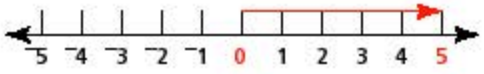
CHAPTER 1
ABSOLUTE VALUE AND INTRO TO RADICALS

1.1 Absolute Value of a Number and the Number Line

The **absolute value** of a number can be thought of as the distance from 0 to the number on a number line. The direction doesn't matter and, as a result, the absolute value of a number is never negative.

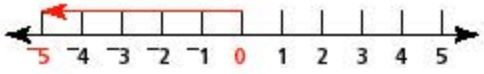
Examples:

The absolute value of 5 is 5.



Note: 5 is 5 units to the right of 0.

The absolute value of -5 is 5.



Note: -5 is 5 units to the left of 0.

The symbol for absolute value consists of two vertical line segments that appear on either side of the number or expression.

Examples:

1. $|5|$ means the absolute value of 5, which is equal to 5.
2. $|-5|$means the absolute value of -5, which is equal to 5.
3. $\left|\frac{1}{2}\right| = \frac{1}{2}$ and $\left|-\frac{1}{2}\right| = \frac{1}{2}$
4. $|0.27| = 0.27$ and $|-0.27| = 0.27$

Keep in mind the following rules.

- The absolute value of a positive number or 0 is the number.
- The absolute value of a negative number is its opposite.

OR

If x is any real number, $|x| = x$ if $x \geq 0$ ($|7| = 7$) **and** $|x| = -x$ if $x < 0$ ($|-7| = -(-7) = 7$)

The absolute value of zero is equal to "0." Why? Just ask yourself: how far is zero from 0.
Answer: zero units. So $|0| = 0$.

We can simplify single absolute value terms containing more than one numerical expression inside the symbol by working inside of it first.

Examples:

(i) $|2-5| = |-3| = 3$

(ii) $|7-8| = |-1| = 1$

(iii) $|12-8| = |4| = 4$

(iv) $|1.2-3| = |-1.8| = 1.8$

Examples with Solutions

Simplify each of the following.

1. $|9-3|$ $|9-3| = |6| = 6$
2. $|4.5-5|$ $|4.5-5| = |-0.5| = 0.5$
3. $|\sqrt{2}-3\sqrt{2}|$ $|\sqrt{2}-3\sqrt{2}| = |-2\sqrt{2}| = 2\sqrt{2}$
4. $\left|\frac{1}{2}-\frac{3}{4}\right|$ $\left|\frac{1}{2}-\frac{3}{4}\right| = \left|-\frac{1}{4}\right| = \frac{1}{4}$
5. $|1--5.5|$ $|1--5.5| = |1+5.5| = |6.5| = 6.5$

Order of Operations (Review)

There is an order of operations with numbers that we will use to simplify expressions with more than one term. When there are several operations needed to simplify an expression, a special order of operations needs to be done. Following are 4 levels to be done in order, beginning with level 1. Remember **BEDMAS:**

1. Level 1 **B** (brackets) ← If more than one set, innermost first
2. Level 2 **E** (exponents)
3. Level 3 **D** (divide) ← Same level, do left to right
   M (multiply)
4. Level 4 **A** (add) ← Same level order doesn't matter
   S (subtract)

If there are no brackets (parentheses), perform all multiplication and division in the order they appear from left to right, before any addition or subtraction.

Examples:

1. $2 + \underline{3 \times 5} + 6 = 2 + \underline{15} + 6 = 23$
2. $15 + \underline{9 \div 3} + \underline{2 \times 4} = 15 + \underline{3} + \underline{8} = 26$

If there are brackets (parentheses) perform all operations inside the brackets before any other operations.

Examples:

1. $\underline{(5+6)} \times 3 = \underline{11} \times 3 = 33$
2. $7 + \underline{(8-2)} \times 4 = 7 + \underline{6} \times 4 = 7 + 24 = 31$
3. $12 \div (2+4) + 7 = \underline{12 \div 6} + 7 = 2 + 7 = 9$
4. $12 \div 2 + (4+7) = \underline{12 \div 2} + 11 = 6 + 11 = 17$

The same rules apply when we are working with terms containing **absolute value**.

Examples:

1. $|-2+3\times5| = |-2+15| = |13| = 13$
2. $7-\left|\frac{1}{2}-1\right| = 7-\left|-\frac{1}{2}\right| = 7-\frac{1}{2} = 6\frac{1}{2}$

Examples with Solutions

1. Simplify.

$2 + 3 \times 4$

Since there are no brackets, perform the multiplication first.

$2 + \underline{3 \times 4} = 2 + 12$

Now add. $2 + 12 = 14$

2. Simplify.

$(2 + 3) \times 4$

Perform operations inside the brackets first.

$\underline{(2 + 3)} \times 4 = 5 \times 4$

Now multiply. $5 \times 4 = 20$

3. Simplify.

$32 \div 4 - (3 \times 2)$

Perform operations inside of brackets first.

$32 \div 4 - \underline{(3 \times 2)} = 32 \div 4 - 6$

Now perform division before adding or subtracting. $\underline{32 \div 4} - 6 = 8 - 6$

Now subtract. $8 - 6 = 2$

4. Simplify.

$1.3 + 2(5 + 0.25) - 4$

Perform operations in brackets first.

$1.3 + 2(5 + 0.25) - 4 = 1.3 + 2(5.25) - 4$

Now perform multiplication before adding or subtracting. $1.3 + 10.5 - 4$

Now add and subtract to get 7.8.

5. Simplify.

$|2 - 3.5| + 2 \times 5$

Work inside of the absolute value sign first to get

$|2 - 3.5| + 2 \times 5 = |-1.5| + 2 \times 5$

Clear the absolute value sign to get

$|-1.5| + 2 \times 5 = 1.5 + 2 \times 5$

Perform multiplication before addition.

$1.5 + 2 \times 5 = 1.5 + 10$

Now perform addition. $1.5 + 10 = 11.5$

6. Simplify.

$|5 - -3.5| - 12 \div 4$

Work inside of the absolute value sign first to get:
$|5 - -3.5| - 12 \div 4 = |5 + 3.5| - 12 \div 4$

Clear the absolute value sign to get:
$|5 + 3.5| - 12 \div 4 = 8.5 - 12 \div 4$

Perform division before subtraction.
$8.5 - 12 \div 4 = 8.5 - 3$

Now perform subtraction. $8.5 - 3 = 5.5$

7. Simplify.

$|7 - 3 \times 4| - |-5|$

Work inside of the absolute value sign first (perform multiplication before subtraction).
$|7 - 3 \times 4| - |-5| = |7 - 12| - |-5|$

Clear the absolute value sign to get:
$|7 - 12| - |-5| = |-5| - |-5| = 5 - 5$

Now perform subtraction. $5 - 5 = 0$

Exercises 1.1

1. Simplify each of the following terms.

a. $|12.5 - 3|$

b. $|15 - 11.5|$

c. $|11.5 - 15|$

d. $|21.2 - 22|$

e. $\left|3\sqrt{2} - 7\sqrt{2}\right|$

f. $\left|\sqrt{27} - 5\sqrt{3}\right|$

g. $\left|\frac{3}{2} - \frac{3}{4}\right|$

h. $\left|2\frac{1}{4} - 3\frac{1}{8}\right|$

i. $|1 - -3.3|$

j. $|2 - -5.8|$

2. Use the order of operations to simplify each of the following expressions.

a. $|1 - 4.5| + 3 \times 7$

b. $\left|2 - \frac{5}{2}\right| + 4 \div 8$

c. $|2.3 - -3.5| - 1.5 \times 4$

d. $|5.7 - -10| - 5 \times 4.2$

e. $|8-2\times4|-|-3|$

f. $|2-3\times4|-|-1.5|$

g. $|-2-3\times5|$

h. $|-15.2-2\times1.5|$

i. $7-\left|\frac{1}{3}-1\right|$

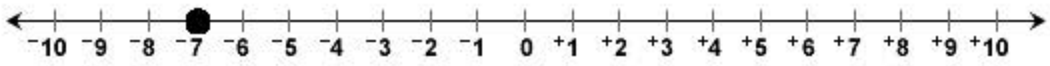
j. $5-\left|\frac{2}{3}-2\right|$

3. Answer each of the following questions.

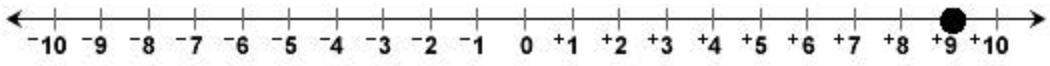
a. Write a term involving absolute value that shows a distance of 25 m below sea level.

b. Riley jumped from a plane that was at an altitude of 850 m. Show the distance he jumped from the plane using the absolute value sign.

c. A point is graphed on the number line below. Graph another point on the line that is the same distance from 0.



d. A point is graphed on the number line below. Graph another point on the line that is the same distance from 0.

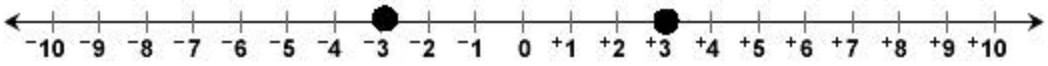


1.2 Equations and Inequalities Involving Absolute Value

When an equation includes a variable that is not equal to zero under the absolute value sign, it is equal to two different values.

Example:

If $|x| = 3$, then we know that $|3| = 3$ and that $|-3| = 3$.
$x = 3$ or $x = -3$ is called the translation of the equation $|x| = 3$.
The graph of the solution to $|x| = 3$ is shown below.

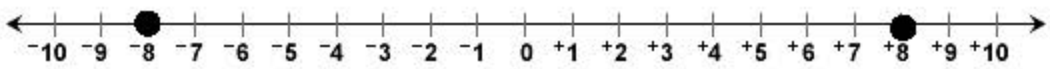


Each point in the graph above is 3 units from zero.

Example:

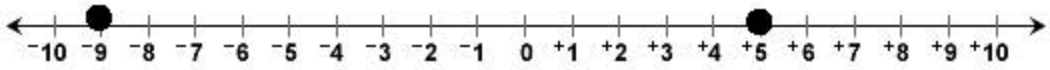
1. Equation: $|x| = 8$ — Translation and Solution: $x = 8$ or $x = -8$

Graph



2. Equation: $|x + 2| = 7$ — Translation and Solution: $x + 2 = 7$ or $x + 2 = -7 \rightarrow x = 5$ or $x = -9$

Graph



Examples with Solutions

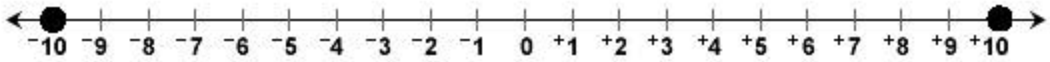
1. Write the translation and find the solution to each of the following equations.

	Equation	Translation	Solution
a.	$\lvert x\rvert = \frac{1}{3}$	$x = \frac{1}{3}$ or $x = \frac{1}{3}$	$x = \frac{1}{3}$ or $x = \frac{1}{3}$
b.	$\lvert x - 1\rvert = 5$	$x - 1 = 5$ or $x - 1 = -5$	$x = 6$ or $x = -4$

c. $|2x-1|=7$ $\quad 2x-1=7$ or $2x-1=-7$ $\quad 2x=8$ or $2x=-6$, so $x=4$ or $x=-3$

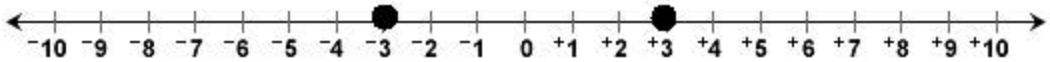
2. The graph of the solution for an absolute value equation is shown below. Write an absolute value equation for each solution.

a.



Solution: $x=10$ or $x=-10$, so $|x|=10$

b.



Solution: $x=3$ or $x=-3$, so $|x|=3$

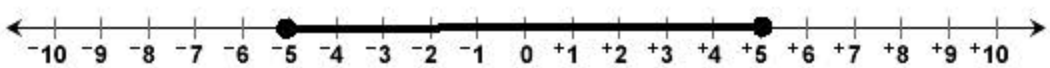
Absolute Value Inequalities

Less Than or Less Than or Equal to

We know that $|x|=5$ means that $x=5$ or $x=-5$. However, if $|x|<5$, then x lies between -5 and 5. This translates to $x>-5$ AND $x<5$. It can be written as $-5<x<5$. The graph of the solution would look like the following:

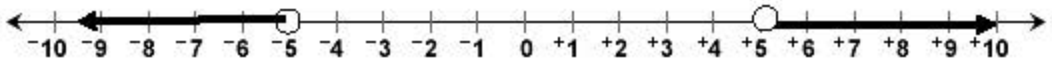
Notice that the endpoints are not part of the solution.

If $|x|\leq 5$ then x lies between -5 and 5 inclusive, this translates to $x\geq -5$ AND $x\leq 5$. It can be written as $-5\leq x\leq 5$. The graph of this solution would look like the following:



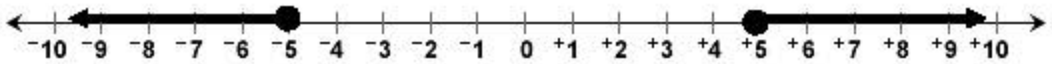
Greater Than or Greater Than or Equal to

If the absolute value inequality were $|x| > 5$, then x is greater than 5 or it is less than -5, this translates into $x < -5$ OR $x > 5$. The graph would look like the following:



As shown, the endpoints for each ray are not part of the solution.

If the absolute value inequality were $|x| \geq 5$, then x is greater than or equal to 5 or it is less than or equal to -5, this translates into $x \geq 5$ OR $x \leq -5$. The graph would look like the following:



Examples with Solutions

1. Write the translation for each of the following inequalities.

   a. $|x| \leq 32$ $\quad -32 \leq x \leq 32$

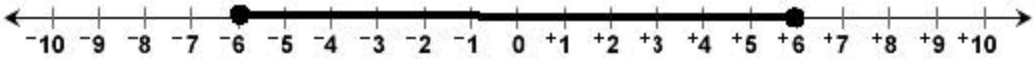
   b. $|x| < \frac{1}{7}$ $\quad -\frac{1}{7} < x < \frac{1}{7}$

   c. $|x| > 3.2$ $\quad x < -3.2$ or $x > 3.2$

   d. $|x| \geq 9$ $\quad x \leq -9$ or $x \geq 9$

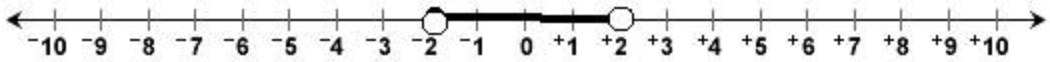
2. Write the translation and inequality for each of the following graphs.

   a.



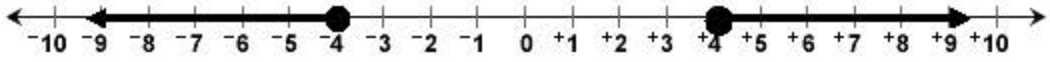
   Translation: $x \leq 6$ and $x \geq -6 \rightarrow -6 \leq x \leq 6$
   Inequality: $|x| \leq 6$

b.



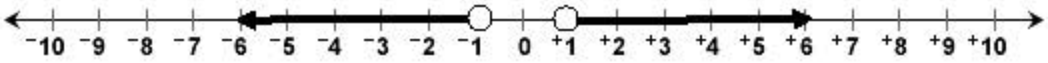
Translation: $x < 2$ and $x > -2 \rightarrow -2 \leq x \geq 2$
Inequality: $|x| < 2$

c.



Translation: $x \geq 4$ or $x \leq -4$
Inequality: $|x| \geq 4$

d.



Translation: $x < -1$ or $x > 1$
Inequality: $|x| > 1$

Exercises 1.2

1. Write the solution for each equation.

a. $|x| = 2.3$

b. $|x| = 21.5$

c. $|x| = \frac{3}{2}$

d. $|x| = \frac{13}{5}$

e. $|x - 1| = 2$

f. $|x + 3| = 7$

g. $\left|\frac{1}{2}x - 1\right| = 2$

h. $\left|\frac{2}{3}x + 1\right| = 2$

i. $|2x-3|=5$

j. $|3x-1|=8$

2. Translate each of the following into an equivalent sentence without the absolute value sign.

a. $|x|<\frac{4}{5}$

b. $|x|<2.7$

c. $|x|\leq\frac{1}{2}$

d. $|x|\leq 1\frac{1}{2}$

e. $|2x|<6$

f. $\left|\frac{1}{2}x\right|<\frac{1}{2}$

g. $|2x+1|<2.5$

h. $|x-3|<3$

i. $|x|\geq 3$

j. $|x|\geq 5.5$

k. $|x|>1$

l. $|x|>\frac{3}{4}$

m. $|2x|>6$

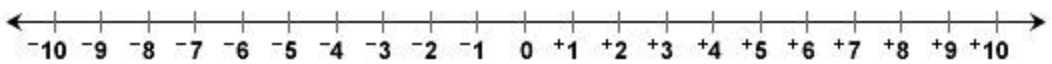
n. $\left|\frac{1}{3}x\right|>2.1$

o. $|x+1|>9$

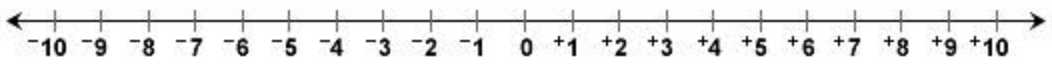
p. $|2x-1|>10$

3. Graph the solution of each of the following on the number line.

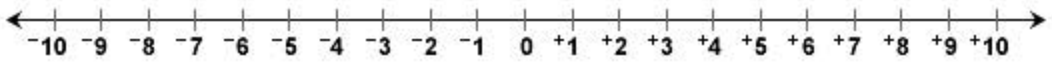
a. $|x|=3$



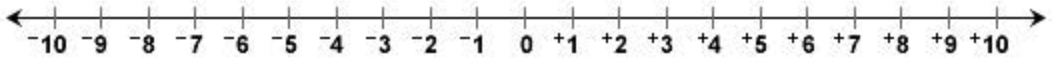
b. $|3x|=1$



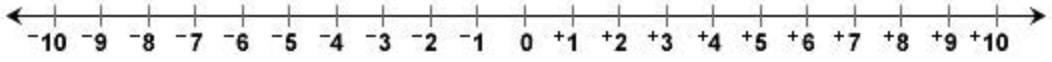
c. $|2x + 1| = 7$



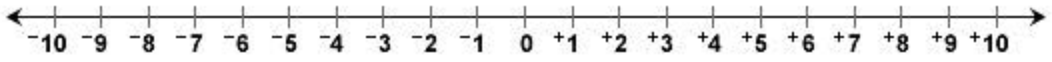
d. $|x| < \frac{1}{2}$



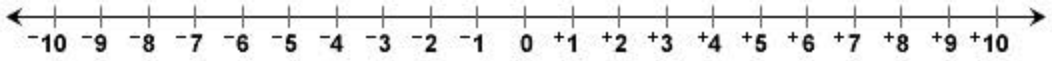
e. $|x| > 3$



f. $|x - 2| \leq 1$



g. $\left|\frac{1}{2}x + 1\right| \geq 2$



4. What is the solution for each of the following absolute value equations?

a. The absolute value of x is equal to 9.

b. The absolute value of two times x is equal to 11.

c. The absolute value of one half x is equal to 9.

d. The absolute value of (x plus two) is equal to 8.6.

e. The absolute value of (two times x minus one) is equal to 9.

5. Translate each of the following into an equivalent sentence without an absolute value sign.

   a. The absolute value of x is less than 5.

   b. The absolute value of x is greater than one half.

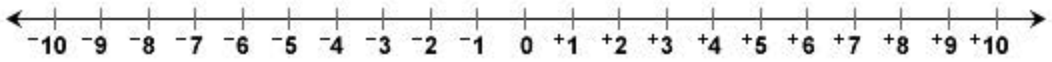
   c. The absolute value of two times x is less than 12.

   d. The absolute value of (x plus two) is greater than 9.5.

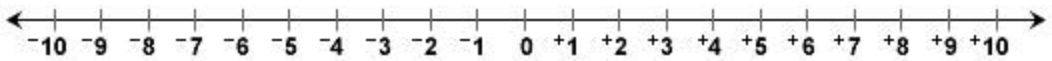
   e. The absolute value of (two times x minus one) is less than 1.

6. Graph the solution for each of the following absolute value inequalities on the number line below.

   a. The absolute value of (two times x plus one) is equal to seven.



   b. The absolute value of one third times x is less than 1.



ANSWERS TO EXERCISES AND CHAPTER TESTS

CHAPTER 1

Exercises 1.1 (page 5)

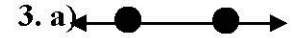
1. a) 9.5 **b)** 3.5 **c)** 3.5 **d)** 0.8 **e)** $4\sqrt{2}$ **f)** $2\sqrt{3}$ **g)** $\frac{3}{4}$ **h)** $\frac{7}{8}$ **i)** 4.3 **j)** 7.8 **2. a)** 24.5 **b)** 1 **c)** -0.2 **d)** -5.3 **e)** -3 **f)** 8.5 **g)** 17 **h)** 18.2 **i)** $6\frac{1}{3}$ **j)** $3\frac{2}{3}$ **3. a)** $|-25|$ **b)** $|-850|$

c) 

d) 

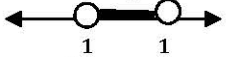
Exercises 1.2 (page 10)

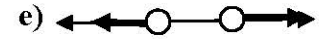
1. a) -2.3, 2.3 **b)** -21.5, 21.5 **c)** $-\frac{3}{2}, \frac{3}{2}$ **d)** $-\frac{13}{5}, \frac{13}{5}$ **e)** -1, 3 **f)** -10, 4 **g)** -2, 6 **h)** $-\frac{9}{2}, \frac{3}{2}$ **i)** -1, 4 **j)** $-\frac{7}{3}$, 3 **2. a)** $-\frac{4}{5} < x < \frac{4}{5}$ **b)** $-2.7 < x < 2.7$ **c)** $-\frac{1}{2} \leq x \leq \frac{1}{2}$ **d)** $-1\frac{1}{2} \leq x \leq 1\frac{1}{2}$ **e)** $-6 < 2x < 6$ **f)** $-\frac{1}{2} < \frac{1}{2}x < \frac{1}{2}$ **g)** $-2.5 < 2x + 1 < 2.5$ **h)** $-3 < x - 3 < 3$ **i)** $x \geq 3$ or $x \leq -3$ **j)** $x \geq 5.5$ or $x \leq -5.5$ **k)** $x > 1$ or $x < -1$ **l)** $x > \frac{3}{4}$ or $x < -\frac{3}{4}$ **m)** $2x > 6$ or $2x < -6$ **n)** $\frac{1}{3}x > 2.1$ or $\frac{1}{3}x < -2.1$ **o)** $x + 1 > 9$ or $x + 1 < -9$ **p)** $2x - 1 > 10$ or $2x + 1 < -10$

3. a) 

b) 

c) 

d) 

e) 

f) 

g) 

4. a) -9, 9 **b)** -5.5, 5.5 **c)** -18, 18 **d)** -10.6, 6.6 **e)** -4, 5 **5. a)** $-5 < x < 5$ **b)** $x > \frac{1}{2}$ or $x < -\frac{1}{2}$ **c)** $-12 < 2x < 12$ **d)** $x + 2 > 9.5$ or $x + 2 < -9.5$ **e)** $-1 < 2x - 1 < 1$

6. a) 

b) 

Exercises 1.3 (page 16)

1. a) index = 3; radicand = 27 **b)** index = 2; radicand = $\frac{4}{9}$ **c)** index = 4; radicand = 625 **d)** index = 2; radicand = $\frac{4}{121}$ **e)** index = 3; radicand = 3.375 **f)** index = 5; radicand = 0.0032 **2. a)** $\sqrt{81}$ **b)** $\sqrt[3]{216}$ **c)** $\sqrt[4]{625}$ **d)** $\sqrt{0.09}$ **e)** $\sqrt[5]{32}$ **f)** $\sqrt[3]{\frac{27}{125}}$ **3. a)** 243 **b)** 0.000027 **c)** $\frac{8}{27}$ **d)** $\frac{81}{4}$ **e)** 0.00032 **f)** 0.00000081 **4. a)** 3 **b)** $\frac{2}{3}$ **c)** 5 **d)** $\frac{2}{11}$ **e)** 0.1 **f)** 0.2 **g)** $\frac{1}{3}$ **h)** $\frac{2}{5}$ **i)** 12 **j)** 4 **k)** 0.3 **l)** 1 **5.** 4 **6.** $\frac{2}{3}$ **7.** $\frac{2}{3}$ **8.** $\frac{2}{3}$

Exercises 1.4 (page 20)

1. a) $\sqrt{5}, 3, \sqrt{11}, 4$ **b)** $\sqrt{3}, \sqrt{3.5}, 2, 3$ **c)** $\sqrt{5}, \sqrt{6}, 2.5, 4$ **d)** $\sqrt{\frac{1}{8}}, \frac{1}{2}, \sqrt{2}, 2$ **e)** $\sqrt[3]{25}, 3, \sqrt[3]{60}, 4$ **f)** $1, \sqrt[3]{2}, \sqrt[3]{7}, 2$ **2. a)** 3.16 **b)** 0.71 **c)** 10.95 **d)** 2.41 **e)** 7.35 **f)** -5.48 **g)** 0.71 **h)** 4.33 **3. a)** 2.76 **b)** 4.82 **c)** 1.74 **d)** 4.65 **e)** 6.67 **f)** 14.12 **g)** 0.63 **h)** -5.90 **4. a)** $2, \sqrt{5}, 2\sqrt{3}$ **b)** $3, 2\sqrt{3}, 3\sqrt{2}, 5$

Exercises 1.5 (page 23)

1. a) $2\sqrt{6}$ **b)** $5\sqrt{2}$ **c)** $5\sqrt{3}$ **d)** $7\sqrt{3}$ **e)** $3\sqrt{7}$ **f)** $2\sqrt{11}$ **g)** $3\sqrt[3]{2}$ **h)** $2\sqrt[3]{5}$ **i)** $5\sqrt[3]{2}$ **j)** $3\sqrt[3]{5}$ **2. a)** $2a\sqrt{3}$ **b)** $ab\sqrt{2b}$ **c)** $9b^2\sqrt{a}$ **d)** $xy\sqrt{6x}$ **e)** $\frac{1}{2}ab\sqrt{a}$ **f)** $abc\sqrt{abc}$ **g)** $2x\sqrt[3]{5}$ **h)** $3y\sqrt[3]{y}$ **i)** $2a\sqrt[3]{2a}$ **j)** $abc\sqrt[3]{2}$ **3. a)** $\sqrt{45}$ **b)** $\sqrt{12n^3}$ **c)** $\sqrt{98ab}$ **d)** $\sqrt{75a^3b}$ **e)** $\sqrt[3]{40}$ **f)** $\sqrt[3]{27a^3b}$ **g)** $\sqrt[3]{\frac{2}{27}}$ **h)** $\sqrt[3]{8a^4b^4}$

Exercises 1.6 (page 27)

1. 11 **2.** 2 **3.** 3 **4.** 18 **5.** 9 **6.** 1 **7.** 7 **8.** 1 **9.** 8 **10.** 2 **11.** 11 **12.** 30 **13.** 20 **14.** 0 **15.** 0 **16.** $4\sqrt{2}$ **17.** $9\sqrt{13}$ **18.** $6\sqrt{6}$ **19.** $5\sqrt{2}$ **20.** $-\sqrt{2}$ **21.** $7\sqrt{3}$ **22.** $\sqrt{2}$ **23.** $4\sqrt{5}$ **24.** $7\sqrt{5}$ **25.** $2\sqrt{3}$ **26.** $2\sqrt{2}$ **27.** $5\sqrt{5}$ **28.** 0 **29.** $-13\sqrt{3}$ **30.** $10\sqrt{6}$ **31.** $6\sqrt{5}$ **32.** $13\sqrt{2} - 2$ **33.** $3\sqrt{x} - 2$ **34.** $-x\sqrt{3}$ **35.** $\sqrt{3x} + 10$ **36.** $-\sqrt{x} + 14$ **37.** $-3\sqrt[3]{a} + 5$ **38.** $-4a\sqrt[3]{2} + 25$

Exercises 1.7 (page 31)

1. a) $20\sqrt{3}$ **b)** $-36\sqrt{2}$ **c)** $12\sqrt{3}$ **d)** $-12\sqrt{10}$