UNIT 1
NUMBERS AND RADICALS

1.1 The Real Number System

1.2 Powers and Roots of Numbers

1.3 Ordering Radicals and Using a Calculator to Find Approximate Values

1.4 Simplifying Radicals by Factoring

1.5 Adding and Subtracting Radicals

1.6 Multiplication and Division of Square Root Radicals

1.7 Laws of Exponents for Rationals

1.8 Applications of Rational Exponents

1.1 The Real Number System

The system of **real numbers** consists of a collection of smaller sets of numbers that has evolved over several centuries. It began with numbers used to count objects, which were used for trading and other commercial purposes. It was extended and refined as a need for numbers to represent parts of objects and locations on the number line became important. Below is a discussion of the sets of numbers that comprise the Real Number System. Each of these sets of numbers builds on those contained in the preceding set.

Graphs of real numbers can be associated with fixed points along the number line. Every point on the line corresponds to a number in the set of "reals." All of these points on the line have tangible "real" values associated with them.

Composition of the Real Number System

The Real Number System is comprised of two major sets of numbers: **rational numbers (Ra)** and **irrational Numbers (Ir)**.

Rational plus Irrational Numbers = Real Numbers

Rational numbers consist of all numbers that can be written in the form $\frac{a}{b}$, b $\neq 0$

Examples: $0.4 = \frac{1}{5}$, $-1\frac{1}{4} = -\frac{5}{4}$, $0 = \frac{0}{4}$

Irrational numbers consist of all other tangible numbers that cannot be written in that form.

Examples: $\sqrt{3}$, $\sqrt{11}$, 1.2013772…

Together, these sets of numbers make up the Real Number System.

Rational Numbers (Ra)

The rational numbers can be broken down further to obtain the following:

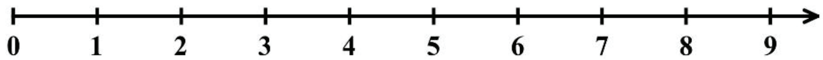
Natural Numbers (N)

Natural numbers can be thought of as counting numbers. They can be used to identify how many objects are contained in a collection. Since a collection of objects has at least one item in it, the natural numbers begin with the number 1 and then proceed to represent additional objects in the set. Counting numbers can be listed as follows: 1, 2, 3, 4, 5, 6, 7, 8, …

Whole Numbers (W)

Whole numbers consist of the natural numbers in addition to the number 0. Although 0 does not represent an object in a set, it is an important addition to the number system. Whole numbers can be listed as follows: 0, 1, 2, 3, 4, 5, 6, 7, 8, …

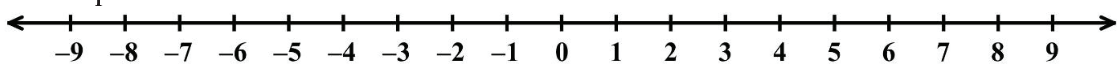
Whole numbers correspond to locations on the number line as follows:



Integers (I)

The set of **Integers** builds on the set of Whole numbers by adding the negative values of each. As a result, it includes numbers such as -1, -2, -3, -4, -5, … Note that there is no negative value for 0. Negative values of Whole numbers are used in many situations, such as to represent a minus temperature (-22° C), distance below sea level (-8 m below the sea), or a golf score that is under par (-4 strokes under par).

Integers correspond to locations on the number line as follows:



Illustrating Real Numbers with a Venn Diagram

As shown in the Venn diagram below, the set of **rational numbers** includes each of the following.

Natural Numbers: **N** = {1, 2, 3, 4, …}

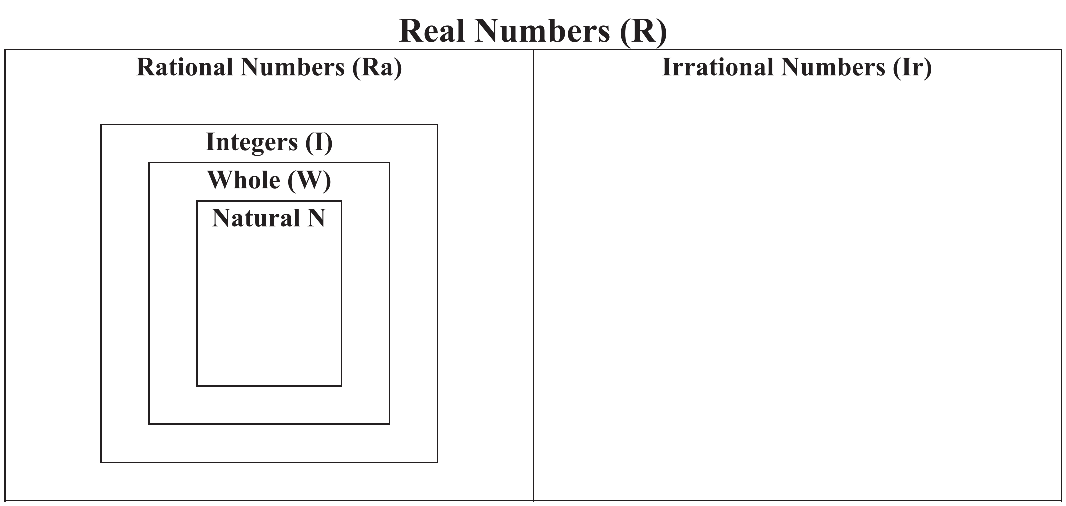
Whole Numbers: **W** = {0, 1, 2, 3, 4, …}

Integers: **I** = {…, -4, -3, -2, -1, 0, 1, 2, 3, 4, …}

Rational: **Ra** = all of the above plus any other number that can be written in the form $\frac{a}{b}$, b ≠ 0

Examples: 0.5, 1.24, $\frac{2}{5}$, -1$\frac{1}{4}$, 7)

Together, all of the **rational** and **irrational** numbers make up the set of **real** numbers.



Identification of Rational and Irrational Numbers

Recall that rational numbers can be shown in several different formats, as long as they can be rewritten in the form $\frac{a}{b}$, b ≠ 0.

1. Natural Numbers, Whole Numbers, and Integers
   Examples: 7, -43, 0, 2761, - 403
2. Proper Fractions, Mixed Numbers, or Improper Fractions
   Examples: $\frac{3}{11}$, -$\frac{2}{9}$, 3$\frac{1}{4}$, $\frac{7}{5}$, -8$\frac{1}{10}$, -$\frac{7}{3}$
3. Decimals – terminating or repeating
   Examples: 0.8, -0.25, 0.22$\overline{3}$, 2.$\overline{61}$

Irrational numbers cannot be shown as common fractions.

1. Decimals that do not terminate or repeat in a pattern (Example: 0.12323569…)
2. Roots of numbers that are not rational (Example: $\sqrt{2}$, $\sqrt{11}$, -3$\sqrt{5}$, …)
3. Special numbers like π

Examples with Solutions

Identify which of the following are rational and which are irrational numbers. Give a reason for your answer.

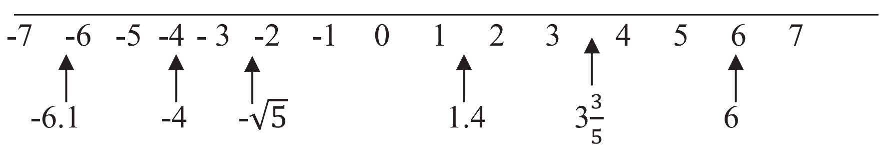
Number	Rational or Irrational?	Reason
1. -1.75	Rational	It can be written as -$\frac{175}{100}$ or $-\frac{7}{4}$.
2. 0.4010347…	Irrational	The decimal doesn’t terminate or repeat the same pattern.
3. -$\sqrt{36}$	Rational	It can be written as -6.
4. 0.2222 …	Rational	It repeats the same pattern and can be written as $\frac{2}{9}$.
5. $\sqrt{17}$	Irrational	The decimal version doesn’t repeat the same pattern $\sqrt{17}$ = 4.123105626…
6. 5.01	Rational	It has a terminating decimal. It could be written as 5$\frac{1}{100}$ or $\frac{501}{100}$.
7. -9$\frac{1}{2}$	Rational	It could be written as $-\frac{19}{2}$.

8. $3125\frac{1}{4}$	Rational	It could be written as $\frac{12501}{4}$.
9. 4.333…	Rational	The decimal repeats the same pattern and is equal to $4\frac{1}{3}$ *or* $\frac{13}{3}$.
10. 7.0100382…	Irrational	The decimal doesn't terminate or repeat the same pattern.

Comparing and Ordering Real Numbers

Each real number corresponds to a point on the number line.

Examples:



It should be noted that numbers **increase** in magnitude as you go from left to right on the line.

Examples: $1 < 3$, $2.1 < 4$; $-7 < -6$; and $2 > 1.8$; $-3 > -5$; $-1 > -10.5$

To compare the magnitudes of rational numbers where one is written in decimal and the other in common fraction form, write both either in decimal or in common fraction form and then compare.

1. Compare 0.1 with $\frac{3}{40}$. Convert both to fractions first. Change 0.1 to $\frac{1}{10}$.
   The common denominator is 40, $\therefore\ \frac{1}{10} = \frac{4}{40}$
   $\frac{4}{40} > \frac{3}{40}$ or $0.1 > \frac{3}{40}$

2. Compare 3.15 with $3\frac{2}{11}$. Convert both to decimals first. Change $3\frac{2}{11}$ to a decimal $\rightarrow 3.\overline{18}$.
   $\therefore\ 3.15 < 3.\overline{18}$

Exercises 1.1

Identify each of the following as rational or irrational numbers. Give a reason for your answer.

	Rational or Irrational	Reason
1. 0.017		
2. $7\frac{1}{2}$		
3. 5.0900134...		
4. 0.3333…		
5. -19.001		
6. $\sqrt{64}$		
7. 0.144357…		
8. 0.313131…		
9. 510.013		
10. $\sqrt{6}$		
11. -5.666…		
12. 4.009		
13. $-343\frac{1}{3}$		

Use a check mark to indicate which set(s) each number belongs to.

		Set of Numbers				
	Number	N	W	I	Ra	Ir
14.	0.8					
15.	-35					
16.	$\frac{17}{7}$					
17.	0.15243…					

	Number	N	W	I	Ra	Ir
18.	$0.\overline{7}$					
19.	3					
20.	$1\frac{3}{8}$					
21.	-155					
22.	0					
23.	$\sqrt{100}$					
24.	0.83					
25.	$\sqrt{20}$					

N = Natural Numbers, W = Whole Numbers, I = Integers, Ra = Rational Numbers, Ir = Irrational Numbers

26. Locate the following numbers on the number line.

$3.2,\ 2\frac{3}{8},\ -\frac{13}{12},\ -\sqrt{5},\ -\sqrt{16}$

-4 -3 -2 -1 0 1 2 3 4

27. Arrange the following numbers from smallest to largest.

a. –1.57, -0.517, -5.17, -5.71

b. $3.4,\ -\frac{11}{3},\ -3.4,\ -3.5$

c. $-\frac{3}{8},\ -\frac{2}{3},\ -0.6,\ -0.4$

28. Put the correct symbol (>, =, <) between each pair of numbers.

a. 0.16 $\frac{7}{40}$

b. -1.8 $-\frac{9}{5}$

c. -2.7 $-\frac{13}{5}$

29. Express each term in common fraction form (as a quotient of two integers).

a. 0.19

b. $-0.\overline{7}$

c. $-1\frac{2}{5}$

d. 3.09

230. Which rational number is greater?

a. $-0.\overline{7}$ or -0.7?

b. -0.25 or $-\frac{1}{3}$?

c. $-\frac{2}{3}$ or $-\frac{4}{5}$?

<u>Extra for Experts</u>

31. List the set of all Integers greater than -4 and less than $\frac{1}{3}$.

32. Is $\sqrt{\frac{4}{25}}$ rational or irrational? Give a reason for your answer.

33. Is the sum of the following numbers rational or irrational? Give a reason for your answer.
0.1 + 0.01 + 0.001

34. Is the sum of the following numbers rational or irrational? Give a reason for your answer.
0.333… + 0.666… + 0.999…

35. If the sum of 3.72 + 12.<u>*ab*</u> is an integer, what digits must go in place of <u>*ab*</u>?

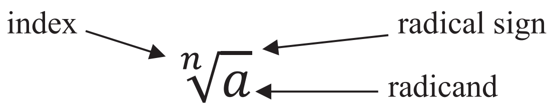
36. If the sum of $\mathbf{-12 + -8\frac{1}{2} +} \boldsymbol{n}$ is a natural number, what is the smallest number that can replace ***n***?

1.2 Powers and Roots of Numbers

If we raise a number to a **positive integer power**, we multiply it by itself that number of times. For example, 5 to the power 3 is 5^3 is equal to $5 \times 5 \times 5 = 125$, or 7 to the power 2 is 7^2 is equal to $7 \times 7 = 49$.

When we go in the opposite direction, instead of raising a number to a power, we find the **root** of a number. For example, the second or square root of 16 is 4 since 4 is one of its two equal factors. The third or cube root of 27 is 3 because it is one of its three equal factors.

We use the **radical sign** to represent the root of a number. The number under the radical sign is called the **radicand** and the root is called the **index**.



Examples:

$\sqrt[2]{64}$ - the square (or second) root of 64 — The radicand is 64. The index is 2. Since square roots are very common, we usually leave the index out so that $\sqrt[2]{64} = \sqrt{64}$

$\sqrt[3]{1000}$ - the cube (or third) root of 1000 — The radicand is 1000. The index is 3.

Raising a number to a Power

The process of **squaring a number** (raising it to the power 2) involves multiplying it by itself for a total of 2 factors.

Examples: $5^2 = 5(5) = 25$, $(\frac{3}{7})^2 = (\frac{3}{7})(\frac{3}{7}) = \frac{9}{49}$

The process of raising a number to the power 3 involves multiplying it by itself twice for a total of 3 factors.

Examples: $2^3 = 2(2)(2) = 8$, $7^3 = 7(7)(7) = 343$

Finding the Root of a number

The process of finding the **square root** of a number is to determine one of its two equal factors; for example, $\sqrt{25} = \sqrt{(5)(5)} = 5$. It should be noted that $\sqrt{25} = \sqrt{(-5)(-5)} = -5$ as well; however, in most cases, when finding the square root of a number we will take only the positive square root of a number which is called the **principal square root**.

The process of finding the **cube or third root** of a number is to determine one of its three equal factors; for example, $\sqrt[3]{125} = \sqrt[3]{(5)(5)(5)} = 5$.

ABORIGINAL APPLICATIONS
THE MOON



Artist: T. Isaac

Aboriginal people view the moon as the protector of the earth and of people at night. It is a symbol of power and is often used to show esteem or respect.

The moon forms the basis for Aboriginal calendars. Different times of the moon are closely linked to nature and reflect events that occur with the changing of the seasons and the phases of life. There are 13 moons in a typical lunar calendar year, compared to 12 months in the western calendar year. There are 28 days from one full moon to the next.

Math Applications

1. The volume of the moon is approximately 2.2×10^{10} km^3 and the volume of the earth is approximately 1.08×10^{12} km^3.

   a. Find the ratio of the moon's volume to that of the earth. Round to 4 decimal places.

   b. The moon is what percent of the volume of the earth? Round to 2 decimal places.

   c. The equatorial radius of the moon is 1738.1 km. Write this distance as a number rounded to the nearest decimal times ten to a power.

Answers

1. a) $(2.2 \times 10^{10}) \div (1.08 \times 10^{12}) = 2.2/108 = 0.0204$
   b) 2.04%
   c) 1.7×10^3

ANSWERS TO EXERCISES AND UNIT TESTS

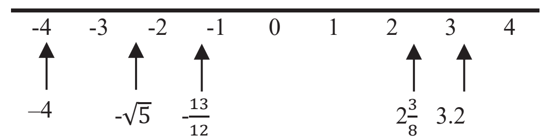
UNIT 1

Exercises 1.1 (page 6)

Number	Rational/ Irrational	Reason
1. 0.017	Rational	Can be written as $\frac{17}{1000}$
2. $7\frac{1}{2}$	Rational	Can be written as $\frac{15}{2}$
3. 5.0900134...	Irrational	Decimal doesn't terminate or repeat
4. 0.333….	Rational	Repeating decimal equal to $\frac{1}{3}$
5. -9.001	Rational	Can be written as $-\frac{9001}{10\,000}$
6. $\sqrt{64}$	Rational	Equal to 8
7. 0.14431353…	Irrational	Decimal doesn't terminate or repeat
8. 0.313131…	Rational	Repeating decimal equal to $\frac{31}{99}$
9. 510.013	Rational	Terminating decimal
10. $\sqrt{6}$	Irrational	Decimal value doesn't terminate or repeat 2.44948974…
11. -5.666…	Rational	Decimal repeats and can be written as $-\frac{17}{3}$
12. 4.009	Rational	Decimal terminates and can be written as $\frac{4009}{1000}$
13. $-343\frac{1}{3}$	Rational	Can be written as $-\frac{1030}{3}$

		Set of Numbers				
	Number	**N**	**W**	**I**	**Ra**	**Ir**
14.	0.8				√	
15.	-35			√	√	
16.	$\frac{17}{7}$				√	
17.	0.15243…					√
18.	$0.\overline{7}$				√	
19.	3	√	√	√	√	
20.	$1\frac{3}{8}$				√	
21.	-155			√	√	
22.	0		√	√	√	
23.	$\sqrt{100}$	√	√	√	√	
24.	0.83				√	
25.	$\sqrt{20}$					√

26.



27. a) -5.71, -5.17, -1.57, -0.517
b) $-\frac{11}{3}$, -3.5, -3.4, 3.4 **c)** $-\frac{2}{3}$, - 0.6, -0.4, $-\frac{3}{8}$
28. a) < **b)** = **c)** < **29. a)** $\frac{19}{100}$ **b)** $-\frac{7}{9}$ **c)** $-\frac{7}{5}$
d) $3\frac{9}{100} = \frac{309}{100}$ **30. a)** -0.7 (it is to the right of $-0.\overline{7}$ on the number line) **b)** -0.25
c) $-\frac{2}{3}$ (change to $-\frac{10}{15}$ and $-\frac{12}{15}$)
31. -3, -2, -1, 0
32. Rational – can be written as $\frac{2}{5}$
33. Rational – can be written as $\frac{111}{1000}$
34. Rational – can be written as 2
35. 28 **36.** $21\frac{1}{2}$

Exercises 1.2 (page 11)
1. a) index = 3; radicand = 27 **b)** index = 2; radicand = $\frac{4}{9}$ **c)** index = 4; radicand = 625
d) index = 2; radicand = $\frac{4}{121}$ **e)** index = 3; radicand = 3.375 **f)** index = 5; radicand = 0.0032 **2. a)** $\sqrt{81}$ **b)** $\sqrt[3]{216}$
c) $\sqrt[4]{625}$ **d)** $\sqrt{0.09}$ **e)** $\sqrt[5]{32}$ **f)** $\sqrt[3]{\frac{27}{125}}$
3. a) 243 **b)** 0.000027 **c)** $\frac{8}{27}$ **d)** $\frac{81}{4}$
e) 0.00032 **f)** 0.00000081 **4. a)** 3 **b)** $\frac{2}{3}$ **c)** 5
d) $\frac{2}{11}$ **e)** 0.1 **f)** 0.2 **g)** $\frac{1}{3}$ **h)** $\frac{2}{5}$ **i)** 12 **j)** 4
k) 0.3 **l)** 1 **5.** 4 **6.** $\frac{2}{3}$ **7.** $\frac{2}{3}$ **8.** $\frac{2}{3}$

Exercises 1.3 (page 15)
1. a) $\sqrt{5}$, 3, $\sqrt{11}$, 4 **b)** $\sqrt{3}$, $\sqrt{3.5}$, 2, 3
c) $\sqrt{5}$, $\sqrt{6}$, 2.5, 4 **d)** $\sqrt{\frac{1}{8}}$, $\frac{1}{2}$, $\sqrt{2}$, 2
e) $\sqrt[3]{25}$, 3, $\sqrt[3]{60}$, 4 **f)** 1, $\sqrt[3]{2}$, $\sqrt[3]{7}$, 2 **2. a)** 3.16