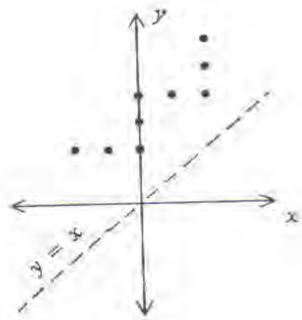
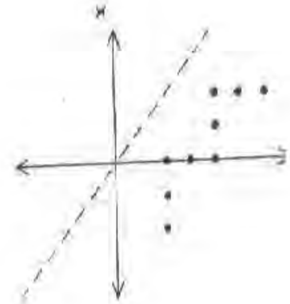




CHAPTER 1

TRANSFORMATIONS

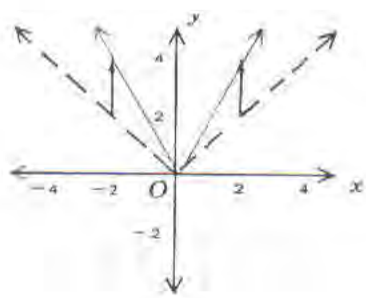
1.1 Relations and Functions (Review)

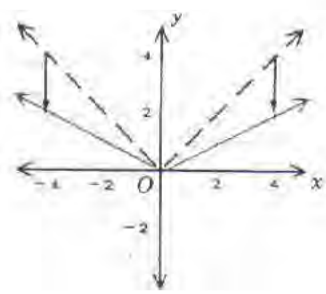
1.2 Even and Odd Functions

1.3 Vertical and Horizontal Translations

1.4 Compressions and Expansions

1.5 Reflections and Inverses





1.1 Relations and Functions (Review)

Before we work with transformations of functions and their related equations, it is important to review what is meant by a **relation** and a **function**. In this section, we will look at relations and functions, in addition to their corresponding equations and graphs.

Relation

A relation is a set of ordered pairs. As a result, every graph is a relation. A relation can be thought of as a set of ordered pairs. All first elements in the set of ordered pairs are called the domain while all second elements determine the range.

Relations can be shown in several ways, among them the following: as a word description, a set of ordered pairs, an equation or inequality, or as a graph. An example of a relation shown in each of these ways is next.

Word description

An output number is determined by multiplying an input number by 2 and then adding 1.

Equation

$y = 2x + 1$

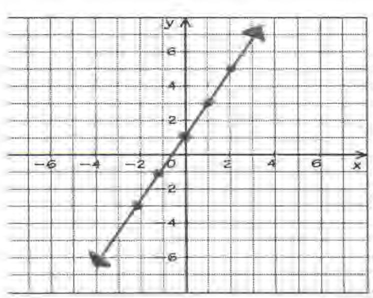
Set of ordered pairs

$(0, 1), (-\frac{1}{2}, 0), (1, 3), (2, 5), (-1, -1), (-2, -3), \ldots$

Table of values

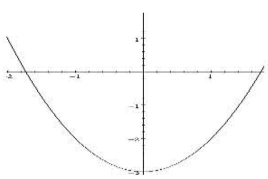
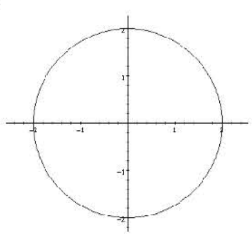
x	0	$-\frac{1}{2}$	1	2	-1	-2	-	-
y	1	0	3	5	-1	-3	-	-

Graph

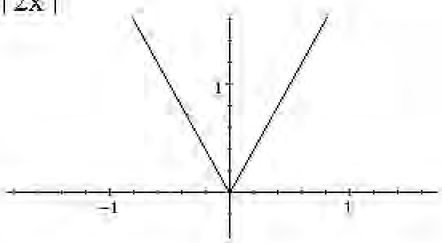


In the preceding graph of **y** = 2**x** + 1
↑ Output ↑ Input

x is the independent variable.
y is the dependent variable.
The domain is all real numbers (all possible values of x).
The range is all real numbers (all possible values of y).

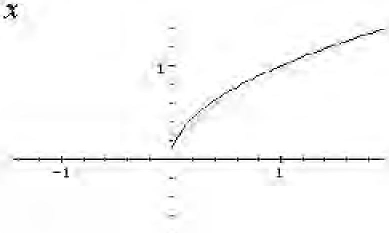
Examples of Relations	**Domain**	**Range**
Ordered Pairs		
(2, 3), (3, 5), (4, 6), (5, 8)	2, 3, 4, 5	3, 5, 6, 8
$(-2, 3), (-\frac{1}{2}, 4), (0, \frac{9}{2}), (2, 6)$	$-2, -\frac{1}{2}, 0, 2$	$3, 4, \frac{9}{2}, 6$
Equations		
$y = 3$	all real numbers	3
$x = y$	all real numbers	all real numbers
$y = \lvert x \rvert$	all real numbers	all reals ≥ 0
$x^2 + y^2 = 1$	$-1 \le x \le 1$	$-1 \le y \le 1$
Graphs		
$y = x^2 - 3$ 	all real numbers	all real numbers ≥ -3
$x^2 + y^2 = 4$ 	$-2 \le x \le 2$	$-2 \le y \le 2$

$y = |2x|$



All real numbers $y \geq 0$

$y = \sqrt{x}$



$x \geq 0$ $y \geq 0$

Function

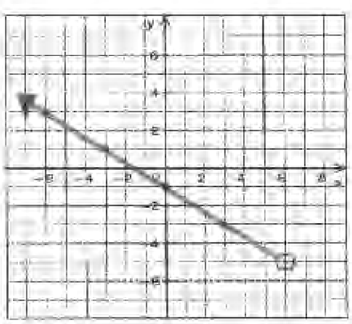
A **function** is a special kind of relation that exists between numbers or variables with the following rule: for every valid input number, there is a single output number. In other words, for every x, there is only one y.

Example: $y = x^2$ is a function since only 1 value of y results from every value of x.
$x = y^2$ is <u>not</u> a function, for example if $x = 4$, y could equal either 2 or -2.

The **domain** of a function is the set of all possible input numbers called the independent variable (all values of x that work).

The **range** is the set of all possible output numbers, called the dependent variable (all values of y that work).

Example: What is the domain and range of the following function?

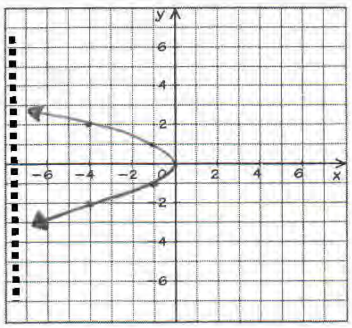


Domain, set of "x's"
$\therefore$ all real numbers < 6

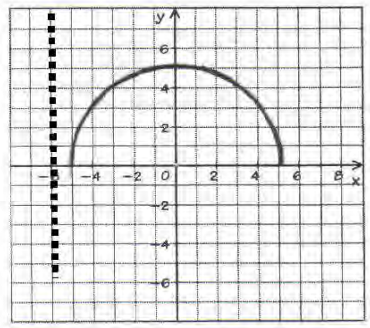
Range, set of "y's"
$\therefore$ all real numbers > -5

A quick way to determine whether a relation is a function or not is to do a vertical line test on the graph of the relation. If only one value of y can be found for every value of x, then we know that the relation is function.

Example: Which of the following are graphs of functions?



The above graph is **not** a function since the vertical dotted line intersected it in more than one location.



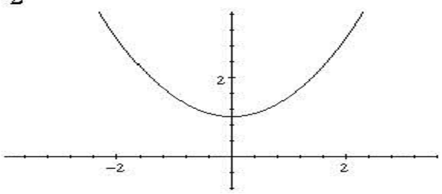
The above graph is a function since any vertical line will intersect it in only one location.

Several examples of relations are shown next, some are functions and others are not.

Examples of Relations	**Function?**	**Reason**
Ordered Pairs		
(2, 3), (3, 5), (4, 6), (5, 8)	Yes	Each value in the domain has a unique value in the range.
(1, 3), (2, 4), (3, 5), (1, 6)	No	When x = 1 there is more than one value of y (e.g. 3 and 6).
Equations		
$y = 2x + 1$	Yes	Each value in the domain has exactly one value in the range.
$x^2 + y^2 = 1$	No	There is more than one value of y for all but two values of x (when x = 0, $y = \pm 1$).

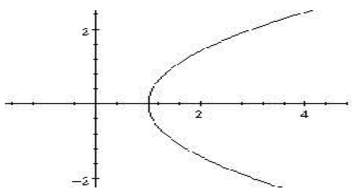
Graphs

$y = \frac{1}{2}x^2 + 1$



Yes — There is exactly one value of y for each value of x.

$x = \frac{1}{2}y^2 + 1$



No — There are two values of y for each value of $x > 1$.

In our review of functions and relations, we stated that a **function** is a special kind of relation that exists between numbers or variables with the following rule: for every valid input number, there is a single output number. In other words, for every x, there is only one y.

To quickly distinguish between a function and a relation when using an equation we use a special notation for a function. It is called **f of x notation**, as described next.

Using f(x) Notation

Functions are usually denoted using "f of x" notation (f(x)), which is another name for the dependent variable, y.

Examples:

$f(2) = 3$	means that function f maps 2 to 3
$f(-1) = 2$	means that function f maps -1 to 2
$f(x) = y$	means that function f maps x to y

Examples with Solutions

$f(x) = 2x - 7$	$f(3)$	$2(3) - 7 = -1$
	$f(-5)$	$2(-5) - 7 = -17$
	$f(x+1)$	$2(x + 1) - 7 = 2x - 5$

$f(x) = x^2 + x - 1$	$f(-2)$	$(-2)^2 + (-2) - 1 = 4 - 2 - 1 = 1$
	$f(x + 1)$	$(x + 1)^2 + (x + 1) - 1 = x^2 + 3x + 1$
	$f(-x)$	$(-x)^2 + (-x) - 1 = x^2 - x - 1$
	$-f(x)$	$-(x^2 + x - 1) = -x^2 - x + 1$
$g(x) = \sqrt{x+2} - 1$	$g(7)$	$\sqrt{7+2} - 1 = \sqrt{9} - 1 = 3 - 1 = 2$
	$g(-1)$	$\sqrt{-1+2} - 1 = \sqrt{1} - 1 = 1 - 1 = 0$
$h(x) = \dfrac{x^2 + x}{x - 1}$	$h(2)$	$\dfrac{2^2+2}{2-1} = \dfrac{6}{1} = 6$
	$h(-1)$	$\dfrac{(-1)^2 + (-1)}{(-1) - 1} = \dfrac{1-1}{-2} = \dfrac{0}{-2} = 0$
	$h(x - 1)$	$\dfrac{(x-1)^2 + (x-1)}{(x-1) - 1} = \dfrac{x^2 - 2x + 1 + x - 1}{x - 2} = \dfrac{x^2 - x}{x - 2}$

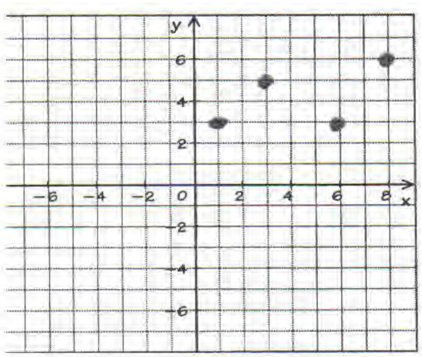
Exercises 1.1

1. Find the domain and range of each function.

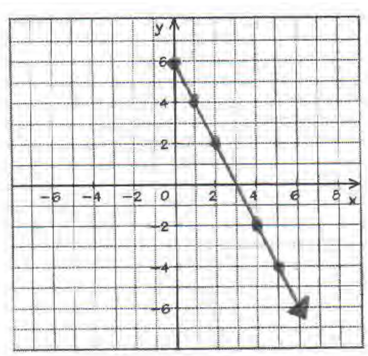
   a. (1, 5), (2, 7), (3, 7), (5, 8)

   b. (-2, 4), (0, 3), (0.5, 4), (1.5, 4)

   c.



   d.



   e. $y = x^2$

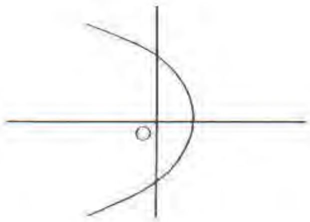
   f. $y = |x| + 1$

2. Which of the following relations are functions?

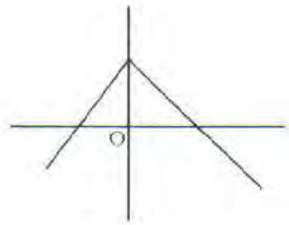
a. (1, -1), (1, 2), (7, 7), (8, 8)

b. (5, 3), (6, 3), (7, 7)

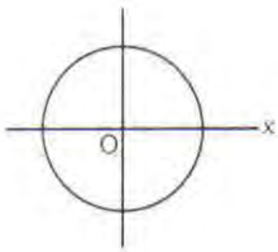
c.



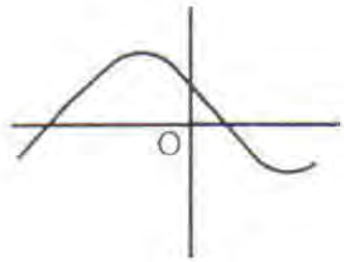
d.



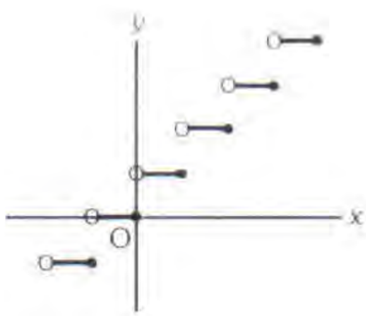
e.



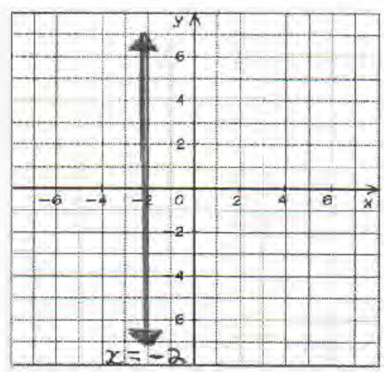
f.



g.



h.



3. If $f(x) = (x + 5)^2$, find

a. the domain

b. the range

c. f(2)

d. f(-2)

e. $f(2x)$

f. $f(x + 1)$

4. If $f(x) = -2(7 - x^2)$ find

a. $f(-2)$

b. $f(0)$

c. $f(\sqrt{2})$

d. $f(x + 1)$

5. If $f(x) = 2x^2 - \frac{1}{2}$, find

a. the domain

b. the range

c. $f(-2)$

d. $f(\frac{1}{2})$

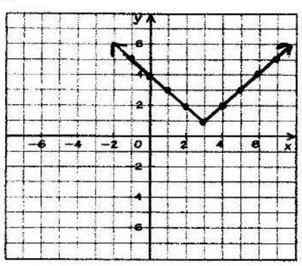
ANSWERS TO EXERCISES AND CHAPTER TESTS

CHAPTER 1

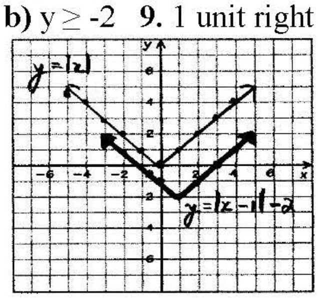
Exercises 1.1 (page 7)
1. a) Domain: 1, 2, 3, 5; Range: 5, 7, 8
b) Domain: -2, 0, 0.5, 1, 5; Range: 3, 4
c) Domain: 1, 3, 6, 8; Range: 3, 5, 6
d) Domain: $x \geq 0$; Range: $y \leq 6$
e) Domain: all reals; Range: $y \geq 0$
f) Domain: all reals; Range: $y \geq 1$
2. a) No **b)** Yes **c)** No **d)** Yes **e)** No **f)** Yes
g) Yes **h)** No **3. a)** Domain: all reals
b) Range: $y \geq 0$ **c)** 49 **d)** 9 **e)** $4x^2 + 20x + 25$
f) $x^2 + 12x + 36$ **4. a)** -6 **b)** -14 **c)** -10
d) $2x^2 + 4x - 12$ **5. a)** Domain: all reals
b) Range: $y \geq -\frac{1}{2}$ **c)** 7.5 **d)** 0

Exercises 1.2 (page 13)
1. a) odd **b)** even **c)** neither **d)** even
e) neither **2. a)** neither **b)** even **c)** neither

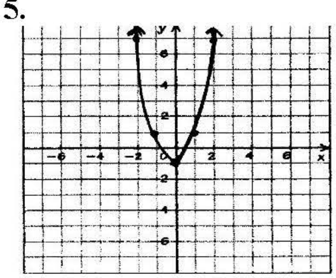
Exercises 1.3 (page 16)
1. $y = f(x - 4)$ **2.** $y = |x - 1| - 1$
3.



4. 5 right and 9 down **5.** 1 down
6. a) (-3, 7) **b)** (0, -2) **c)** (-4, 1) **d)** (-5, 1)
7. $y = |x - 3|$ **8. a)** 3 units left, 3 units down
b) $y \geq -2$ **9.** 1 unit right and 2 units down

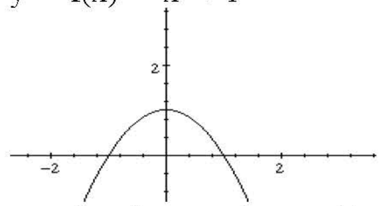


Exercises 1.4 (page 22)
1. a) Replace x with $\frac{1}{3}$x. **b)** Replace x with $\frac{3}{7}$x.
2. a) Replace x with 4x. **b)** Replace x with $\frac{5}{2}$x.
3. a) horizontal expansion by a factor of 2
b) horizontal compression by a factor of $\frac{1}{3}$
4. $y = 9x^2 + 4$
5.

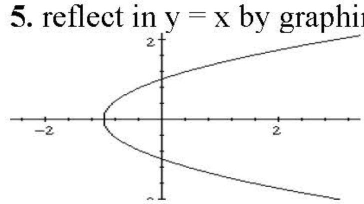


6. a) vertical expansion by a factor of 3
b) vertical compression by a factor of $\frac{1}{4}$
c) horizontal compression by a factor of $\frac{1}{4}$
d) horizontal expansion by a factor of 3
e) vertical expansion by a factor of 3 and a horizontal compression by a factor of $\frac{1}{2}$
f) vertical compression by a factor of $\frac{1}{2}$ and a horizontal expansion by a factor of 3
7. a) $y = 2|x|$ or $y = |2x|$ **b)** $y = \frac{1}{2}x^2$
c) $y = 2(x + 2)$ **d)** $y = 0.5x^3$ **e)** $y = 2(\frac{4}{x^2+1})$
8. a) 0, -1, 2 **b)** 0, -2, 4

Exercises 1.5 (page 30)
1. $y = -(2x^2 + 3x - 5) = -2x^2 - 3x + 5$
(change to the graph of $y = -f(x)$)
2. $y = 5(-x)^2 + 3(-x) - 2 = 5x^2 - 3x - 2$
(change to the graph of $y= f(-x)$)
3. $x = 4y^2 + 3y - 1$ (change to $x = f(y)$)
4. reflect in x-axis by graphing
$y = -f(x) = -x^2 + 1$



5. reflect in y = x by graphing $x = y^2 - 1$



6. It is reflected through the y-axis.
7. It is reflected through y = x.