Chapter 4 Algebraic Manipulation

Suggested Approach

Algebraic manipulation is crucial for students in learning mathematics and science. Careful elaboration of the concepts is necessary in order to build a strong foundation. Students should be encouraged not to skip steps when presenting their solutions.

An algebraic expression can be considered as a machine, with the terms as its parts. Analogously, like terms and unlike terms are like parts and unlike parts respectively of the machine. We may use an activity to ask students to identify coefficients of given terms, like terms and unlike terms. Models can be used to introduce the idea of simplification of like terms.

Numerical expressions and geometrical interpretation can be used to introduce distributive property. Teachers may present various cases and help students discuss how to handle brackets in algebraic expressions.

Teachers may draw the analogy between prime factorization of a whole number and the factorization of an algebraic expression. For factorization by grouping terms, teachers may provide some guided practice to students and let them discover the skill by themselves.

4.1 Like Terms and Unlike Terms

It should be emphasized that the sign of a term is attached to its coefficient and not the variable. Terms with the same variables may not be like terms. For instance, (x and x^3) and (a^2b and ab^2) are two pairs of unlike terms.

4.2 Distributive Law, Addition, and Subtraction of Linear Algebraic Expressions

Students should be careful when removing brackets. The vertical form of addition and subtraction will be used in long multiplication and division. Thus, it is helpful to teach this method as well.

4.3 Simplification of Linear Algebraic Expressions

This section is confined to the expansion of linear algebraic expressions. Students should learn the skill of both distribution from the left and distribution from the right.

4.4 Factorization by Extracting Common Factors

Factorization by extracting the common factor may be considered as the reverse process of expansion. After sufficient practice, students should be able to locate the appropriate factor. They should develop the habit of expanding the factorization result and see whether the original expression can be obtained. Instead of factoring $ax + ay + a$ as $a(x + y + 1)$, some students drop the 1 and get the wrong answer $a(x + y)$.

4.5 Factorization by Grouping Terms

Students should be encouraged to try different combinations of grouping terms. They should be made aware that some algebraic expressions cannot be factorized.

Chapter 3 Introduction To Algebra

Extend Your Learning Curve

Hand-Shaking Problem

There are n persons attending a party. Each person shakes hands with every other person just once. Let T be the total number of handshakes.

(a) Complete the following table.

n	2	3	4	5	6
T					

(b) Establish a formula connecting n and T.

Solution

(a)

n	2	3	4	5	6
T	**1**	**3**	**6**	**10**	**15**

(b) Suppose the n persons in the party are arranged in order.

The 1st person will shake hands with $n - 1$ persons.
The 2nd person will shake hands with the remaining $n - 2$ persons, without repeating.
The 3rd person will shake hands with the remaining $n - 3$ persons, without repeating.

The $(n - 1)$th person will just shake hands with the nth person, without repeating.
$\therefore$ the total number of handshakes is given by
$T = (n - 1) + (n - 2) + (n - 3) + ... + 3 + 2 + 1$ ....(1)

By reversing the above expression, we have

$T = 1 + 2 + 3 + ... + (n - 3) + (n - 2) + (n - 1)$..........................(2)

(1) + (2), $2T = n + n + n + ...$ for $(n - 1)$ terms
$2T = n(n - 1)$
$\therefore\ T = \frac{1}{2}n(n - 1)$

17. The price $\$P$ for a birthday cake of radius r centimeters and height h centimeters is given by the formula

$$P = \frac{1}{25}r^2h.$$

(a) Find the price for a cake of radius 10 cm and height 8 cm.

(b) If the height of the cake in **(a)** is increased to 10 cm, what is the increase in price?

Solution

(a) When $r = 10$ and $h = 8$,

$$\begin{aligned} P &= \frac{1}{25}r^2h \\ &= \frac{1}{25} \times 10^2 \times 8 \\ &= 32 \end{aligned}$$

The price of the cake is \$32.

(b) When $r = 10$ and $h = 10$,

$$\begin{aligned} P &= \frac{1}{25}r^2h \\ &= \frac{1}{25} \times 10^2 \times 10 \\ &= 40 \end{aligned}$$

Increase of price = \$(40 – 32)
= \$8

The increase in price is \$8.

Brainworks

18. **(a)** When two resistors of resistance a ohms and b ohms are connected to two points, X and Y, by using different wires in a circuit as shown in Diagram 1, the equivalent resistance R ohms is given by the formula

$$R = \frac{ab}{a+b}.$$

Find the value of R when $a = 20$ and $b = 30$.

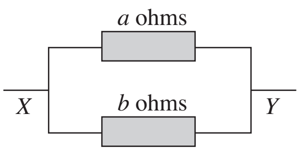


Diagram 1

(b) Suppose 3 resistors of 20 ohms, 30 ohms, and 15 ohms are connected to the points X and Y in the circuit as shown in Diagram 2. Using the formula in **(a)**, find their equivalent resistance.

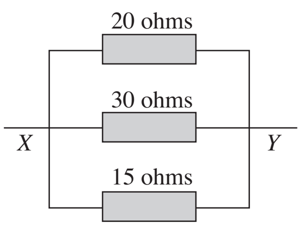


Diagram 2

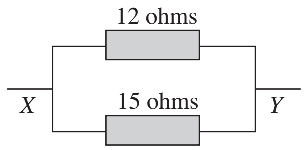
Solution

(a) When $a = 20$ and $b = 30$,

$$\begin{aligned} R &= \frac{ab}{a+b} \\ &= \frac{20 \times 30}{20 + 30} \\ &= 12 \end{aligned}$$

The value of R is 12.

(b) Combining the 20 ohms and 30 ohms resistors first, we have an equivalent circuit as shown.
Take $a = 12$ and $b = 15$,



$$\begin{aligned} R &= \frac{ab}{a+b} \\ &= \frac{12 \times 15}{12 + 15} \\ &= \frac{180}{27} \\ &= \frac{20}{3} \\ &= 6\frac{2}{3} \end{aligned}$$

The equivalent resistance is $6\frac{2}{3}$ ohms.

Exercise 3.3

Basic Practice

1. The length of a rectangular room is 10 feet more than its width. Suppose the width of the room is w feet, what is the length of the room in terms of w?

Solution

Length of the room = $(w + 10)$ ft

2. The price of a burger is \$6 less than that of a pizza. Given that the price of the pizza is $\$p$, express the price of the burger in terms of p.

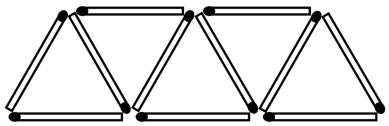
Solution

Price of burger = $\$(p - 6)$

Extend Your Learning Curve

Matchstick Triangle Patterns

Johnny uses matchsticks to form a pattern of triangles as shown below.



Suppose m matchsticks are required to form n triangles.

(a) Copy and complete the following table.

n	1	2	3	4	5	6
m	3					

(b) Find a formula connecting m and n.
(c) How many matchsticks are required to form 100 triangles?
(d) How many triangles can be formed with 2,005 matchsticks?
(e) Suppose the area of a triangle is $\sqrt{3}$ cm^2. Find the total area of the triangles formed in **(d)**. Give your answer correct to the closest whole number.

Suggested Answers

(a)

n	1	2	3	4	5	6
m	3	**5**	**7**	**9**	**11**	**13**

(b) Except for the first triangle, each triangle in a pattern of n triangles is formed by adding 2 matchsticks.
$\therefore\ m = 3 + 2(n - 1)$ or $m = 2n + 1$

(c) When $n = 100$,
$m = 2(100) + 1$
$= 201$
$\therefore$ 201 matchsticks are required to form 100 triangles.

(d) When $m = 2{,}005$,
$2{,}005 = 2n + 1$
$2{,}004 = 2n$
$\therefore\ n = 1{,}002$
1,002 triangles can be formed with 2,005 matchsticks.

(e) Total area of 1,002 triangles $= 1{,}002 \times \sqrt{3}$
$= 1{,}736$ cm^2 (correct to the closest whole number)

Exercise 6.1

Basic Practice

1. Express each ratio in the simplest form.

(a) 18 : 27 **(b)** 144 : 132

(c) $1\frac{1}{2} : 4\frac{1}{2}$ **(d)** $2\frac{2}{3} : 1\frac{1}{5}$

(e) 0.250 : 0.375 **(f)** $0.48 : 2\frac{2}{15}$

(g) 1.6 feet : 36 inches

(h) 850 grams : 3.4 kilograms

(i) $1\frac{1}{3}$ hours : 20 minutes

(j) 80¢ : $2

Solution

(a) $18 : 27 = \frac{18}{9} : \frac{27}{9}$
$= 2 : 3$

(b) $144 : 132 = \frac{144}{12} : \frac{132}{12}$
$= 12 : 11$

(c) $1\frac{1}{2} : 4\frac{1}{2} = \frac{3}{2} : \frac{9}{2}$
$= 1 : 3$

(d) $2\frac{2}{3} : 1\frac{1}{5} = \frac{8}{3} : \frac{6}{5}$
$= 8 \times 5 : 6 \times 3$
$= 40 : 18$
$= 20 : 9$

(e) $0.250 : 0.375 = 250 : 375$
$= \frac{250}{125} : \frac{375}{125}$
$= 2 : 3$

(f) $0.48 : 2\frac{2}{15} = \frac{48}{100} : \frac{32}{15}$
$= \frac{48}{100} \times \frac{300}{16} : \frac{32}{15} \times \frac{300}{16}$
$= 9 : 40$

(g) 1.6 feet : 36 inches = 19.2 inches : 36 inches
$= \frac{19.2}{2.4} : \frac{36}{2.4}$
$= 8 : 15$

(h) 850 grams : 3.4 kilograms = 850 g : 3,400 g
$= 1 : 4$

(i) $1\frac{1}{3}$ hours : 20 minutes = 80 minutes : 20 minutes
$= 4 : 1$

(j) 80¢ : $2 = 80¢ : 200¢
$= \frac{80}{40} : \frac{200}{40}$
$= 2 : 5$

2. Given that $a : b : c = 20 : 35 : 15$,

(a) simplify $a : b : c$,

(b) find $a : b$,

(c) find $c : b$.

Solution

(a) $a : b : c = 20 : 35 : 15$
$= \frac{20}{5} : \frac{35}{5} : \frac{15}{5}$
$= 4 : 7 : 3$

(b) $a : b = 4 : 7$

(c) $c : b = 3 : 7$

3. Given that $x : y : z = 5\frac{1}{2} : 4.62 : 33$,

(a) simplify $x : y : z$, **(b)** find $y : x$,

(c) find $x : z$.

Solution

(a) $x : y : z = 5\frac{1}{2} : 4.62 : 33$
$= \frac{11}{2} : \frac{462}{100} : 33$
$= \frac{1}{2} : \frac{42}{100} : 3$
$= \frac{1}{2} \times 50 : \frac{42}{100} \times 50 : 3 \times 50$
$= 25 : 21 : 150$

(b) $y : x = 21 : 25$

(c) $x : z = 25 : 150 = 1 : 6$

4. Find the ratio of $a : b : c$.

(a) $a : b = 3 : 4$, $b : c = 4 : 9$

(b) $a : b = 5 : 3$, $b : c = 4 : 1$

(c) $a : b = \frac{1}{2} : 1$, $b : c = 1 : \frac{1}{3}$

(d) $a : b = 3 : 7$, $b : c = 3 : 7$

Solution

(a) $a : b = 3 : 4$
$b : c = 4 : 9$
$\therefore\ a : b : c = 3 : 4 : 9$

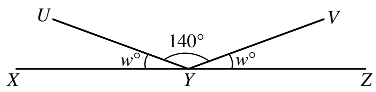
(b) $a : b = 5 : 3$
$= 20 : 12$
$b : c = 4 : 1$
$= 12 : 3$
$\therefore\ a : b : c = 20 : 12 : 3$

(c) $a : b = \frac{1}{2} : 1$
$b : c = 1 : \frac{1}{3}$
$\therefore\ a : b : c = \frac{1}{2} : 1 : \frac{1}{3}$
$= \frac{1}{2} \times 6 : 1 \times 6 : \frac{1}{3} \times 6$
$= 3 : 6 : 2$

Try It!

Section 8.2

1. In the diagram, *XYZ* is a straight line. Find the value of *w*.



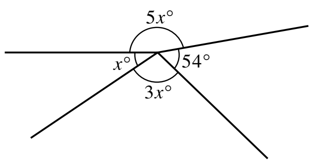
Solution

$w° + 140° + w° = 180°$ (adj. ∠s on a st. line)

$2w = 40$

$w = 20$

2. Find the value of *x* in the diagram.



Solution

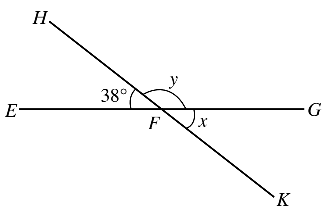
$5x° + x° + 3x° + 54° = 360°$ (∠s at a point)

$9x + 54 = 360$

$9x = 306$

$x = 34$

3. In the diagram, *EFG* and *HFK* are straight lines. Find $m\angle x$ and $m\angle y$.



Solution

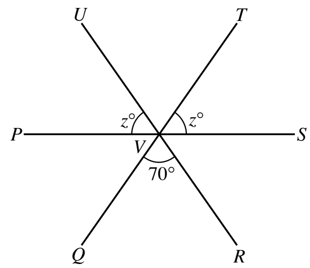
$m\angle x = m\angle EFH$ (vert. opp. ∠s)

$= 38°$

$m\angle y + 38° = 180°$ (adj. ∠s on a st. line)

$m\angle y = 142°$

4. In the diagram, *PS*, *QT* and *RU* are straight lines, intersecting at *V*. Find the value of *z*.



Solution

$m\angle UVT = 70°$ (vert. opp. ∠s)

$z° + m\angle UVT + z° = 180°$ (adj. ∠s on a st. line)

$z° + 70° + z° = 180°$

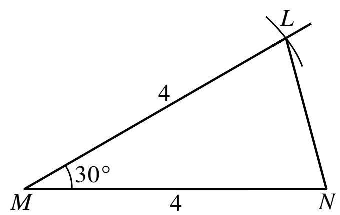
$2z = 110$

$z = 55$

Section 8.4

5. Construct △*LMN* with *LM* = 4 cm, *MN* = 4 cm, and ∠*LMN* = 30°. Measure and write down the length of *LN*.

Solution



Construction Steps:

1. Construct a line segment *MN* 4 cm long.
2. Draw a ray with the end point *M* and making an angle of 30° with *MN* using a protractor.
3. With *M* as centre and 4 cm as radius, draw an arc to cut the ray at *L*.
4. Join *L* and *N*. △*LMN* is the required triangle.
   LN = 2 cm