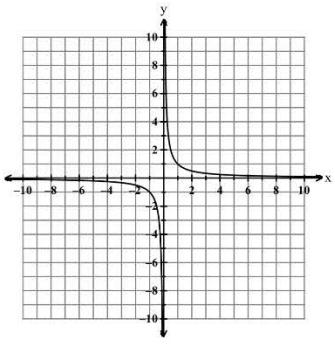
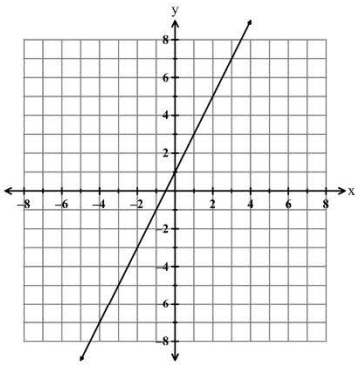
$$y = f(x)$$

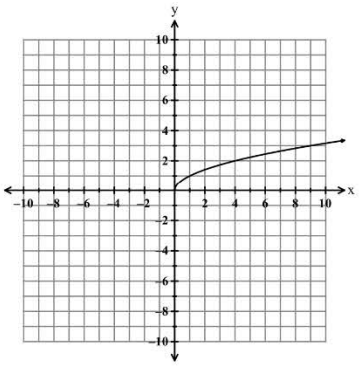
CHAPTER 1

RELATIONS AND FUNCTIONS



Domain





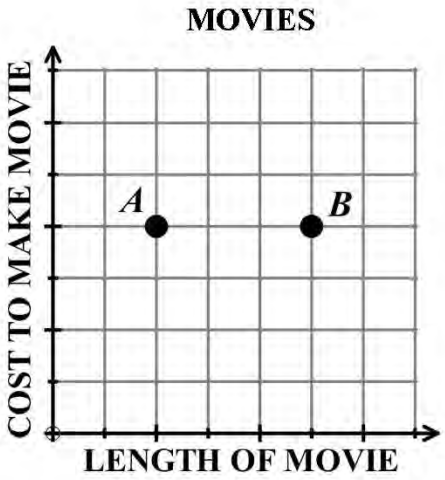
Range

1.1 Graphs

Graphs can be powerful tools to describe information. They illustrate the information in a way that makes profiles, comparisons, or trends easy to identify.

Examples:

1. Write a statement that describes the information contained in the graph below.



Movies A and B cost the same to make, but movie B is longer than movie A.

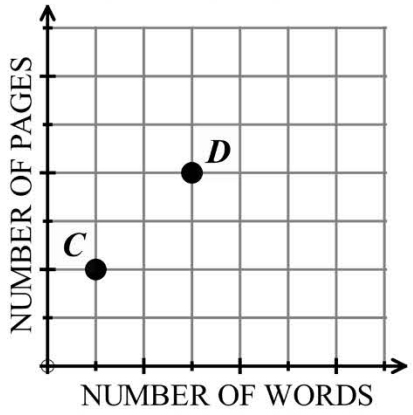
2. A student rides a bike to school, first up and then down a steep hill. Which graph below represents the student's speed as compared to the time elapsed? Explain your answer.

The answer is graph #2 because the speed decreases as the student is going uphill and increases while going downhill.

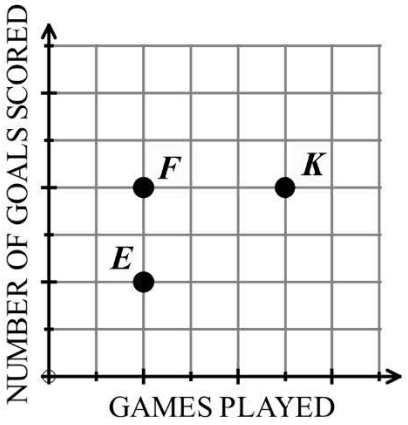
Exercises 1.1

Write a statement that describes the information contained in each graph below.

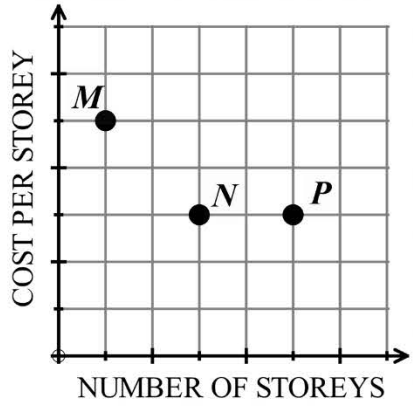
1. NOVELS



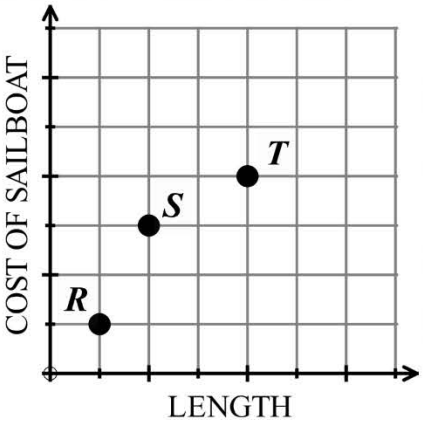
2. HOCKEY PLAYERS



3. BUILDING COST

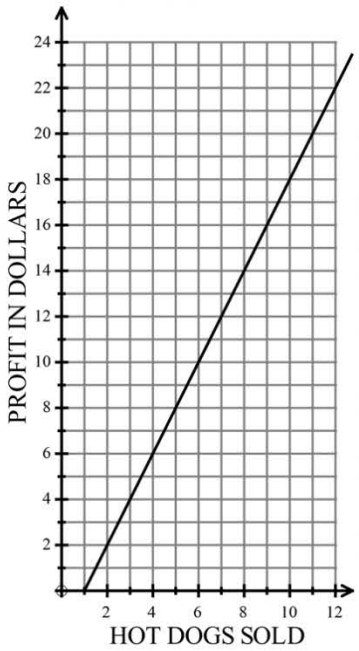


4. SAILBOATS

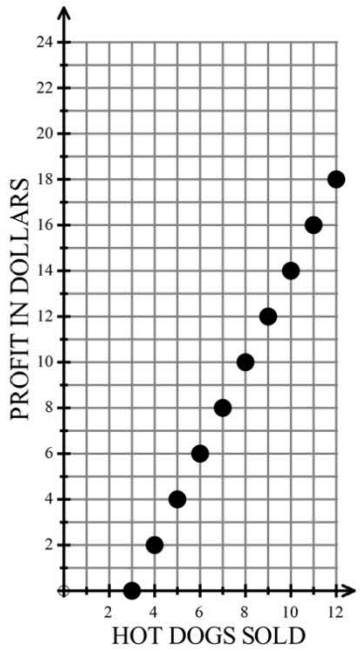


5. A school team is raising money for uniforms. Each day, the team buys a dozen hot dogs with buns for $6. The team then sells the hot dogs for $2 each. Which graph best represents the team's profit for one day as a function of the number of hot dogs sold? Explain your answer.

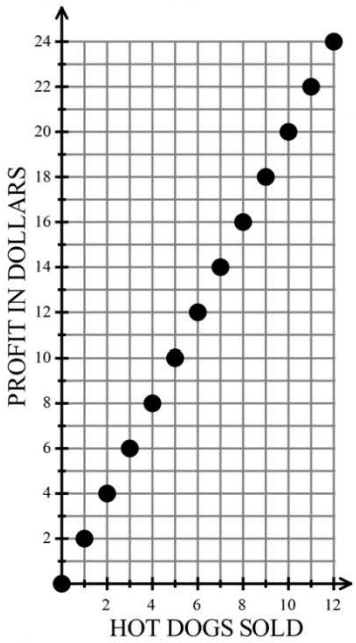
Graph #1



Graph #2

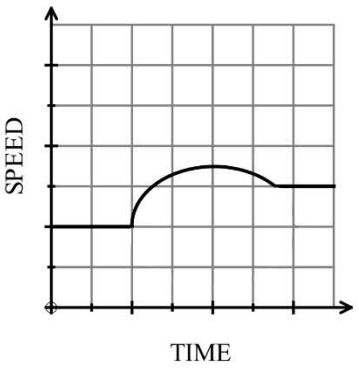


Graph #3

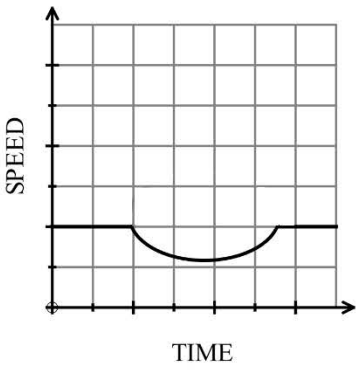


6. A truck is traveling at a constant speed on a highway. The truck increases its speed to pass a slower moving car, and then the truck resumes its original speed. Which graph best describes this motion? Explain your answer.

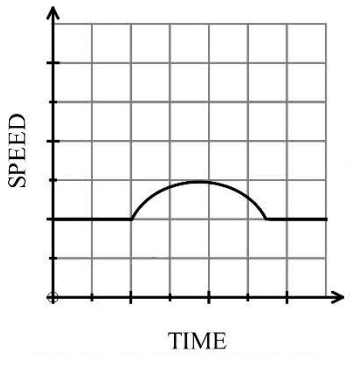
Graph #1



Graph #2

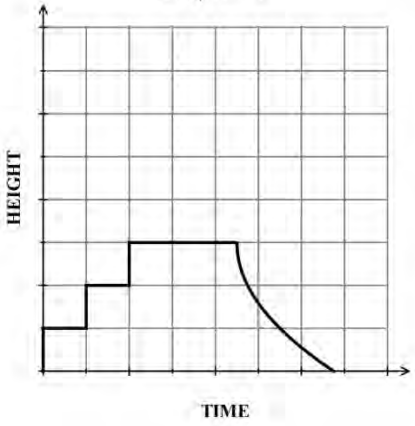


Graph #3

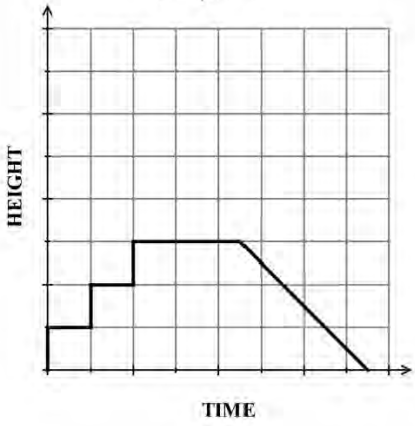


7. In a backyard playground, a child climbs 3 steps to a slide, stops at the top, then slides down. Which graph represents the height of the child as a function of time? Explain your answer.

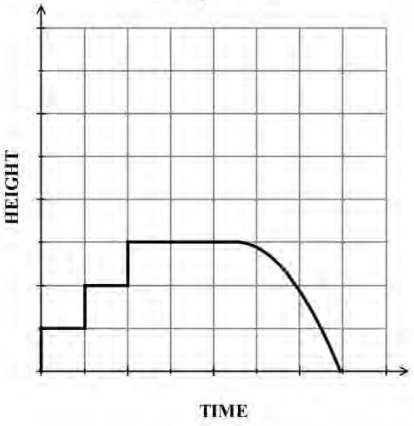
Graph #1



Graph #2

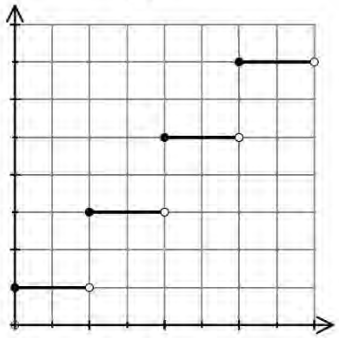


Graph #3

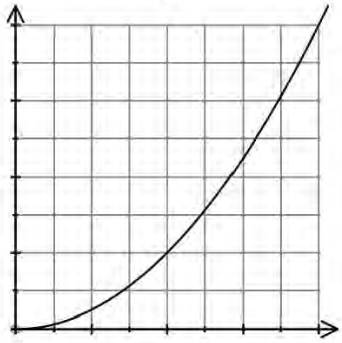


8. Match each graph below with an appropriate statement that might describe the relation.

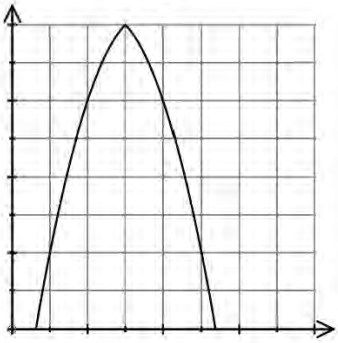
A



B



C



(i) The height of a kicked soccer ball changes with time.
(ii) The cost of parking depends on the length of time parked.
(iii) The braking distance of a car changes with time.

1.2 Relations

When one type of information is connected to another type of information, we say they are related. Some examples are time and distance travelled, or height and weight. When the value of one piece of information changes, so does the value of the other. These are called **relations**.

A **relation** can be shown as a set of ordered pairs. The order of the values (or elements) in the pairs of numbers is important. The set of all the first values in these ordered pairs is called the **domain** of the relation. The set of all the second values or elements in these ordered pairs is called the **range** of the relation.

Example: State the domain and range of the relation (1, 3), (2, 6), (3, 9).

The domain is {1, 2, 3}. These are the first values in each ordered pair.
The range is {3, 6, 9}. These are the second values in each ordered pair.

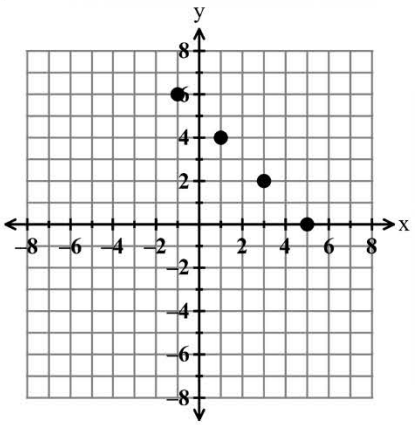
Relations can be described in different ways. In addition to ordered pairs, they could be shown in tables or graphs or described with words, equations, or diagrams.

Examples:

1. Table of values

x	y
1	4
3	2
5	0
-1	6

2. Graph



3. Words

The sum of two numbers is five.

4. Equation

$x + y = 5$
or $y = -x + 5$
or $x = 5 - y$

5. Arrow Diagram

$1 \rightarrow 4$
$3 \rightarrow 2$
$5 \rightarrow 0$
$-1 \rightarrow 6$

Example:

Students wanted to see a complete reflection of themselves in a mirror that was fixed to a flat wall. They had to use a mirror that was long enough, as in the table below.

Height of student (h) cm	Length of mirror (l) cm
120	60
110	55
100	50
90	45
80	40

a) Write the relation as a set of ordered pairs (h, l).
b) What is the (i) domain and (ii) range.
c) Show an arrow diagram of this relation.
d) Describe this relation in words.
e) Draw a graph of this relation.
f) Write an equation to describe this relation.

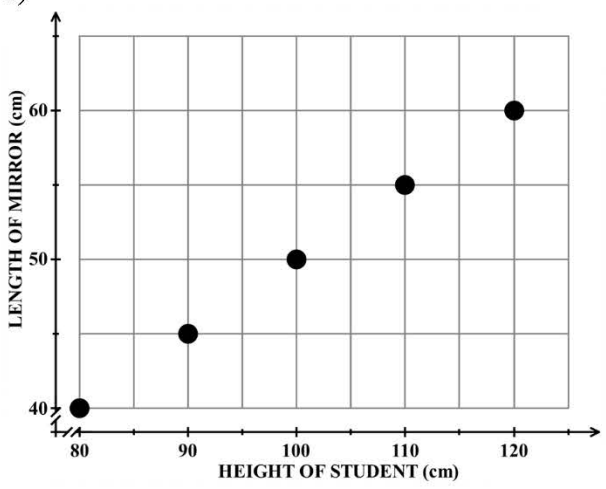
a) (120, 60), (110, 55), (100, 50), (90, 45), (80, 40)

b) (i) Domain = {120, 110, 100, 90, 80}
(ii) Range = {60, 55, 50, 45, 40}

c) 120 → 60
110 → 55
100 → 50
90 → 45
80 → 40

d) The length of the mirror is one-half the height of the student.

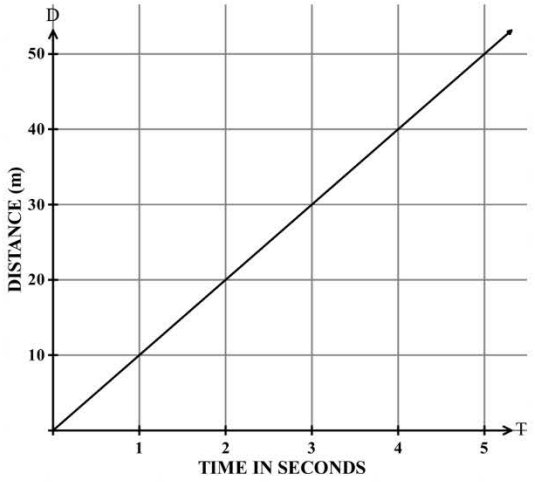
e)



f) $l = \frac{1}{2}h$ or $h = 2l$

Some graphs are solid lines (made up of an infinite number of points), while other graphs are only points. If we were graphing distance travelled (in metres) verses time (in seconds), the graph might look like the following.

Graph #1

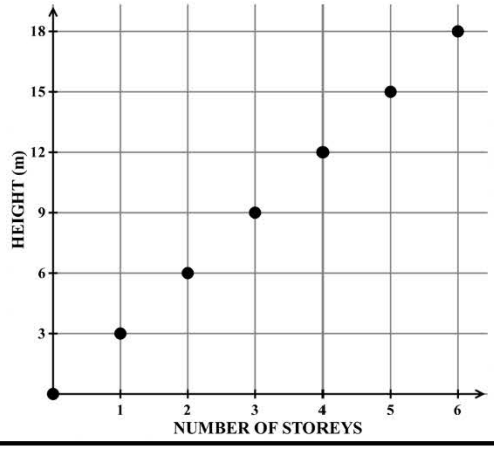


We probably would need points from a table of values, such as the following, to draw this graph.

T	D
0	0
1	10
2	20
3	30
4	40
5	50

We would join the points because time is continuous. (We could have 0.5 seconds, 3.5 seconds, etc.) On the other hand, if we were graphing the height of buildings (in metres) verses the number of storeys of the building, the graph might look like the following.

Graph #2



We probably would need points from a table of values to draw this graph.

n	m
0	0
1	3
2	6
3	9
4	12
5	15
6	18

We would **not** join the points because the number of storeys must be a whole number. We can't have a $\frac{1}{2}$ storey or $3\frac{1}{2}$ storeys.

Thus, in graph #1 the domain is all real numbers greater than 0. In graph #2, the domain are the integers from 0 to 6, inclusive.

Example:

A table of values for the graph for $y = 2x - 3$ could be as follows:

x	3	2	1	0	-1
y	3	1	-1	-3	-5

If the domain is all real numbers, would the points be joined on the graph? Give a reason.

The points would be joined because the domain is all real number, which includes fractional parts between the selected points in the table of values.

To determine whether a relation is linear or not, we will have to investigate one of the following.

a) Table of values
b) The set of ordered pairs
c) The graph
d) The words in a particular context

Examples with Solutions

Solution

1. Is the relation in the following table of values linear?

x	0	1	2	3	4	5
y	0	4	8	12	16	20

Method 1:
We notice that each y-value is 4 times the corresponding x-value. $\therefore y = 4x$

Method 2:
Each x-value changes by 1 and each y-value changes by 4. $\therefore$ The table of values is a linear relation.

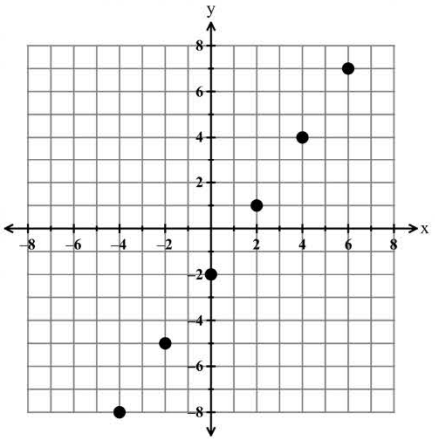
2. Is the relation in the following set of ordered pairs linear?

(0, 90), (50, 129), (100, 168), (150, 207), (200, 246)

Each x-value changes by 50
Each y-value changes by 39
$\therefore$ The set of ordered pairs is linear

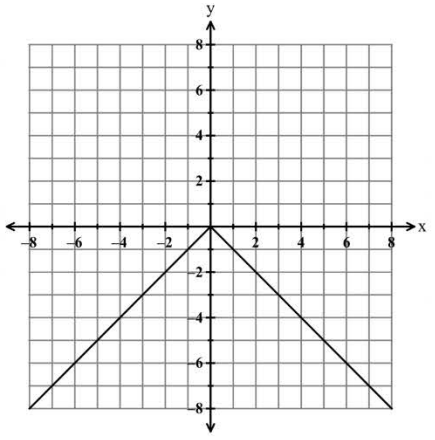
3. Which of the following graphs are linear relations? Give reasons for your answer.

a.



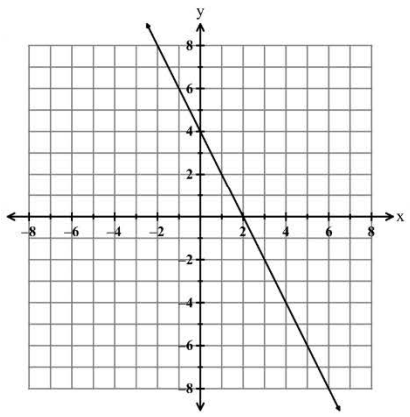
The graph is linear. Each x-value changes by 2 and each y-value changes by 3

b.



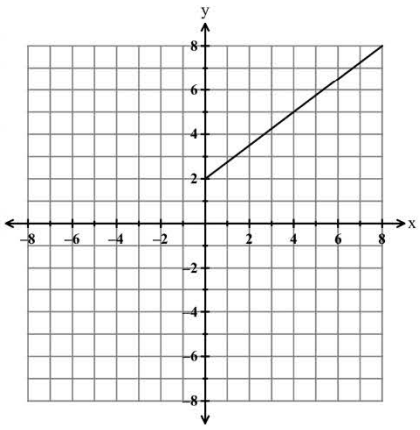
The graph is not linear, because at some points of the graph the y-value change is different (some are +1 and some are -1).

c.



The graph is linear. As each x-value changes by 1, each y-value changes by -2.

d.



The graph is linear. As each x-value changes by 4, each y-value changes by 3.

4. State whether the relationship between each pair of variables is an example of a linear relation.

a. The circumference of a circle and its radius

b. The tax paid and the price when buying an item

c. The area of a circle and its radius

d. The cost of gas and the number of litres bought

e. The perimeter of a square and the length of a side

a. $C = 2\pi R$. As R changes, C will be 2π times larger. So the relationship is linear.

b. Taxes are a percentage of an item. As the cost of the item goes up, the tax goes up as well. So the relationship is linear.

c. $A = \pi R^2$. As R changes, A will change by the square of R times π, and not uniformly. $\therefore$ the relationship is not linear.

d. Gas is sold at a cost per litre. As the number of litres increase, the price increases. So the relationship is linear.

e. $P = 4S$. As S increases, P increases by a factor of 4. So the relationship is linear.

ANSWERS TO EXERCISES AND CHAPTER TESTS

CHAPTER 1

Exercises 1.1 (page 3)

1. Novel D has more pages and more words than Novel C. **2.** Hockey players E and F played the same number of games, but player F scored more goals. Player K played more games than player F but scored the same number of goals. **3.** Building M has the least number of storeys, but its cost per storey is the highest. Building P has more storeys than building N, but its cost per storey is the same as building N. **4.** Sailboat T is longer than Sailboat S, which is longer than Sailboat R. Sailboat T is the most expensive and Sailboat R is the least expensive. **5.** Graph #1: Each hotdog sold is a discrete element, so the graph must be dots, not a line, and 4 hotdogs must be sold before any profit is made. **6.** Graph #3: As the truck passes the car, there is an increase of speed (so it is not #2) and then the speed returns to the original speed (so it is not #1). **7.** Graph #3: As the child slides down, there is an increase in speed, which results in height decreasing quicker than in #1 and #2. **8.** Graph A = (ii), Graph B = (iii), and Graph C = (i)

Exercises 1.2 (page 12)

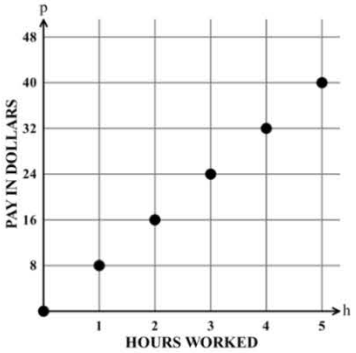
1. Domain: {2, 3, 4, 5}; Range: {4, 5, 6, 7}
2. Domain: {10, 9, 8, 7}; Range: {7, 6, 5, 4}
3. Domain: {5, 6, 7, 8}; Range: {10, 12, 14, 16}
4. Domain: {14, 10, 8, 6}; Range: {7, 5, 4, 3}
5. Domain: {3}; Range: {3, 4, 5, 6}
6. Domain: {4, 5, 6, 7}; Range: {1}
7. Domain: {-1, 0, 1, 2, 3}; Range: {4}
8. Domain: {2, 3, 4, 5, 6}; Range: {0, 1, 2, 3, 4}
9. Domain: {-2, 0, 3, 5}; Range: {0, 1, 2, 3, 4}
10. Domain: {-4, -2, 0, 2, 4}; Range: {-2, 0, 2, 4, 6} **11.** Domain: {-4, -2, 0, 2, 4}; Range: {5, 3, 1, -1, -3} **12. a)** (0, 0), (1, 8), (2, 16), (3, 24), (4, 32), (5, 40) **b)** Domain: {0, 1, 2, 3, 4, 5}; Range: {0, 8, 16, 24, 32, 40}

c)

$$\begin{array}{ccc} 0 & \rightarrow & 0 \\ 1 & \rightarrow & 8 \\ 2 & \rightarrow & 16 \\ 3 & \rightarrow & 24 \\ 4 & \rightarrow & 32 \\ 5 & \rightarrow & 40 \end{array}$$

d) For every whole hour worked, Sambra gets paid $8.00.

e)



f) $p = 8h$ or $h = \frac{1}{8}p$

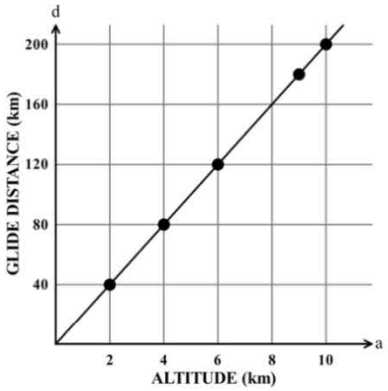
13. a) (2, 40), (4, 80), (6, 120), (9, 180), (10, 200) **b)** Domain: {2, 4, 6, 9, 10}; Range: {40, 80,120,180,200}

c)

$$\begin{array}{ccc} 2 & \rightarrow & 40 \\ 4 & \rightarrow & 80 \\ 6 & \rightarrow & 120 \\ 9 & \rightarrow & 180 \\ 10 & \rightarrow & 200 \end{array}$$

d) A plane can glide 20 km for every 1 km in altitude.

e)



f) $d = 20a$ or $a = \frac{1}{20}d$