Chapter 10 Pythagorean Theorem

Suggested Approach

Students can explore Pythagorean Theorem using the GSP activity in Class Activity 1. There are over 300 proofs of Pythagorean Theorem. Teachers may illustrate some proofs, or ask students to think and search for different proofs. The emphasis of this chapter is on its application to solve problems reducible to right-angled triangles. Students should understand the relationship between Pythagorean Theorem and its converse, and have ample practice on application problems.

10.1 Pythagorean Theorem

Students should be able to identify the right angle in a right-angled triangle before applying Pythagorean Theorem. The simple Pythagorean Triples such as (3-4-5, 5-12-13) should be introduced. Refer to Question 12 of Exercise 10.1.

10.2 The Converse of Pythagorean Theorem

Teachers may first introduce the idea of the converse of a theorem. Students should understand that the converse of Pythagorean Theorem is a tool used to prove that a triangle is a right-angled triangle.

10.3 Applications of Pythagoras Theorem

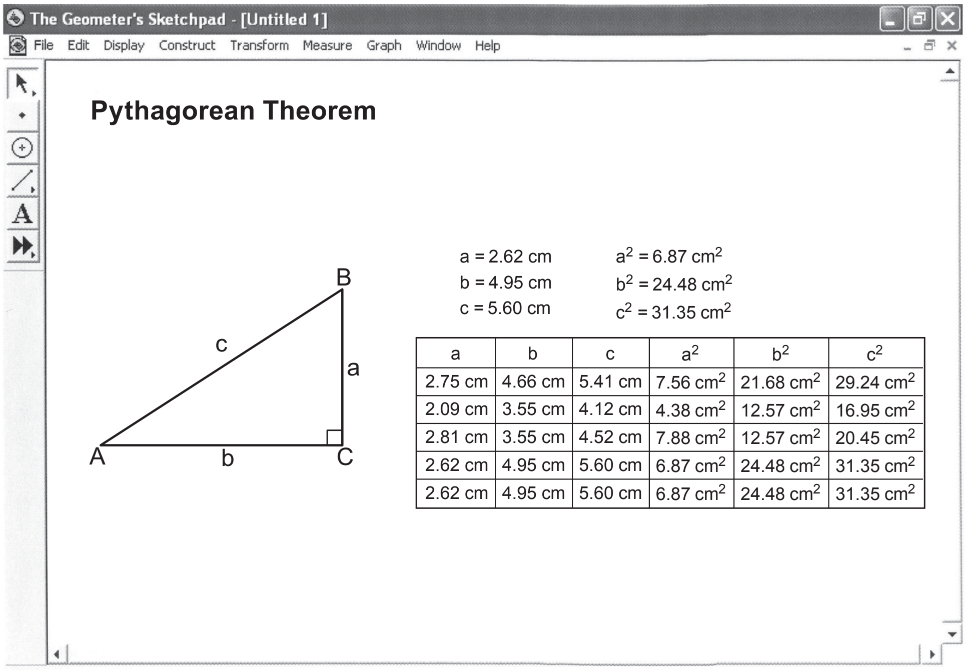
Pythagorean Theorem can be applied to find the length of a side of a right-angled triangle when two other sides are given. In practical problems, students should identify the right angles first. Sometimes, we need to construct additional lines to form right-angled triangles.

Besides arousing and increasing students' interest in the topic with proofs of the Pythagorean Theorem, students should also be encouraged to explore the role of the theorem in the spiral of Theodorus.

Chapter 10 Pythagorean Theorem

Class Activity 1

Objective: To explore the relationship between the sides of a right-angled triangle.



a	b	c	a^2	b^2	c^2
2.75 cm	4.66 cm	5.41 cm	7.56 cm²	21.68 cm²	29.24 cm²
2.09 cm	3.55 cm	4.12 cm	4.38 cm²	12.57 cm²	16.95 cm²
2.81 cm	3.55 cm	4.52 cm	7.88 cm²	12.57 cm²	20.45 cm²
2.62 cm	4.95 cm	5.60 cm	6.87 cm²	24.48 cm²	31.35 cm²
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Tasks

(a) Draw a right-angled $\triangle ABC$ with $\angle C = 90°$.
(*Hint*: Use the command **Construct** | **Perpendicular line** to draw a line passing through *C* and perpendicular to *AC*.)

(b) Label the sides opposite $\angle A$, $\angle B$, and $\angle C$ as *a*, *b*, and *c* respectively.

(c) Measure the lengths of *a*, *b*, and *c*.

(d) Find the squares of *a*, *b*, and *c*.

(e) Select all the measurements in steps **(c)** and **(d)**. Then select the command **Graph** | **Tabulate** to create a table of values of a, b, c, a^2, b^2, and c^2.

(f) Drag the vertices of $\triangle ABC$ around to obtain another set of values of the sides and their squares. Double-click the table to add the current measurements as a new row to it.

(g) Repeat step **(f)** for two or more sets of measurements.

Question

What is the relationship between a^2, b^2, and c^2 in the right-angled $\triangle ABC$?

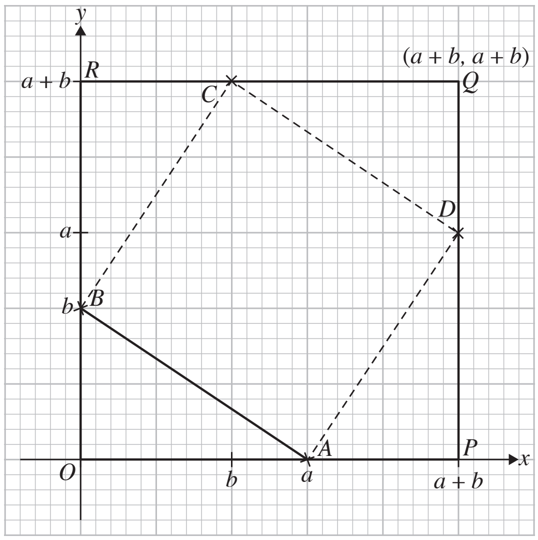
$a^2 + b^2 = c^2$

Class Activity 2

Objective: To explore one of the proofs of the Pythagorean Theorem.

Tasks

(a) A triangle with vertices $O(0, 0)$, $A(a, 0)$ and $B(0, b)$, and a square with vertices $O(0, 0)$, $P(a + b, 0)$, $Q(a + b, a + b)$, and $R(0, a + b)$, where a and b are real numbers, are drawn on the Cartesian plane in the diagram.



(b) On RQ, mark the point C with coordinates $(b, a + b)$ and on PQ, mark the point D with coordinates $(a + b, a)$. Join the points A, B, C, and D to obtain figure $ABCD$.

(c) Why is the figure $ABCD$ a square? Explain your answer.

All sides of figure *ABCD* are hypotenuses of triangles with the same dimensions as $\triangle OAB$.

(d) Giving your answers in terms of a and b, find

(i) the area of the square $OPQR$,

Area of $OPQR$ = base × height

$= (a + b) \times (a + b)$

$= (a + b)^2$

(ii) the area of each triangle.

Area of each triangle $= \frac{1}{2} \times$ base × height

$= \frac{1}{2} \times a \times b$

$= \frac{1}{2} ab$

(e) Suppose the length of AB is c units, find an expression in terms of a, b, and c for the area of $ABCD$.

$AB^2 = OA^2 + OB^2$ (Pythagorean Theorem)

$= a^2 + b^2$

$\therefore AB = \sqrt{a^2 + b^2}$

If $AB = c$ units,

$c = \sqrt{a^2 + b^2}$

Area of $ABCD = AB \times AD$

$= \sqrt{a^2 + b^2} \times \sqrt{a^2 + b^2}$

$= (\sqrt{a^2 + b^2})^2$

$= a^2 + b^2$

$= c^2$

(f) Relating your expression in **(e)** with $\triangle OAB$, what conclusion can you draw?

Area of $ABCD$ = Area of $OPQR$ – Area of 4 triangles

$= (a + b)^2 - 4\left(\frac{1}{2}ab\right)$

$= a^2 + 2ab + b^2 - 2ab$

$= a^2 + b^2$

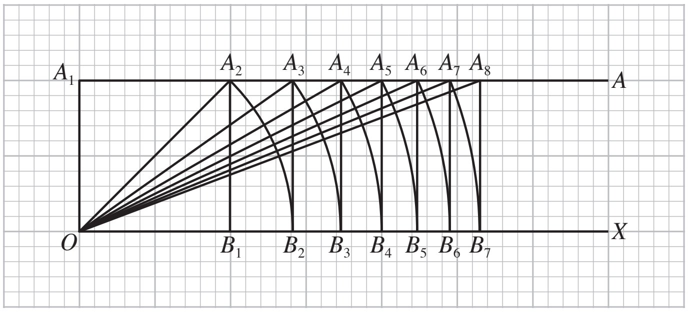
$\therefore a^2 + b^2 = c^2$

Class Activity 3

Objective: To apply Pythagorean Theorem to construct lengths of irrational square-root values on the number line diagram.

Tasks

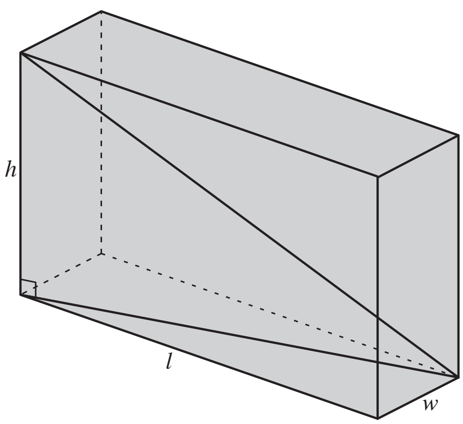
(a) On a sheet of graph paper and taking 2 cm to represent 1 unit, draw a unit square $OA_1A_2B_1$ as shown below. Extend the lines OB_1 and A_1A_2 to OX and A_1A respectively. Line OX is considered as a number line.



Extend Your Learning Curve

Diagonal of a Prism

Research and write a brief report on this technology and its possible applications. A rectangular prism is a 3-dimensional figure with a rectangular base and top of the same size which are parallel to each other, and each of the other sides is also a rectangle. A rectangular prism with edge lengths, l, w, and h is shown below.



(a) Find the length of the diagonal which is drawn from one corner of the base to the opposite corner of the top, as shown in the figure, in terms of l, w, and h.

(b) What is the length of the diagonal joining the opposite corners of a prism whose base is 3 cm by 4 cm and height is 12 cm?

Suggested Answers:

(a) (length of diagonal of base)$^2 = l^2 + w^2$ (Pythagorean Theorem)

length of diagonal of base $= (\sqrt{l^2 + w^2})$ units

length of diagonal from base to top $= \sqrt{(\sqrt{l^2 + w^2})^2 + h^2}$

$= \sqrt{l^2 + w^2 + h^2}$ units

(b) Let $h = 12$ cm, $w = 3$ cm and $l = 4$ cm.

length of diagonal $= \sqrt{l^2 + w^2 + h^2}$

$= \sqrt{4^2 + 3^2 + 12^2}$

$= \sqrt{169}$

$= 13$ cm

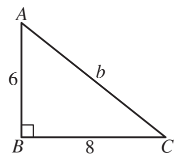
Exercise 10.1

In this exercise, give your answers rounded to 3 significant figures where necessary.

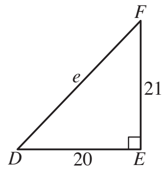
Basic Practice

1. Find the unknown side of each of the following triangles, given that the measurements are in centimeters.

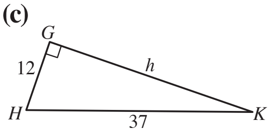
(a)



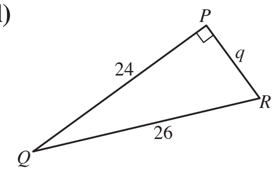
(b)



(c)



(d)



Solution

(a) $b^2 = 6^2 + 8^2$ (Pythagorean Theorem)
$= 100$
$b = \sqrt{100}$
$= 10$ cm

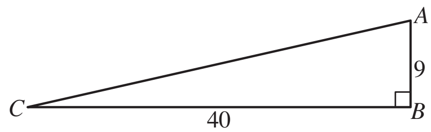
(b) $e^2 = 20^2 + 21^2$ (Pythagorean Theorem)
$= 841$
$e = \sqrt{841}$
$= 29$ cm

(c) $37^2 = 12^2 + h^2$ (Pythagorean Theorem)
$h^2 = 1{,}225$
$h = \sqrt{1{,}225}$
$= 35$ cm

(d) $26^2 = 24^2 + q^2$ (Pythagorean Theorem)
$q^2 = 100$
$q = \sqrt{100}$
$= 10$ cm

2. In $\triangle ABC$, $\angle B = 90°$, $AB = 9$ in., and $BC = 40$ in. What is the length of AC?

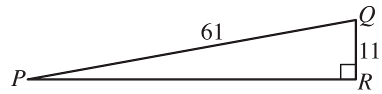
Solution



$AC^2 = 40^2 + 9^2$ (Pythagorean Theorem)
$= 1{,}681$
$AC = \sqrt{1{,}681}$
$= 41$ in.

3. $\triangle PQR$ has a right angle at $\angle R$ with $PQ = 61$ cm and $QR = 11$ cm. What is the length of PR?

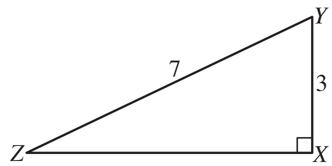
Solution



$PR^2 = 61^2 - 11^2$ (Pythagorean Theorem)
$= 3{,}600$
$PR = \sqrt{3{,}600}$
$= 60$ cm

4. In $\triangle XYZ$, $\angle X = 90°$, $YZ = 7$ ft, and $XY = 3$ ft. Find the length of XZ.

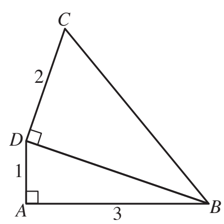
Solution



$7^2 = 3^2 + XZ^2$ (Pythagorean Theorem)
$XZ^2 = 40$
$XZ = \sqrt{40}$
$= 6.32$ ft (rounded to 3 sig. fig.)

Further Practice

5. In the figure, $\angle BAD = \angle BDC = 90°$, $AB = 3$ cm, $AD = 1$ cm, and $CD = 2$ cm. Find the length of
(a) BD,
(b) BC.



Solution

(a) In $\triangle ABD$,
$BD^2 = 1^2 + 3^2$ (Pythagorean Theorem)
$= 10$
$BD = \sqrt{10}$
$= 3.16$ cm (rounded to 3 sig. fig.)

(b) In $\triangle BCD$,
$BC^2 = CD^2 + BD^2$ (Pythagorean Theorem)
$= 2^2 + 10$
$BC = \sqrt{14}$
$= 3.74$ cm (rounded to 3 sig. fig.)