Introduction To Algebra

Basic Practice

1. Simplify the following.

(a) $(2w)^2$
(b) $3p \times 4p$
(c) $3q^2 \times 5q$
(d) $2r \times (4r)^2$
(e) $12x^2 \div 4$
(f) $24y^3 \div 2y$
(g) $21w^2 \div 7w^2$
(h) $18z^2 \div (3z)^2$

2. Simplify the following.

(a) $2x \times 3y$
(b) $18y \div 3x$
(c) $6x \div 2y \times 3w$
(d) $8y \times 3y \div 2x$
(e) $p \times 5q - 2 \times 3r$
(f) $3x + 8y \div 2z$
(g) $(3p)^2 + 5q \times 2r$
(h) $(5b)^2 - 3c \times 2d$

3. When $x = 3$ and $y = 5$, evaluate the following expressions.

(a) $4x - 5y$
(b) $8y + 2x$
(c) $3y^2 + (2x)^2$
(d) $2y^3 - (2x)^3$
(e) $\frac{x}{y}$
(f) $\frac{4x}{y^2}$
(g) $\frac{x+y}{x-y}$
(h) $\frac{x^2+y^2}{(x-y)^3}$

4. When $x = -2$, $y = -5$, and $z = 3$, evaluate the following expressions.

(a) $2.5x - 3y + 4z$
(b) $3x + \frac{2z}{y}$
(c) $3xy - yz$
(d) $2y \times (z^2 - xy)$
(e) $x^2 + y^2 + z^2$
(f) $\frac{2x^3}{(z+y)^2}$
(g) $x^3 + y^3 + z^3$
(h) $-3x^3 - y^3 + \frac{1}{9}z^3$

5. Find the value of

(a) $\sqrt[3]{\frac{2p}{q}}$ when $p = 16$ and $q = \frac{1}{2}$,
(b) $p(R^2 - r^2)$ when $p = \frac{22}{7}$, $R = 25$, and $r = 24$,
(c) kx^t when $k = 5$, $x = 7$, and $t = 2$,
(d) $(kx + 2y)^z$ when $k = 3.5$, $x = 4$, $y = -5$, and $z = 3$,
(e) $\frac{k}{(\sqrt{x})^3}$ when $k = 3$ and $x = \frac{1}{4}$,
(f) $\sqrt{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}$ when $a = \frac{1}{21}$, $b = -\frac{1}{5}$, and $c = \frac{1}{9}$.

Further Practice

11. **(a)** Find the sum of

(i) $8x + 15y$ and $6x - 10y$,

(ii) $7a - 3b$, $-4a + 9b$, and $-9a - 10b$,

(iii) $2(4p - 5q)$ and $3(-4q + 3p)$,

(iv) $\frac{1}{4}$ of $(8x - 12y)$ and $\frac{3}{2}$ of $(4x + 10y)$.

(b) Subtract

(i) $4s + 9t$ from $3s - t$,

(ii) $8r - 5w$ from $7w + 12r$,

(iii) $-\frac{2}{3}(3x + 9y)$ from $\frac{1}{2}(8x + 14y)$.

(c) Subtract $7m - 8n$ from the sum of $7n - 8m$ and $20m - 9n$.

12. Simplify each of the following.

(a) $(3m - 7) + 2(4m - 5n) - 3(1 - 2n)$

(b) $(3a + 5b - 7) + (4a - 6b + 5)$

(c) $(4p - 7q - 9) - (p + 5 + 3q)$

(d) $\left(-\frac{1}{2}x + \frac{2}{3}y - \frac{3}{4}\right) - \left(\frac{3}{2}x - \frac{7}{3}y + \frac{1}{4}\right)$

(e) $5(x + 4y - 1) + 4(-4x + 6y - 2)$

(f) $-5(3p - 2q - 8) - 4(-10 + 3p - q)$

(g) $3\left(\frac{1}{6}a + \frac{1}{4}b - 2\right) + 4\left(\frac{5}{8}a + \frac{9}{16}b - 1\right)$

(h) $\frac{8}{5}\left(\frac{5}{2}s - \frac{3}{4}t - \frac{5}{8}\right) - \frac{2}{3}\left(12s + \frac{6}{5}t - 3\right)$

13. Simplify each of the following.

(a) $4[-2a + 4 - 2(a + 3)]$

(b) $6w - 5 + 3[(4 - 3w) - 2(w - 8)]$

(c) $4 - 7c - 2[(c + 4) + 2(2c - 5)]$

(d) $2s + 9 - 3(s - 5) - 2[3(3 - s) + 2(4 - 3s)]$

(e) $3[5 - 3w - 5(2w + 1)]$

(f) $-y + 3x + 2[3x - y + 2(y - 2x)]$

(g) $4(3p + 7q) - 5[4p - (q + 4p) + 5q]$

(h) $-21m + 8n - 3[2(m - 2n) - 3(3m - 2n)]$

14. **(a)** **(i)** Simplify the expression $3a + 9 - 5a - 6$.

(ii) Hence, find the value of the expression when $a = 2.5$.

(b) **(i)** Simplify the expression $2(4b - 7c) - 3(2c - 3b)$.

(ii) Hence, find the value of the expression when $b = -6$ and $c = \frac{1}{2}$.

(c) **(i)** Simplify the expression $\frac{x}{3}(6y - 9) - \frac{x}{2}(8y - 6)$.

(ii) Hence, find the value of the expression when $x = 5$ and $y = -3$.

(d) **(i)** Simplify the expression $\frac{3}{5}p - \frac{1}{4}q + \frac{3}{10}(2p - q)$.

(ii) Hence, find the value of the expression when $p = 15$ and $q = -10$.

(e) **(i)** Simplify the expression $40 - z - 3[2(4 + 3z) - 3(3z - 1)]$.

(ii) Hence, find the value of the expression when $z = 4$.

15. Express each of the following in its simplest form.

(a) $\frac{2x + 1}{3} + \frac{x - 3}{4}$

(b) $\frac{4y - 3}{3} - \frac{y - 5}{2}$

(c) $\frac{4z + 2}{4} + \frac{1 - 5z}{5}$

(d) $\frac{3(2 - 3w)}{2} + \frac{6(4w - 3)}{5}$

(e) $\frac{3(4p + 5)}{5} - \frac{2(3p + 1)}{3}$

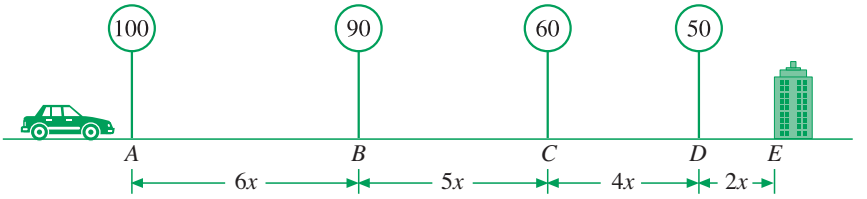
(f) $\frac{q + 5}{2} + \frac{2q + 7}{5} - 1$

(g) $\frac{2(2p - q)}{3} - \frac{3(q + 4p)}{2} + \frac{1}{4}$

(h) $12\left(\frac{m + 2m}{3} - \frac{m - 3n}{6} - \frac{m + n}{2}\right)$

Enrichment

26.



In the figure, $ABCDE$ is a portion of a road from the exit A of an expressway to a building E. $AB = 6x$ km, $BC = 5x$ km, $CD = 4x$ km, and $DE = 2x$ km. A car drives at the speed limits, i.e., 100 km/hr, 90 km/hr, 60 km/hr, and 50 km/hr in each section from A to E respectively. Let T minutes be the time taken by the car to reach E from A.

(a) Express T in terms of x.

(b) When $x = 0.45$, find the value of T.

27. The sides of $\triangle ABC$ are $AB = (3x + 4)$ cm, $BC = (4x - 5)$ cm, and $CA = (x + 13)$ cm.

(a) Express the perimeter of $\triangle ABC$ in terms of x. Give the answer in factored form.

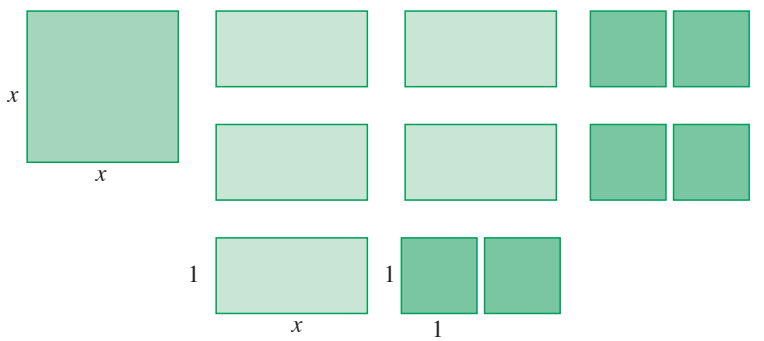
(b) A square $PQRS$ has the same perimeter as $\triangle ABC$. Express the length of PQ in terms of x.

(c) When $x = 7$, find

(i) the perimeter of $\triangle ABC$,

(ii) the area of $PQRS$.

28.



(a) The figure shows 1 square tile of x by x units, 5 rectangular tiles of x by 1 unit, and 6 square tiles of 1 by 1 unit. Arrange the tiles to form a rectangle and state its dimensions.

(b) Hence, or otherwise, express $x^2 + 5x + 6$ in the form $(x + a)(x + b)$, where a and b are integers.

(c) Express $x^2 + 8x + 15$ in the form $(x + p)(x + q)$, where p and q are integers.

29. The volumes of two glasses of water are $(7ax - 3bx + 6ay - 4by)$ cm^3 and $(11bx + 5ax - 6by - 21ay)$ cm^3 respectively. Let V cm^3 be the total volume of water in the two glasses.

(a) Express V in terms of a, b, x, and y in factored form.

(b) If both x and y are doubled, determine whether V will be doubled.

Challenging Practice

24. The following table shows Kenneth's results in 4 tests.

Test Number	Score	Maximum Possible Score
1	6.5	10
2	12	20
3	19	25
4	28	40

(a) In which test was Kenneth's performance the best? Explain your answer.
(b) For each test, grade 'A' is given if the score is more than or equal to 70% of the maximum possible score. Find, as a percentage, the number of times Kenneth was given grade 'A'.
(c) Suppose that 67.5% of the students in Kenneth's class were given grade 'A' at least once in the 4 tests. Find the number of students who were not given grade 'A' in any of the tests if there are 40 students in the class.

25. **(a)** A fruit crate contains a mix of 80 apples and oranges. If 21.25% of the fruits are rotten, find the number of rotten fruits.
(b) Suppose that 30% of the apples and $\frac{1}{5}$ of the oranges are rotten. Find the number of
(i) rotten apples,
(ii) rotten oranges.
(c) Hence, express the number of apples as a percentage of
(i) the number of fruits,
(ii) the number of oranges.

26. Eligible clients of a bank are offered 2 repayment schemes for a one-year loan.
Scheme *A*: Pay \$50 and 105% of the loan at the end of the one-year period
Scheme *B*: Pay 103% of the sum of \$200 and the loan at the end of the one-year period
(a) **(i)** Which is a better scheme for Mr. Martin to use if he is eligible for the loan and wants to borrow \$10,000?
(ii) How much will he save if he selects the better scheme?
(b) Mr. Carter, another eligible client, also borrowed from the bank. Find his loan amount if his payment by either of the schemes is the same.

27. **(a)** If X is 25% less than Y, by how many percent is Y more than X?
(b) If X is 25% more than Y, by how many percent is Y less than X?
(c) If X is decreased by 10% and then increased by 10%, find the percentage change in X.
(d) If Y is increased by 10% and then decreased by 10%, find the percentage change in Y.