

# CHAPTER 1

## OPERATIONS WITH DECIMALS AND PERCENT

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1.1 Divisibility of Numbers

1.2 Addition and Subtraction of Decimals

1.3 Multiplication of Decimals

1.4 Division of Decimals

1.5 Percent

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## 1.1 Divisibility of Numbers

### Dividing One Number into Another

We can use certain rules to quickly determine whether or not a number is divisible by a second number. Does the second number divide evenly into the first with no remainder?

Examples:

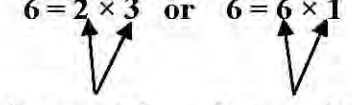
Is 10 divisible by 2? Yes, 2 divides into 10 five times with no remainder.

Is 18 divisible by 3? Yes, 3 divides into 18 six times with no remainder.

An important reason for looking at rules for divisibility is to see if one number is divisible by another without have to do too much calculation. This will be very helpful when we factor large numbers. First, let's review the **factor** of a number and then look at some **rules for divisibility**.

### Factor of a Number

Recall that when we break a number down to show it as the product of other numbers, those other numbers are called **factors**.

Example:  $6 = 2 \times 3$  or  $6 = 6 \times 1$   
  
factors of 6      factors of 6

Each of the factors divides evenly into the original number without any remainder. For example, 2 divides into 6, 3 divides into 6, 1 divides into 6, and 6 divides into 6). We say that 6 is **divisible** by 1, 2, 3, and 6.

### Divisibility of a Number

If a number divides evenly into a second number, we say the second number is **divisible** by the first number. But we also can say that the first number is a **factor** of the second.

Example: 5 divides evenly into 10, so 10 is divisible by 5 and 5 is a factor of 10.

Note that every whole number is divisible by 1 and itself.

Example: 5 is divisible by 1 and 5, and 30 is divisible by 1 and 30.

We know that every even whole number is also divisible by 2.

Example: 4 is divisible by 2 as well as 1 and 4, 22 is divisible by 2 as well as 1 and 22.

### Division by Zero

When we divide one number by another we could think of dividing it into equal parts. For example, if we divide 10 by 5, we can think of dividing 10 into two equal parts of 5 (or two parts of 5 each). Or if we divide 10 by 1 there is one part of 10.

What if we try to divide by 0? If we divide 10 by 0, how many parts of 0 are there? Or how many zeros would give us 10? This is not meaningful, so we say division by 0 is undefined. If you check 10 divided by 0 with a calculator, it will give you an error message. This message may look different on different calculators, but many will show it as a symbol looking like - E -.

### Divisibility Rules for 2, 3, 4, 5, 6, 8, 9, and 10

The following rules let you test if one number can be evenly divided by another, without having to do too much calculation.

A number is divisible by:	If the following is true:	Example:
2	The last digit is even (0, 2, 4, 6, 8)	342 is divisible by 2. 543 is not divisible by 2.
3	The sum of the digits is divisible by 3	426 ( $4 + 2 + 6 = 12$ ) and ( $12 \div 3 = 4$ ). <b>Yes</b> , it is divisible by 3. 322 ( $3 + 2 + 2 = 7$ ) and ( $7 \div 3 = 2\frac{1}{3}$ ). <b>No</b> , it is not divisible by 3.
4	The last 2 digits are divisible by 4	2108 ( $08 = 8$ ) and ( $8 \div 4 = 2$ ). <b>Yes</b> , 2108 is divisible by 4. 2627 ( $27 \div 4 = 2.25$ ). <b>No</b> , it is not.
5	The last digit is 0 or 5	175 is (it ends in a 5). 809 is not divisible by 5. It doesn't end in either a 5 or a 0.
6	The number is divisible by both 2 and 3	252 (it is even so it is divisible by 2, and $2 + 5 + 2 = 9$ , and $9 \div 3 = 3$ ). <b>Yes</b> 136 (it is even so it is divisible by 2, but $1 + 3 + 6 = 10$ , which is not divisible by 3). <b>No</b>
8	The last three digits are divisible by 8	5928 ( $928 \div 8 = 116$ ) <b>Yes</b> 6660 ( $660 \div 8 = 82.5$ ) <b>No</b>

<b>9</b>	The sum of all the digits are divisible by 9	<b>4932</b> ( $4 + 9 + 3 + 2 = 18$ ) and $18 \div 9 = 2$ . <b>Yes</b> <b>4192</b> ( $4 + 1 + 9 + 2 = 16$ ) and $16 \div 9 = 1\frac{7}{9}$ . <b>No</b>
<b>10</b>	The number ends in 0	<b>4560</b> <b>Yes</b> , it ends in zero. <b>4312</b> <b>No</b> , it doesn't end in zero.

### Examples with Solutions

- Which of the following numbers are divisible by 5?

**8, 15, 30, 48, 65**

According to the rule, a number must end in either a 5 or a 0 to be divisible by 5. The numbers are 15, 30, and 65.
- Which of the following numbers are divisible by 3?

**53, 159, 305, 2061**

According to the rule, a number is divisible by 3 if the sum of the digits in the number is divisible by 3. The numbers are 159 and 2061.
- Which of the following numbers are divisible by 6?

**2243, 4542, 3152, 3144**

According to the rule, a number is divisible by 6 if it is divisible by both 2 and 3 (It is even and the sum of digits is divisible by 3).  
4542, 3152 and 3144 are even so they are divisible by 2. Only 4542 and 3144 are also divisible by 3, since the sum of their digits is divisible by 3. The numbers are 4542 and 3144.
- Which of the following numbers are divisible by 8?

**413, 2336, 4033, 56728**

According to the rule, a number is divisible by 8 if the number formed by the last 3 digits is divisible by 8. The numbers are 2336 and 56728.
- If the following number is divisible by 9, what number must the last digit be equal to?

**456?**

According to the rule, a number is divisible by 9 if the sum of the digits in the number is divisible by 9. The number must be 3 since  $(4 + 5 + 6 + \underline{3} = 18$ , which is divisible by 9).

6. If the following number is divisible by 8, what digits could the last digit be equal to?

340?

According to the rule, a number is divisible by 8 if the number formed by the last 3 digits is divisible by 8. The digits could be 0 or 8 (giving 3400 and 3408 since 400 and 408 are both divisible by 8).

### Representing Divisors in Different Ways

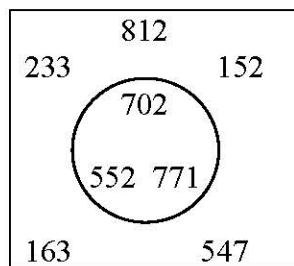
We could organize divisors of a number (or what a number is divisible by) using different types of graphs, such as **Venn diagrams** and **Carroll diagrams**.

#### Venn Diagram

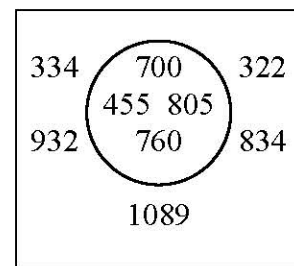
A **Venn diagram** is a picture of what numbers belong and what numbers do not belong to a set with certain properties. It helps to clarify what numbers do belong and what numbers do not. We usually show all of the numbers that we could be interested in inside of a rectangle (called the universe), and then we draw a circle around those that have the properties we are after.

Examples:

- The following numbers that are divisible by 3: 233, 152, 163, 702, 552, 812, 771, 547
- The following numbers that are divisible by 5: 334, 700, 455, 322, 932, 805, 760, 1089, 834



In the Venn diagram, numbers divisible by 3 are in the circle and those that aren't divisible by 3 are in the rectangle (universe) but not in the circle.



In the Venn diagram, numbers divisible by 5 are in the circle and those that aren't divisible by 5 are in the rectangle (universe) but not in the circle.

### Carroll Diagram

A **Carroll diagram** displays “yes/no” information in a table form.

Examples:

- Which of the following numbers are divisible by 6?  
345, 1062, 432, 1209, 531, 644

Recall that to be divisible by 6 a number must be even (divisible by 2) and the sum of its digits must be divisible by 3.

	Yes	No
<b>Even</b>	1062, 432, 644	345, 1209, 531
<b>Sum of Digits divisible by 3</b>	1062, 432, 345, 531, 1209	644
<b>Both Even and Divisible by 3</b>	1062, 432	

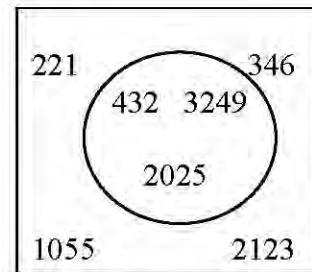
Numbers that are both even and divisible by 3 are 1062 and 432.

### Examples with Solutions

- Use a Venn diagram to show each of the following sets of numbers.

- Numbers that are divisible by 9 from the following numbers.

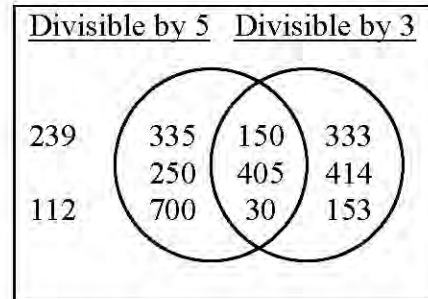
**432, 221, 3249, 346**  
**1055, 2025, 2123**



Numbers are divisible by 9 if the sum of their digits is divisible by 9.

- b. Numbers that are divisible by both 5 and 3 from the following numbers.

**30, 150, 153, 250, 333, 335, 405,  
414, 700, 239, 112**



Numbers divisible by 5 end in a 5 or a 0 and numbers are divisible by 3 if the sum of their digits is divisible by 3.

The following numbers are divisible by both 5 and 3: 150, 405, 30.

Note that these numbers are located where the diagrams overlap or intersect.

2. Fill in the blanks in the Carroll diagram and determine which numbers are divisible by both 5 and 9.

**35, 70, 135, 216, 225, 430, 621, 4338**

Recall that

- If a number is divisible by 5, it ends in a 5 or a 0.
- If a number is divisible by 9, the sum of the digits is divisible by 9.

	Yes	No
<b>Divisible by 5</b>	35, 70, 135, 225, 430	216, 4338, 621
<b>Divisible by 9</b>	135, 216, 225, 4338, 621	35, 70, 430
<b>Divisible by both 5 and 9</b>	135, 225	

Numbers that are divisible by both 5 and 9 are 135 and 225.

## Exercises 1.1

Use the rules for divisibility to answer the following questions. Do not perform the actual division.

1. Which of the following numbers are divisible by 2?      2. Which of the following numbers are divisible by 5?

23, 44, 105, 112, 1012, 3700

51, 15, 35, 60, 205, 222, 1080

3. Which of the following numbers are divisible by 4?  
22, 44, 320, 105, 210, 244
4. Which of the following numbers are divisible by 3?  
33, 65, 72, 135, 220, 451, 513

5. Which of the following numbers are divisible by 10?  
25, 50, 750, 555, 2130

State whether the following questions are true or false.

6. All numbers divisible by 9 are also divisible by 3.
7. All numbers divisible by 3 are also divisible by 9.
8. All numbers divisible by 10 are also divisible by 5.
9. All numbers divisible by 4 are also divisible by 2.
10. All numbers divisible by 2 are also divisible by 4.
11. List all numbers between 230 and 241 that are divisible by 3.
12. List all numbers between 112 and 151 that are divisible by 5.
13. Find all numbers between 100 and 115 that are divisible by 6.



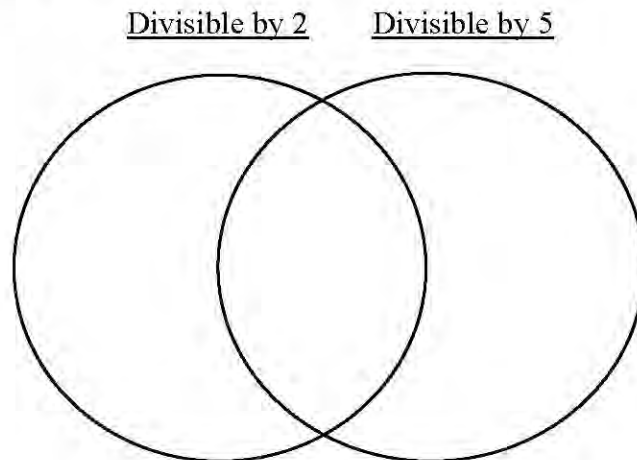
14. Fill in the blanks in the Carroll diagram and determine which of the following numbers are divisible by both 3 and 4.

**129, 612, 642, 705, 712, 828, 908, 1016**

	Yes	No
Divisible by 3		
Divisible by 4		
Divisible by both 3 and 4		

15. Fill in the Venn diagram below to show which numbers are divisible by both 5 and 2.

**4, 12, 15, 30, 45, 60, 75, 100, 105, 110, 150, 156**



### Extra for Experts

16. Find all numbers less than 50 that are divisible by 2, 3 and 4.
17. Find all numbers less than 150 that are divisible by both 5 and 6.

Find all possible values for the missing digit in the questions that follow.

18. What value(s) for the missing digit would make the number below divisible by 5?

**321?**

19. What value(s) for the missing digit would make the number below divisible by 6?

**425?**

20. What value(s) for the missing digit would make the number below divisible by 3?

**421?**

21. What value(s) for the missing digit would make the number below divisible by 8?

**2013?**

22. What value(s) for the missing digit would make the number below divisible by 5? (The missing digits are the same number.)

**1???**

# ANSWERS TO EXERCISES AND CHAPTER TESTS

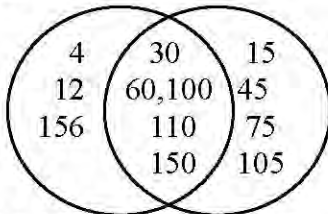
## CHAPTER 1

## Exercises 1.1 (page 7)

1. 44, 112, 1012, 3700    2. 15, 35, 60, 205, 1080  
 3. 44, 320, 244    4. 33, 72, 135, 513  
 5. 50, 750, 2130    6. True    7. False    8. True  
 9. True    10. False    11. 231, 234, 237, 240  
 12. 115, 120, 125, 130, 135, 140, 145, 150  
 13. 102, 108, 114  
 14.

	Yes	No
Divisible by 3	129, 612, 642, 705, 828	712, 908, 1016
Divisible by 4	612, 712, 828, 908, 1016	129, 642, 705
Divisible by both 3 and 4	612, 828	

15.



16. 12, 24, 36, 48    17. 30, 60, 90, 120  
 18. 5 or 0    19. 4    20. 2, 5, 8    21. 6    22. 0, 5

## Exercises 1.2 (page 14)

1. a) 162    b) 376    c) 227    d) 4053  
 2. a) 23, greater than    b) 87, less than  
 c) 2984, greater than    d) 27, less than  
 3. a) 31.3    b) 233.3    c) 3102.82    d) 2829.05  
 e) 124.7    f) 259.33    g) 14.8    h) 269.33  
 i) 41.03    j) 9042.15    k) 453.01    l) 71.871  
 4. a) 19.1    b) 129.01    c) 2084.92    d) 3225.013  
 e) 1144.1    f) 1097.944    g) 41.15    h) 427.43  
 i) 34.36    j) 2.91    k) 95.24    l) 2784.92  
 5. \$66.50    6. 4.8 m    7. \$40.25    8. \$70.95  
 9. 401.9 km    10. 222.31 kg    11. \$113.82  
 12. 6.73 m    13. 4.1 m    14. \$18.36    15. a) 7  
 b) 3    16. 80.4 and 65.2

## Exercises 1.3 (page 19)

1. a) 945    b) 200    c) 1000    d) 2210  
 2. a) 187.5    b) 100.8    c) 3662.64    d) 24 828

3. a) 84.9    b) 173.02    c) 116.61    d) 100.232  
 e) 130.231    f) 1126.5    g) 42.813    h) 16 412.3  
 i) 21.606    j) 100.835    k) 260.48    l) 16.443  
 4. 301 km    5. 21.4 m    6. \$6.45    7. \$43.75  
 8. 18.24 kg    9. 12.1 kg    10. 102.6 kg    11. 2.5  
 12. \$8.50    13. 650 g    14. \$2.70    15. \$16.70

## Exercises 1.4 (page 23)

1. a) 7    b) 11    c) 13    d) 3    2. a) 21.5    b) 52  
 c) 3.6    d) 5.6    e) 2.5    f) 4.2    g) 5.6    h) 74  
 i) 70.4    j) 24.5    k) 55.5    l) 80.2    3. 2.225  
 4. 334.156 25    5. 140.3 g    6. 40    7. 27.3 cm  
 8. \$10.50    9. 0.625 kg    10. 8 L    11. 38.79 kg  
 12. \$53.15    13. \$29.75    14. 6 and 7    15. 19  
 16. 5.6    17. 2.3

## Exercises 1.5a (page 28)

1. a)  $\frac{16}{100}$     b)  $\frac{37}{100}$     c)  $\frac{4}{100}$     d)  $\frac{98}{100}$     e)  $\frac{3}{100}$   
 f)  $\frac{0.7}{100}$  or  $\frac{7}{1000}$     g)  $\frac{150}{100}$     h)  $\frac{233}{100}$     i)  $\frac{322}{100}$   
 j)  $\frac{15.5}{100}$  or  $\frac{155}{1000}$     k)  $\frac{4.6}{100}$  or  $\frac{46}{1000}$     l)  $\frac{10.7}{100}$  or  $\frac{107}{1000}$   
 2. a) 3%    b) 14%    c) 76%    d) 8%    e) 104%  
 f) 123%    g) 367%    h) 1.5%    i) 12.3%  
 j) 56.5%    k) 0.6%    l) 10.7%    3. a) 0.21  
 b) 0.09    c) 0.67    d) 0.01    e) 1.75    f) 1.4  
 g) 0.014    h) 0.075    i) 0.013    4. a) 9%    b) 11%  
 c) 133%    d) 104%    e) 12.5%    f) 8.6%  
 g) 175%    h) 45%    i) 218%

## Exercises 1.5b (page 31)

1. a) 25    b) 80    c) 4.5    d) 45    e) 63    f) 90  
 g) 112.5    h) 187.5    i) 218.4    j) 216    k) 227.5  
 l) 22.4    2. 84    3. 4.2    4. 100    5. 216    6. 72  
 7. \$2.17    8. \$76.30    9. \$113    10. 525  
 11. \$812.50    12. \$90    13. 24    14. \$167.50  
 15. \$11 475    16. 64    17. 275

## Exercises 1.5c (page 35)

1. a) 52.5    b) 6.75    c) 24    d) 150    e) 210  
 f) 20%    g) 25%    h) 40    2. 75%    3. \$468.75  
 4. \$375    5. \$40    6. 15%    7. \$166    8. \$8250  
 9. 250    10. 36    11. 800 kg

## Extra Practice – Chapter 1 (page 37)

1. a) 32, 84, 90, 110    b) 85, 90, 110, 2105, 4200  
 c) 644, 812, 1024    d) 36, 87, 225, 1011