Chapter 14 Proportions

Suggested Approach

Teachers can show why it is relevant to learn how to calculate distance and area based on a map using some daily life examples. Students can integrate the knowledge learned in this topic with what they learn on map reading.

Teachers can also introduce direct and indirect proportions by using daily life examples. Students are expected to recognize these relations from tables, graphs, equations, and verbal descriptions of proportional relationships. Further real-life problems that can be modeled by these relations could be explored. Students should learn to differentiate cases such as "x and y are in direct (or inverse) proportion" and "x^2 and y are in direct (or inverse) proportion".

14.1 Scale Drawings

Students should understand that the scale factor is the ratio of a side of the image to the corresponding side of the object. They are expected to draw simple scale drawings of models and floor plans, and get actual dimensions from scale drawings.

14.2 Map Scale and Calculation of Area

Students should note that a map scale can be represented as a ratio or a fraction. They should learn to calculate the actual distance from a given distance on a map and vice versa. Teachers, when teaching the concept of ratio of actual area to the area on a map, can use squares and rectangles to illustrate the relationship with the scale of the map.

14.3 Direct Proportion

When learning direct proportion, students are expected to observe patterns of the variation between two quantities and express the generalization in algebraic form. Students should be able to relate linear graphs learned earlier with direct relation of two quantities. Real-life examples will make learning more effective and interesting.

14.4 Inverse Proportion

Teachers need to emphasize to their students that they have to draw a linear graph of y against $\frac{1}{x}$ to show that x and y are in inverse proportion. This is because some students may have a misconception that a straight line with a negative slope in x-y plane indicates an inverse proportion between x and y.

Chapter 14   Proportions

Class Activity 1

Objective: To understand the idea of direct proportion.
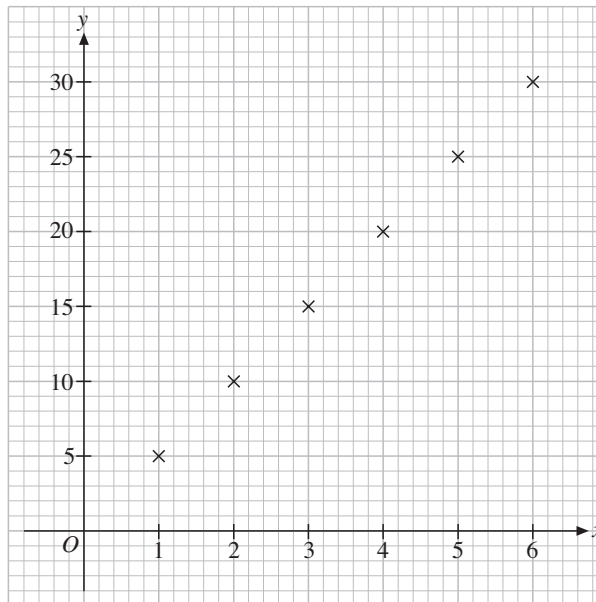
Questions

The following table shows the relationship between the number of books bought (x) and the total cost of the books ($\$y$).

Number of books (x)	1	2	3	4	5	6
Total cost ($\$y$)	5	10	15	20	25	30

(a)   Copy and complete the following table.

x	1	2	3	4	5	6
y	5	10	15	20	25	30
$\dfrac{y}{x}$	5	5	5	5	5	5

(b)   On a sheet of graph paper, plot the corresponding points (x, y) in **(a)** using the scale for both axes as shown.

(c)   What can you say about the points you have plotted in **(b)**?

The points are on a straight line.

(d)   Write down an equation connecting x and y.

$y = 5x$

(e)   What is the value of y when $x = 8$?

When $x = 8$, $y = 5 \times 8 = 40$.

(f)   Does the graph of the equation in **(d)** pass through the origin $(0, 0)$?

Yes.

Solution

(a) Map scale = 1 : 1,500,000

$\quad\quad\quad$ = 1 cm : 1,500,000 cm

$\quad\quad\quad$ = 1 cm : 15 km

Actual north-south dimension of California

= 84 × 15 km

= 1,260 km

(b) Actual east-west dimension of California

= 480 km

= 480 × 1,000 × 100 cm

= 48,000,000 cm

Actual north-south dimension of California

= 1,260 km

= 1,260 × 1,000 × 100 cm

= 126,000,000 cm

Let the scale of the required map be 1 : n.

We should have

$\quad\quad$ $(21 - 2 \times 1)n > 48,000,000$

and $\quad (28 - 2 \times 1)n > 126,000,000$,

i.e., $n > 2,526,315.79$ and $n > 4,846,153.85$.

Hence, one appropriate scale would be 1 : 5,000,000.

Exercise 14.3

Basic Practice

1. In each of the following tables, determine whether x and y are in direct proportion.

(a)

x	1	2	3	4
y	3	6	9	12

(b)

x	2	4	6	8
y	10	20	25	40

(c)

x	3	6	7	10
y	21	42	49	70

(d)

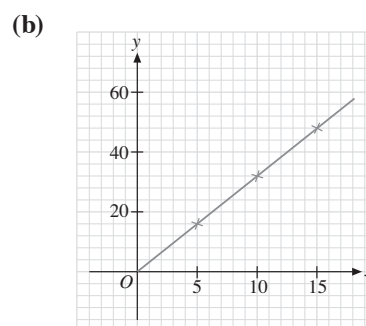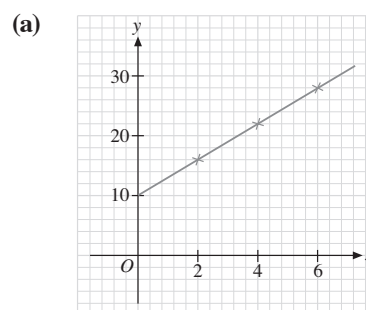| x	8	12	15	20
y	30	48	60	80

Solution

(a) $\dfrac{1}{3} = \dfrac{2}{6} = \dfrac{3}{9} = \dfrac{4}{12}$

$\therefore$ x and y are in direct proportion.

(b) $\dfrac{2}{10} \neq \dfrac{6}{25}$

$\therefore$ x and y are NOT in direct proportion.

(c) $\dfrac{3}{21} = \dfrac{6}{42} = \dfrac{7}{49} = \dfrac{10}{70}$

$\therefore$ x and y are in direct proportion.

(d) $\dfrac{8}{30} \neq \dfrac{12}{48}$

$\therefore$ x and y are NOT in direct proportion.

2. In each of the following graphs, determine whether x and y are in direct proportion.

(a)

(b)

Solution

(a) The straight line graph does not pass through the origin.

$\therefore$ x and y are NOT in direct proportion.

(b) The straight line graph passes through the origin.

$\therefore$ x and y are in direct proportion.

3. In each of the following equations, determine whether x and y are in direct proportion.

(a) $y = 4x$ $\quad\quad\quad\quad\quad$ **(b)** $y = x + 2$

(c) $y = x^2$ $\quad\quad\quad\quad\quad$ **(d)** $y = \dfrac{1}{2}x$

Solution

(a) $y = 4x$ is in the form of $y = kx$ with $k = 4$.

$\therefore$ x and y are in direct proportion.

(b) $y = x + 2$ cannot be written as $y = kx$.

$\therefore$ x and y are NOT in direct proportion.

(c) $y = x^2$ cannot be written as $y = kx$.

$\therefore$ x and y are NOT in direct proportion.

(d) $y = \dfrac{1}{2}x$ is in the form of $y = kx$ with $k = \dfrac{1}{2}$.

$\therefore$ x and y are in direct proportion.

4. If two quantities, x and y, are in direct proportion, find the values of p and q in the following table.

x	12	18	q
y	8	p	24

Solution

$$\frac{12}{8} = \frac{18}{p}$$

$$\therefore\ p = 18 \times \frac{8}{12}$$

$$= 12$$

$$\frac{12}{8} = \frac{n}{24}$$

$$q = 24 \times \frac{12}{8}$$

$$= 36$$

5. It is given that w is directly proportional to t. When $t = 4$, $w = 20$. Find
  (a) the value of w when $t = 6$,
  (b) the value of t when $w = 45$.

Solution

(a) Let $w = kt$, where k is a constant.
   When $t = 4$, $w = 20$.

$$20 = k \times 4$$
$$k = 5$$
$$w = 5t$$

   When $t = 6$,
$$w = 5 \times 6$$
$$= 30$$

(b) When $w = 45$,
$$45 = 5t$$
$$t = 9$$

6. It is given that A is directly proportional to r^2 and $r > 0$. When $r = 5$, $A = 75$. Find
  (a) the value of A when $r = 4$,
  (b) the value of r when $A = 147$.

Solution

(a) Let $A = kr^2$, where k is a constant.
   When $r = 5$, $A = 75$.

$$75 = k \times 5^2$$
$$k = 3$$
$$\therefore\ A = 3r^2$$

   When $r = 4$,
$$A = 3 \times 4^2$$
$$= 48$$

(b) When $A = 147$,
$$147 = 3r^2$$
$$r^2 = 49$$
$$r = \sqrt{49}$$
$$= 7$$

Further Practice

7. The following table shows the total price (P) for x copies of books.

Copies of books (x)	1	2	3	4	5
Total price (P)	15	30	45	60	75

(a) Show that x and P are in direct proportion.
(b) Draw the graph of P against x.
(c) Describe the graph in (b).
(d) Find the equation connecting x and P.
(e) Hence, find the total price for 8 copies of books.

Solution

(a) $$\frac{1}{15} = \frac{2}{30} = \frac{3}{45} = \frac{4}{60} = \frac{5}{75}$$
   $\therefore\ x$ and P are in direct proportion.

(b) The diagram below shows the graph of P against x.
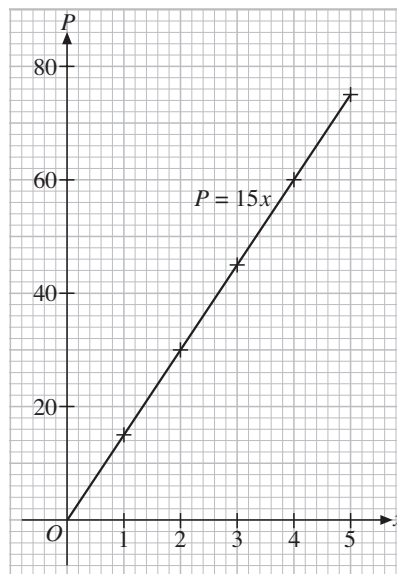
(c) The graph in (b) is a straight line that passes through the origin and its slope is positive.

(d) As $\dfrac{x}{P} = \dfrac{1}{15}$,

$P = 15x$.

(e) When $x = 8$,

$P = 15 \times 8$

$= 120$

The total price for 8 copies of books is $120.

8. The following table shows the mass (m g) of a pinewood cube of side x cm.

Length of a side (x cm)	2	4	5	8	10
Mass (m g)	5.2	41.6	81.25	332.8	650

(a) Is m proportional to x?
(b) Is m proportional to x^3?
(c) Find an equation connecting m and x.
(d) Hence, find the mass in grams of a pinewood cube of side 9 cm.

Solution

(a) $\dfrac{2}{5.2} \neq \dfrac{4}{41.6}$

$\therefore$ m is NOT proportional to x.

(b) $\dfrac{5.2}{2^3} = \dfrac{41.6}{4^3} = \dfrac{81.25}{5^3} = \dfrac{332.8}{8^3} = \dfrac{650}{10^3} = 0.65$

$\therefore$ m is proportional to x^3.

(c) $\dfrac{P_1}{n_1} = 0.65$

$\therefore$ $m = 0.65x^3$

(d) When $x = 9$,

$m = 0.65 \times 9^3$

$= 473.85$

The required mass is 473.85 g.

Math@Work

9. The cost of renting a car is directly proportional to the number of days the car is being rented for. The cost of renting a car for 4 days is $240. Find the cost of renting a car for 7 days.

Solution

Let the price of renting a car for n days be $P.

Then $\dfrac{P_1}{n_1} = \dfrac{P_2}{n_2}$.

When $n_1 = 4$, $P_1 = 240$, $n_2 = 7$,

$\dfrac{240}{4} = \dfrac{P_2}{7}$

$P_2 = 7 \times \dfrac{240}{4}$

$= 420$

The cost of renting a car for 7 days is $420.

10. The mass of a metal plate is directly proportional to its volume. When its volume is 20 cm^3, its mass is 210 g. If the volume of the metal plate is 50 cm^3, what is its mass?

Solution

Let the mass of a metal plate of V cm^3 be m g.

Then $\dfrac{V_1}{m_1} = \dfrac{V_2}{m_2}$.

When $V_1 = 20$, $m_1 = 210$, $V_2 = 50$,

$m_2 = 50 \times \dfrac{210}{20}$

$= 525$

The mass of the metal plate of volume 50 cm^3 is 525 g.

11. When a car is traveling steadily along a highway, its consumption of gasoline is directly proportional to the distance traveled. A car travels 100 mi on 2.7 gal of gasoline. Find, giving your answer correct to 1 decimal place,
(a) the gasoline consumption of the car for a distance of 74 mi,
(b) the maximum distance that the car can travel with 1 gal of gasoline.

Solution

(a) Let the consumption of gasoline be y gal when the distance traveled is d mi.

Then $y = kd$, where k is a constant.

When $d = 100$, $y = 2.7$.

$2.7 = k(100)$

$k = 0.027$

$\therefore$ $y = 0.027d$

When $d = 74$,

$y = 0.027 \times 74$

$= 2.0$ (correct to 1 d.p.)

The required gasoline consumption is 2.0 gal.

(b) When $y = 1$,

$1 = 0.027d$

$d = 37.0$ (correct to 1 d.p.)

The required maximum distance traveled is 37.0 mi.

12. The period (the time taken for one complete oscillation) of a simple pendulum is directly proportional to the square root of its length. When its length is 1.02 m, its period is 2.01 seconds. Find
  (a) the period of the pendulum when its length is 0.8 m,
  (b) the length of the pendulum when its period is 1.0 second.
  Give your answers correct to 2 decimal places.
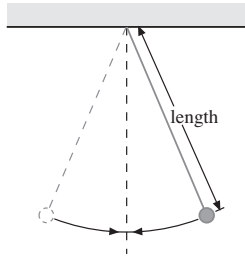
Solution
  (a) Let the period of a simple pendulum of length L cm be T seconds.

  Then $T = k\sqrt{L}$, where k is a constant.

  When $L = 1.02$, $T = 2.01$.

  $$2.01 = k\sqrt{1.02}$$

  $$k = \frac{2.01}{\sqrt{1.02}}$$

  i.e., $T = \dfrac{2.01}{\sqrt{1.02}}\sqrt{L}$

  When $L = 0.8$,

  $$T = \frac{2.01}{\sqrt{1.02}} \times \sqrt{0.8}$$

  $$= 1.78 \quad \text{(correct to 2 d.p.)}$$

  The required period of the pendulum is 1.78 seconds.

  (b) When $T = 1$,

  $$1 = \frac{2.01}{\sqrt{1.02}}\sqrt{L}$$

  $$\sqrt{L} = \frac{\sqrt{1.02}}{2.01}$$

  $$L = \frac{1.01}{2.01^2}$$

  $$= 0.25 \quad \text{(correct to 2 d.p.)}$$

  The required length of the pendulum is 0.25 m.

13. The vertical falling distance of a ball is directly proportional to the square of the time of falling. The ball falls 80 m in 4 s.
  (a) Find the vertical falling distance of the ball when the time taken is 6 s.
  (b) If the ball is dropped from a height of 245 m, find the time it takes to hit the ground.

Solution
  (a) Let the vertical falling distance be y m in t seconds.
  Then $y = kt^2$, where k is a constant.
  When $t = 4$, $y = 80$.
  $$80 = k \times 4^2$$
  $$k = 5$$
  $$\therefore \ y = 5t^2$$
  When $t = 6$,
  $$y = 5 \times 6^2$$
  $$= 180$$
  The required vertical falling distance is 180 m.

  (b) When $y = 245$,
 $$245 = 5t^2$$
  $$t^2 = 49$$
  $$t = \sqrt{49}$$
  $$= 7$$
  The required time taken is 7 seconds.

Brainworks

14. **(a)** In our daily life, we often encounter a wide variety of quantities involving direct proportion. Describe two such quantities.
  (b) Draw a graph to show their relationship.
  (c) Find an equation connecting the quantities.

Solution
  (a) If the admission fee per person for a concert is $40, then the total admission fee, T, is directly proportional to the number of people, n, attending the concert.

  (b) The graph of T against n is shown below.
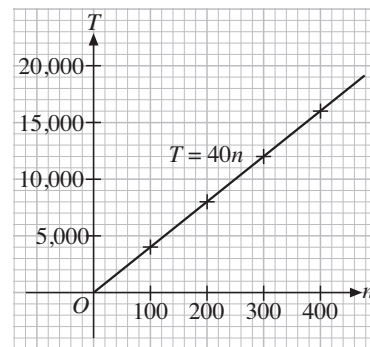
  (c) The equation connecting T and n is $T = 40n$.
  Note: Students may provide other relevant cases.