numbers
- 1.175136981
- Integers
- $-5$
- 0
- Natural Numbers
- 56



## USING VARIABLES

What if someone sent you a message that looked like this: Δ@\ 1/> #@>/> ~>##>/7 &\$ 51#@?

How would you know what it says? What must you have in order to figure out the meaning of the message? A key! You would need to know what each number or symbol

stands for. Otherwise, the message is meaningless to you. Here is the key to this code:

Decode the message to reveal a question. You can find the answer to the question in your lesson!

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	<	+	2	>	!	3	@	&	4	=	~	5	\$	%	6	*	/	7	#	÷	8	Δ	∅	\	9

### Objectives

- Identify a variable, term, or expression.
- Use substitution to simplify expressions and formulas.

### Vocabulary

**constant**—a number; a term containing no variables

**expression**—one term or multiple terms connected by an addition or subtraction sign

**formula**—an expression that uses variables to state a rule

**term**—a number or a variable, or the product of a number and variable(s)

**variable**—a letter or symbol used to represent an unknown number

### Using Letters to Represent Numbers

Did you decode the message from the beginning of the lesson? It asked, “Why are there letters in math?” Let’s find out why.

Algebra is kind of like a secret code that you have to work out. And, as you learn more and more parts of the key, you’ll be able to solve harder and harder codes! But first, you’ll need to learn a few of the words that you’ll be seeing a lot from now on.

Just like the given code used symbols to represent letters, algebra uses letters to represent numbers. These letters stand for numbers that are unknown and are called *variables*, because they can change.

Numbers always represent themselves, so they are called *constants*.

---

**Vocabulary!** The word *variable* describes something that is able to change. The word *constant* describes something that never changes or always stays the same.

---

We can also combine variables and constants. When we combine them using multiplication or division, the result is called a *term*. Here are some examples of terms:

$$7x, abc, \frac{x}{2}, \frac{3y}{5}, r, 8$$

A term can be just a number, just a variable, or any combination of numbers and variables that uses multiplication or division.

In looking at the examples of a term, however, you may be wondering why there are no multiplication symbols. As you start using variables in math, it becomes very easy to confuse the letter ( $x$ ) with the multiplication symbol ( $\times$ ). So, as a convention in math, we use a raised dot ( $\cdot$ ), parentheses, or write variables side-by-side to indicate multiplication between two numbers. For example,  $7x$  means “7 times  $x$ ,” and  $abc$  means “ $a$  times  $b$  times  $c$ .”

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**Key point!** Letters written side-by-side, or a number and letters written side-by-side, indicate multiplication.

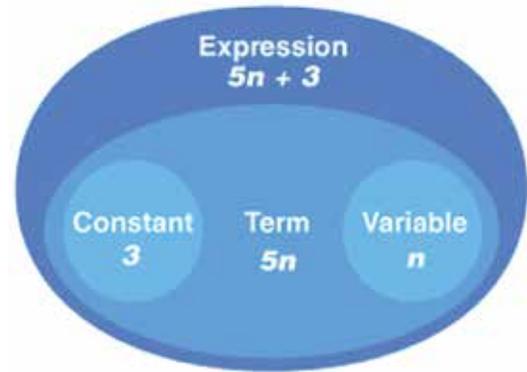
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An *expression* is one term or a combination of multiple terms using an addition or subtraction sign. Here are some examples of expressions:

$$3x + 5, x - 11, \frac{1}{2}ab + 8 - y, \frac{a+2}{5}, s, 9, -2mn$$

Notice that even a single term is considered an expression, not just multiple terms.

Did you notice how the vocabulary words we just looked at seemed to build on each other? Starting with constants and variables, we add operations to show relationships and to form terms and expressions. The Expression diagram will help you visualize this.




---

**This might help!** The diagram shows how the vocabulary words are related. Constants and variables are types of terms. And, you can combine constants, variables, and terms to make expressions.

---

You may be wondering, “What is an expression good for?” Well, expressions show a relationship between ideas. For example, the expression,  $5n + 3$ , could represent the following scenario:

- *To park in a downtown parking garage, it costs a flat fee of \$3, plus an additional \$5 per hour.*

If you know the number of hours a person parked, you could evaluate what their total cost would be.

---

**Vocabulary!** Evaluating an expression means to find the numerical answer for an expression.

---

For now, you will be given expressions and asked to evaluate them. Later on, you’ll be able to determine the relationships and write the expressions yourself!

Evaluating an expression is very simple. The first step is to replace the appropriate variables with any known values. After that,

you just need to perform the computation. Let's look at a couple of examples.

### Example:

- ▶ Evaluate the expression:  
 $m \div 4$ , if  $m = 68$ .

### Solution:

- $m \div 4$  Original equation.
- $68 \div 4$  Replacing the variable with a known value, substitute  $m = 68$ .
- 17 Perform the division.

### Example:

- ▶ Evaluate the expression:  $abc$ , if  $a = 3$ ,  $b = 4$ , and  $c = 6$ .

### Solution:

- $abc$  Original equation.
- $3 \cdot 4 \cdot 6$  Replace the variables with their known values.
- 72 Perform the multiplication.

One special type of expression is a *formula*. A formula uses variables to state a commonly known or frequently used rule. For example, to find the area of a rectangle, the rule is to multiply the length of the rectangle by the width of the rectangle. So, the formula is  $A = lw$ . Formulas use logical variables to stand for the different parts, such as the first letter of what they represent. In this case,  $A$  stands for Area,  $l$  for length, and  $w$  for width. Here are some other common formulas:

<b>Distance</b>	$d = rt$	$d$ = distance; $r$ = rate; $t$ = time
<b>Simple Interest</b>	$i = prt$	$i$ = interest; $p$ = principal; $r$ = rate; $t$ = time
<b>Area of a Triangle</b>	$A = \frac{1}{2}bh$	$A$ = area; $b$ = base; $h$ = height
<b>Volume</b>	$V = Bh$	$V$ = volume; $B$ = area of the base; $h$ = height
<b>Circumference of a Circle</b>	$C = 2\pi r$	$C$ = circumference; $r$ = radius

Evaluating formulas is done the same way as evaluating other expressions. Simply substitute any given values in for the correct variables and complete the computation!

### Example:

- ▶ Find the distance a man traveled, if his rate ( $r$ ) was 50 miles per hour, and his time traveled ( $t$ ) was 3 hours.
- ▶ Use the formula  $d = rt$ .

### Solution:

- $d = rt$  Formula for distance traveled.
- $d = (50)(3)$  Replace variables with known values.
- $d = 150$  Perform the multiplication.
- ▶ Answer: The man traveled 150 miles.

### Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson.

- Constants, variables, and operations are used to form terms and expressions.
- Formulas use variables to state a commonly known rule.
- Expressions and formulas can be evaluated using substitution.

# SELF TEST 1: Relationships

Complete the following activities (6 points, each numbered activity).

**1.01** Select all of the symbols that would make the comparison true.  $-7 \underline{\quad} -4$

  $<$   $\leq$   $=$   $>$   $\geq$   $\neq$ 

**1.02** Select all of the symbols that would make the comparison true.  $2.5 \underline{\quad} 2.05$

  $<$   $\leq$   $=$   $>$   $\geq$   $\neq$ 

**1.03** Select all of the symbols that would make the comparison true.  $-|-9| \underline{\quad} -9$

  $<$   $\leq$   $=$   $>$   $\geq$   $\neq$ 

**1.04** All of the following are rational numbers *except* \_\_\_\_\_.

  $\frac{2}{3}$   $1.\overline{33}$   $-16$   $3.14159\dots$ 

**1.05** Which of the following is a true statement?

 The opposite of  $-45$  is equal to the absolute value of  $-45$ . The opposite of  $45$  is equal to the absolute value of  $-45$ . The opposite of  $45$  is equal to the absolute value of  $45$ . The opposite of  $-45$  is not equal to the absolute value of  $45$ .

**1.06** The following chart shows the times of runners in the 100 meter sprint.

Rahn	11.33 sec
Miguel	11.5 sec
Tyrone	11.09 sec
George	11.28 sec

Who won the race?

 Rahn Tyrone Miguel George

**1.07** Which of the following statements is false?

 If a number is a natural number, then it is rational. If a number is a fraction, then it is rational. If a number is a whole number, then it is rational. If a number is an integer, then it is irrational.

1.08 Which of the following numbers is between  $-\frac{3}{4}$  and  $\frac{5}{8}$ ?

- 1                     
   $-\frac{1}{2}$                      
   $\frac{7}{8}$                      
  0.9

1.09 Which of the following statements is false?

- $-|-5| = -5$                      
   $-(-5) = 5$                      
   $-|5| = -5$                      
   $|-5| = -5$

1.010 Given the statement  $-12 \leq -15$ , which of the following is correct?

- It is a true statement, because -12 is less than -15.                     
  It is a false statement, because -15 is less than -12.
- It is a false statement, because -12 is not equal to -15.                     
  It is true, because 12 is less than 15.

1.011 The distance between -7 and 2 on the number line is \_\_\_\_.

- 5                     
  -5                     
  9                     
  10

1.012  $\pi$  is an example of \_\_\_\_.

- a rational number                     
  an integer
- an irrational number                     
  a natural number

1.013 Which of the following statements is *not* true based on the given graph?



- $a \leq b$                      
   $c > 0$                      
   $|b| = c$                      
   $-a = c$

1.014 Which of the following lists is ordered from least to greatest?

- 5, 0, 0.8,  $1, 1\frac{1}{2}$                      
  -5, 0, 0.8,  $1\frac{1}{2}, 1$
- 3, -5, 0, 0.8, 1                     
   $1, 0.8, \frac{1}{2}, 0, -5$

1.015 If  $h = 12$  and  $g = 4$ , which of the following has a value of 3?

- $h - g$                      
   $\frac{h}{g}$                      
   $h \div 3$                      
   $g + 1$

**1.016**  $X$  is all of the following *except* \_\_\_\_\_.

a constant

a term

a variable

an expression

**1.017** Evaluate the formula  $V = Bh$ , if  $B = 24$  and  $h = 6$ .

$V = 4$

$V = 12$

$V = 30$

$V = 144$

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102							
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