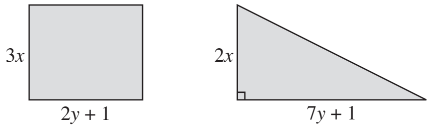
If $x = 4$, $4 - 1 = 3 > 0$
$\therefore$ pack A offers a better value if $x = 4$.

24. The length and width of a rectangle are $(2y + 1)$ cm and $3x$ cm respectively. The base and height of a triangle are $(7y + 1)$ cm and $2x$ cm respectively.



(a) Find, in terms of x and y,
  (i) the area of the rectangle,
  (ii) the area of the triangle.
(b) Subtract the area of the rectangle from the area of the triangle expressing your answer in terms of x and y.
(c) **(i)** Factor your answer in **(b)**.
  (ii) If y is a prime number, show that the area of the triangle is always greater than or equal to the area of the rectangle.

Solution

(a) **(i)** Area of rectangle $= 3x(2y + 1)$
$= (6xy + 3x)$ cm^2
  (ii) Area of triangle $= \frac{1}{2}(2x)(7y + 1)$
$= (7xy + x)$ cm^2
(b) $7xy + x - 6xy - 3x = (xy - 2x)$ cm^2
(c) **(i)** $xy - 2x = x(y - 2)$
  (ii) The smallest prime number is 2.
$\therefore$ difference in area = 0
For $y > 2$, area of triangle minus area of rectangle > 0 since $x > 0$.
$\therefore$ area of triangle is always greater than or equal to area of rectangle.

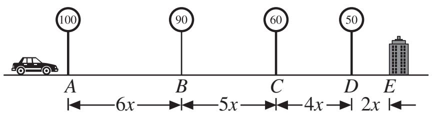
25. Johnny and Marcus ran the same distance to determine who runs faster. Their coach recorded the time taken by each of them to complete the distance. Suppose that the time taken by Johnny subtracted from the time taken by Marcus is $(2xy - 5y - 15 + 6x)$ seconds.
(a) Factor $2xy - 5y - 15 + 6x$.
(b) Given that y is positive and $x = 1.5$, determine who ran faster. Explain your answer.

Solution

(a) $2xy + 6x - 5y - 15 = 2x(y + 3) - 5(y + 3)$
$= (2x - 5)(y + 3)$
(b) $x = 1.5$
$\therefore (3 - 5)(y + 3) = -2(y + 3)$
Since $y > 0$, $-2(y + 3) < 0$.
Since time taken by Marcus minus time taken by David < 0, Marcus ran faster.

Enrichment

26.



In the figure, $ABCDE$ is a portion of a road from the exit A of an expressway to a building E. $AB = 6x$ km, $BC = 5x$ km, $CD = 4x$ km, and $DE = 2x$ km. A car drives at the speed limits, i.e. 100 km/hr, 90 km/hr, 60 km/hr, and 50 km/hr in each section from A to E respectively. Let T minutes be the time taken by the car to reach E from A.
(a) Express T in terms of x.
(b) When $x = 0.45$, find the value of T.

Solution

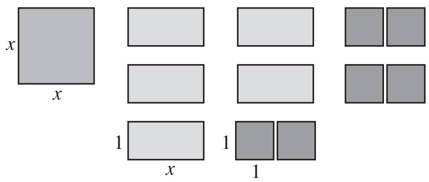
(a) $T = \left(\frac{6x}{100} + \frac{5x}{90} + \frac{4x}{60} + \frac{2x}{50}\right) \times 60$
$= \frac{2x}{9} \times 60$
$= \frac{40x}{3}$
(b) When $x = 0.45$,
$T = \frac{40}{3} \times 0.45$
$= 6$

27. The sides of $\triangle ABC$ are $AB = (3x + 4)$ cm, $BC = (4x - 5)$ cm, and $CA = (x + 13)$ cm.
(a) Express the perimeter of $\triangle ABC$ in terms of x. Give the answer in factored form.
(b) A square $PQRS$ has the same perimeter as $\triangle ABC$. Express the length of PQ in terms of x.
(c) When $x = 7$, find
  (i) the perimeter of $\triangle ABC$,
  (ii) the area of $PQRS$.

Solution

(a) Perimeter of $\triangle ABC = (3x + 4) + (4x - 5) + (x + 13)$
$= 8x + 12$
$= 4(2x + 3)$ cm
(b) Length of $PQ = 4(2x + 3) \div 4$
$= (2x + 3)$ cm
(c) **(i)** When $x = 7$,
perimeter of $\triangle ABC = 4[2(7) + 3]$
$= 68$ cm
  (ii) When $x = 7$,
$PQ = 2 \times 7 + 3$
$= 17$ cm
$\therefore$ area of $PQRS = 17 \times 17$
$= 289$ cm^2

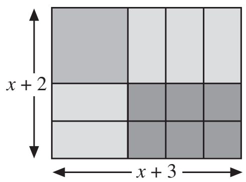
28.



(a) The figure shows 1 square tile of x by x units, 5 rectangular tiles of x by 1 unit, and 6 square tiles of 1 by 1 unit. Arrange the tiles to form a rectangle and state its dimensions.
(b) Hence, or otherwise, express $x^2 + 5x + 6$ in the form $(x + a)(x + b)$, where a and b are integers.
(c) Express $x^2 + 8x + 15$ in the form $(x + p)(x + q)$, where p and q are integers.

Solution

(a)



Its dimensions are $(x + 3)$ by $(x + 2)$ units.

(b) $x^2 + 5x + 6 = (x + 3)(x + 2)$

(c) $x^2 + 8x + 15 = (x + 5)(x + 3)$

29. The volumes of two glasses of water are $(7ax - 3bx + 6ay - 4by)$ cm^3 and $(11bx + 5ax - 6by - 21ay)$ cm^3 respectively. Let V cm^3 be the total volume of water in the two glasses.

(a) Express V in terms of a, b, x, and y in factored form.

(b) If both x and y are doubled, determine whether V will be doubled.

Solution

(a)
$$\begin{aligned} V &= (7ax - 3bx + 6ay - 4by) + \\ &\quad (11bx + 5ax - 6by - 21ay) \\ &= (7ax + 5ax) + (6ay - 21ay) + (-3bx + 11bx) + \\ &\quad (-4by - 6by) \\ &= 12ax - 15ay + 8bx - 10by \\ &= 3a(4x - 5y) + 2b(4x - 5y) \\ &= (3a + 2b)(4x - 5y) \end{aligned}$$

(b) When the new value of $x = 2x$
and the new value of $y = 2y$,
$$\begin{aligned} \text{new value of } V &= (3a + 2b)[4(2x) - 5(2y)] \\ &= (3a + 2b)(8x - 10y) \\ &= 2(3a + 2b)(4x - 5y) \\ &= 2V \end{aligned}$$
$\therefore$ V is doubled.

Chapter 5 Simple Equations In One Variable

Basic Practice

1. Solve the following equations.

(a) $x + 12 = 17$ **(b)** $x - 8 = 12$

(c) $14 - x = 11$ **(d)** $2x = 10$

(e) $-3x = 27$ **(f)** $\frac{x}{7} = 4$

(g) $\frac{x}{5} = -9$ **(h)** $\frac{x}{3} + 1 = -5$

(i) $\frac{x}{4} - 3 = 17$ **(j)** $20 - \frac{x}{7} = 18$

Solution

(a)
$$\begin{aligned} x + 12 &= 17 \\ x + 12 - 12 &= 17 - 12 \\ x &= 5 \end{aligned}$$

(b)
$$\begin{aligned} x - 8 &= 12 \\ x - 8 + 8 &= 12 + 8 \\ x &= 20 \end{aligned}$$

(c)
$$\begin{aligned} 14 - x &= 11 \\ 14 - x - 14 &= 11 - 14 \\ -x &= -3 \\ (-1)(-x) &= (-1)(-3) \\ x &= 3 \end{aligned}$$

(d)
$$\begin{aligned} 2x &= 10 \\ \frac{2x}{2} &= \frac{10}{2} \\ x &= 5 \end{aligned}$$

(e)
$$\begin{aligned} -3x &= 27 \\ \frac{-3x}{-3} &= \frac{27}{-3} \\ x &= -9 \end{aligned}$$

(f)
$$\begin{aligned} \frac{x}{7} &= 4 \\ \frac{x}{7} \times 7 &= 4 \times 7 \\ x &= 28 \end{aligned}$$

(g)
$$\begin{aligned} \frac{x}{5} &= -9 \\ \frac{x}{5} \times 5 &= -9 \times 5 \\ x &= -45 \end{aligned}$$

(h)
$$\begin{aligned} \frac{x}{3} + 1 &= -5 \\ \frac{x}{3} + 1 - 1 &= -5 - 1 \\ \frac{x}{3} &= -6 \\ \frac{x}{3} \times 3 &= -6 \times 3 \\ x &= -18 \end{aligned}$$

(i)
$$\begin{aligned} \frac{x}{4} - 3 &= 17 \\ \frac{x}{4} - 3 + 3 &= 17 + 3 \\ \frac{x}{4} &= 20 \\ \frac{x}{4} \times 4 &= 20 \times 4 \\ x &= 80 \end{aligned}$$

(j)
$$\begin{aligned} 20 - \frac{x}{7} &= 18 \\ 20 - \frac{x}{7} - 20 &= 18 - 20 \\ -\frac{x}{7} &= -2 \\ -\frac{x}{7} \times (-7) &= (-2) \times (-7) \\ x &= 14 \end{aligned}$$

2. Solve the following equations.

(a) 3c – 9 = –24

(b) 35 – 12w = 29

(c) –4p – 45 = 15

(d) 3(2x + 5) = 45

(e) 5(2y – 9) = 60

(f) –8(7 – 3z) = 64

(g) 2(5p + 14) = 3(1 – 5p)

(h) 3(2w – 11) = 5(6 – 3w)

(i) 7(2q – 5) – 4(7 – 4q) = 27

(j) 5(3m – 7) – 2(–8 + 7m) = 13 – 3(6 – 5m)

Solution

(a)
$$\begin{aligned} 3c - 9 &= -24 \\ 3c &= -15 \\ c &= -5 \end{aligned}$$

(b)
$$\begin{aligned} 35 - 12w &= 29 \\ 12w &= 6 \\ w &= \frac{1}{2} \end{aligned}$$

(c)
$$\begin{aligned} -4p - 45 &= 15 \\ 4p &= -60 \\ p &= -15 \end{aligned}$$

(d)
$$\begin{aligned} 3(2x + 5) &= 45 \\ 2x + 5 &= 15 \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

(d) Amount charged

$= \frac{100}{107} \times \$(15{,}000 + 425.25)$

= \$14,416 (correct to the nearest dollar)

Enrichment

29. Funds were collected in a school to assist the family of a student. Aaron donated 20% of his pocket money. Barbara donated 25% of her pocket money. Carlo donated $33\frac{1}{3}\%$ of her pocket money. It is given that each of them donated the same amount of money.

(a) Find the percentage of the total amount of pocket money of these 3 students that had been donated.

(b) If the total amount of their donation was \$72, find the amount of

(i) Aaron's pocket money,

(ii) Carlo's pocket money,

(iii) Aaron's pocket money as a percentage of the amount of Carlo's pocket money.

Solution

(a) Let the amount of money each of the students donated be $\$d$.

Aaron's pocket money $= \$d \div 20\%$
$= \$5d$

Barbara's pocket money $= \$d \div 25\%$
$= \$4d$

Carlo's pocket money $= \$d \div 33\frac{1}{3}\%$
$= \$3d$

$\therefore$ the required percentage $= \frac{d \times 3}{5d + 4d + 3d} \times 100\%$

$= \frac{3}{12} \times 100\%$

$= 25\%$

(b) **(i)** Total donation $= \$3d$

$\therefore\ 3d = 72$

$d = 24$

Aaron's pocket money $= \$5 \times 24$
$= \$120$

(ii) Carlo's pocket money $= \$3 \times 24$
$= \$72$

(iii) The required percentage $= \frac{120}{72} \times 100\%$

$= 166\frac{2}{3}\%$

30. A box contains 150 marbles, of which 60% are green and the remaining are red.

(a) How many green marbles have to be removed from the box so that the percentage of green marbles becomes 52%?

(b) If the number of green marbles is increased by 20% and the number of red marbles is decreased by 10%, find

(i) the percentage change in the number of marbles in the box,

(ii) the number of red marbles as a percentage of the number of green marbles.

Solution

(a) Let n be the number of green marbles removed.

Number of green marbles in the box $= 150 \times 60\%$
$= 90$

Number of red marbles in the box $= 150 - 90$
$= 60$

We require

$90 - n = [(90 - n) + 60] \times 52\%$

$90 - n = 78 - 0.52n$

$0.48n = 12$

$n = 25$

i.e. 25 green marbles have to be removed.

(b) **(i)** New number of green marbles

$= 90 \times (1 + 20\%)$

$= 108$

New number of red marbles $= 60 \times (1 - 10\%)$
$= 54$

New total number of marbles $= 108 + 54$
$= 162$

$\therefore$ percentage increase in the number of marbles

$= \frac{162 - 150}{150} \times 100\%$

$= 8\%$

(ii) The required percentage $= \frac{54}{108} \times 100\%$
$= 50\%$

31. In the 7th grade classes in a school, there are 24 more boys than girls. Let n be the number of 7th grade boys.

(a) Express the percentage of boys in the 7th grade classes in terms of n.

(b) If the percentage of boys in the 7th grade classes is 55%, find the total number of 7th grade students.

Solution

(a) Number of 7th grade girls $= n - 24$

Total number of 7th grade students $= n + (n - 24)$
$= 2n - 24$

$\therefore$ percentage of boys $= \frac{n}{2n - 24} \times 100\%$

$= \frac{100n}{2n - 24}\%$

(b) $\frac{100n}{2n - 24}\% = 55\%$

$100n = 55(2n - 24)$

$100n = 110n - 1320$

$10n = 1320$

$n = 132$

$\therefore$ total number of 7th grade students

$= 2 \times 132 - 24$

$= 240$

32. The marked price of a mobile phone was \$840, including 5% sales tax. The sales tax was increased to 7%. Find

(a) the percentage increase in the sales tax,

(b) the percentage increase in the marked price of the mobile phone,

(c) the percentage of discount on the new marked price so that the mobile phone was sold at the old price \$840.

Give your answers correct to 2 decimal places if necessary.

Solution

(a) Percentage increase in sales tax $= \frac{7\% - 5\%}{5\%} \times 100\%$

$= 40\%$

(b) Price without sales tax = \$840 ÷ (1 + 5%)
= \$800

Marked price with new sales tax = \$800 × (1 + 7%)
= \$856

$\therefore$ percentage increase in the marked price

$= \frac{856 - 840}{840} \times 100\%$

$= 1.90\%$ (correct to 2 d.p.)

(c) Discount = \$856 – \$840
= \$16

$\therefore$ percentage of discount

$= \frac{16}{856} \times 100\%$

$= 1.87\%$ (correct to 2 d.p.)

Chapter 8 Angles, Triangles, And Quadrilaterals

Basic Practice

1. In each case, find the angle that is complementary to the given angle.

(a) 35° **(b)** 21°
(c) 63° **(d)** 79°
(e) 15.4° **(f)** 48.5°

Solution

(a) $90° - 35° = 55°$
(b) $90° - 21° = 69°$
(c) $90° - 63° = 27°$
(d) $90° - 79° = 11°$
(e) $90° - 15.4° = 74.6°$
(f) $90° - 48.5° = 41.5°$

2. In each case, find the angle that is supplementary to the given angle.

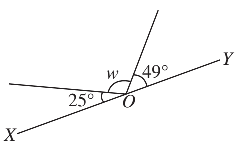
(a) 74° **(b)** 123°
(c) 86° **(d)** 142°
(e) 155.6° **(f)** 94.5°

Solution

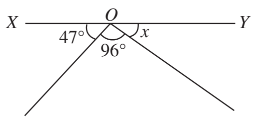
(a) $180° - 74° = 106°$
(b) $180° - 123° = 57°$
(c) $180° - 86° = 94°$
(d) $180° - 142° = 38°$
(e) $180° - 155.6° = 24.4°$
(f) $180° - 94.5° = 85.5°$

3. In each figure, *XOY* is a straight line. Find the measure of each unknown marked angle.

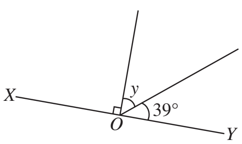
(a)



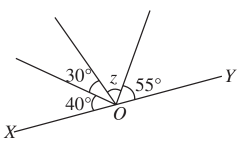
(b)



(c)



(d)



Solution

(a) $m\angle w + 25° + 49° = 180°$ (adj. $\angle$s on a st. line)
$m\angle w = 180° - 74°$
$= 106°$

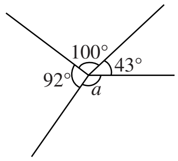
(b) $m\angle x + 47° + 96° = 180°$ (adj. $\angle$s on a st. line)
$m\angle x = 180° - 143°$
$= 37°$

(c) $m\angle y + 90° + 39° = 180°$ (adj. $\angle$s on a st. line)
$m\angle y = 180° - 129°$
$= 51°$

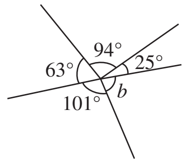
(d) $m\angle z + 40° + 30° + 55° = 180°$ (adj. $\angle$s on a st. line)
$m\angle z = 180° - 125°$
$= 55°$

4. Find the measure of the unknown marked angle in each figure.

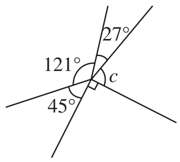
(a)



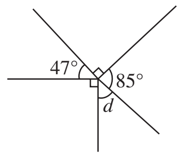
(b)



(c)



(d)



Solution

(a) $m\angle a + 92° + 100° + 43° = 360°$ ($\angle$s at a point)
$m\angle a = 360° - 235°$
$= 125°$

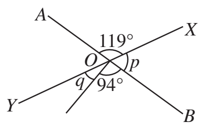
(b) $m\angle b + 101° + 63° + 94° + 25° = 360°$ ($\angle$s at a point)
$m\angle b = 360° - 283°$
$= 77°$

(c) $m\angle c + 90° + 45° + 121° + 27° = 360°$ ($\angle$s at a point)
$m\angle c = 360° - 283°$
$= 77°$

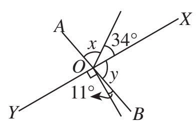
(d) $m\angle d + 90° + 47° + 90° + 85° = 360°$ ($\angle$s at a point)
$m\angle d = 360° - 312°$
$= 48°$

5. In each of the figures, the straight lines *AB* and *XY* intersect at point *O*. Find the measure of each of the following unknown marked angles.

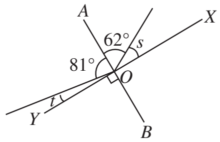
(a)



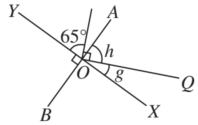
(b)



(c)



(d)



Solution

(a) $m\angle p + 119° = 180°$ (adj. $\angle$s on a st. line)
$m\angle p = 180° - 119°$
$= 61°$