$$M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$m=\frac{y_2-y_1}{x_2-x_1}$$

CHAPTER 1

LINE SEGMENTS

$l_1 \perp l_2$

$m_1 \bullet m_2 = -1$

$$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

1.1 Distance Between Two Points

The distance between two Points P_1 (x_1, y_1) and P_2 (x_2, y_2) is given by the formula

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Examples:

1. Find the distance between (-3, 4) and (6, -5).

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(-3, 4) (6, -5)

(x_1, y_1) (x_2, y_2)

$$= \sqrt{(6 - -3)^2 + (-5 - 4)^2}$$

$$= \sqrt{9^2 + (-9)^2} = \sqrt{81+81}$$

$$= \sqrt{162} = \sqrt{81 \bullet 2}$$

$$= 9\sqrt{2}$$

$$D \approx 12.73$$

2. Given vertices at points A (6, -4), B (1, 5), and C (-3, 1), what kind of triangle do they form?

Distance $AB = \sqrt{(6-1)^2 + (-4-5)^2}$

$= \sqrt{5^2 + (-9)^2}$

$= \sqrt{25+81}$

$AB = \sqrt{106}$

$AB \approx 10.3$

Distance $AC = \sqrt{(6 - -3)^2 + (-4-1)^2}$

$= \sqrt{9^2 + (-5)^2}$

$= \sqrt{81+25}$

$AC = \sqrt{106}$

$AC \approx 10.3$

Distance $BC = \sqrt{1 - -3)^2 + (5-1)^2}$

$= \sqrt{4^2 + 4^2}$

$= \sqrt{16+16}$

$= \sqrt{32}$

$BC = 4\sqrt{2}$

$BC \approx 5.7$

Since AB = AC, the triangle is isosceles.

Exercises 1.1

1. Find the distance between each pair of points.

   a. (7, 4) and (8, 10)

   b. (-6, 4) and (3, -2)

   c. (-5, 6) and (-4, -8)

   d. (-3, -4) and (-2, -9)

2. Given the coordinates of the vertices of each triangle, what kind of triangle is it?

   a. A (-2, 7), B (-7, 7), and C (-7, -5)

   b. A (0, 1), B (-3, -3), and C (3, -3)

   c. A (-4, 0), B (4, 0), and C (0, $4\sqrt{3}$)

3. The distance between two points is $\sqrt{20}$. If one coordinate is (2, 5) and the other is (x, 1), find x.

4. The coordinates of two rectangles are:
   Rectangle A: (-1, -5), (-1, -2), (5, -2), (5, -5) and Rectangle B: (4, 7), (-4, 7), (-4, -1), (4, -1)
   For each rectangle, find the perimeter, the area, and the length of the diagonal.

   a. Rectangle A
   (i) the perimeter

   (ii) the area

   (iii) the length of the diagonal

   b. Rectangle B
   (i) the perimeter

   (ii) the area

   (iii) the length of the diagonal

5. A circle with a centre at (-3, 4) has a point (4, -6) on its circumference. Determine the radius.

1.2 Mid-point

Given any two points $P_1(x_1 y_1)$ and $P_2(x_2, y_2)$, use the following formula to find the coordinates of the mid-point of the line segment between them.

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Examples:

1. Find the mid-point of the line segment joining (-4, 6) and (8, -2).

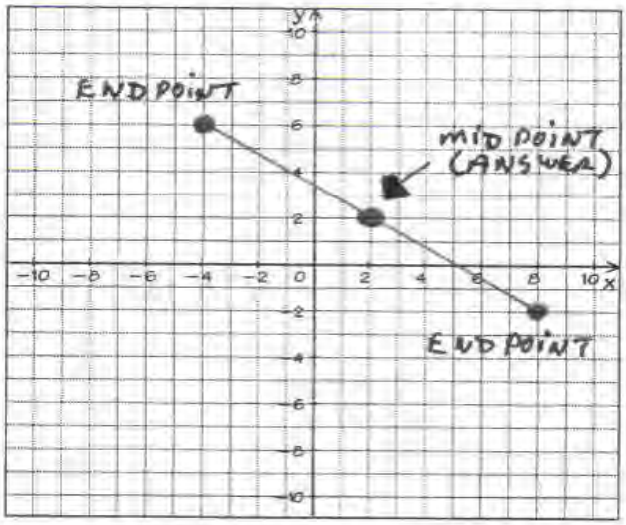
   Solution by formula:

   The mid-point is $\left(\frac{(x_1+x_2)}{2}, \frac{(y_1+y_2)}{2}\right)$.
   Remember (-4, 6) and (8, -2) correspond to (x_1, y_1) and (x_2, y_2), so $\left(\frac{-4+8}{2}, \frac{6-2}{2}\right) = \left(\frac{4}{2}, \frac{4}{2}\right)$.

   The mid-point is (2, 2)

   Solution by graphing:

   Graph the endpoints. Then measure the length of the line and take half of it to find the midpoint.



2. Find the other end-point of a line segment with one end-point at (-4, -8) and a mid-point at (1, -3).

Solution by formula:

Mid-point is $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}) = (1, -3)$ and $(x_1, y_1) = (-4, -8)$.

$\frac{x_1+x_2}{2} = 1$ and $x_1 = -4$

$\frac{-4+x_2}{2} = 1$

$-4 + x_2 = 2$

$x_2 = 6$

$\frac{y_1+y_2}{2} = -3$ and $y_1 = -8$

$\frac{-8+y_2}{2} = -3$

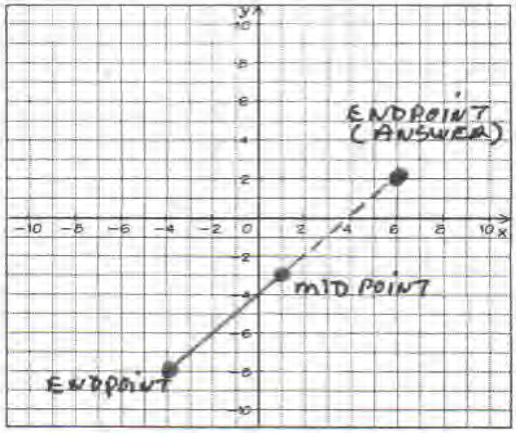
$-8 + y_2 = -6$

$y_2 = 2$

The other end-point is (6, 2).

Solution by graphing:

Graph the endpoint and midpoint. Then extend the line joining these points the same distance to find the other endpoint.



3. A parallelogram has the following vertices: A (-3, 3), B (4, 1), C (-2, -2), and D (-9, 0). Find the point of intersection where the diagonals bisect each other.

Solution by formula:

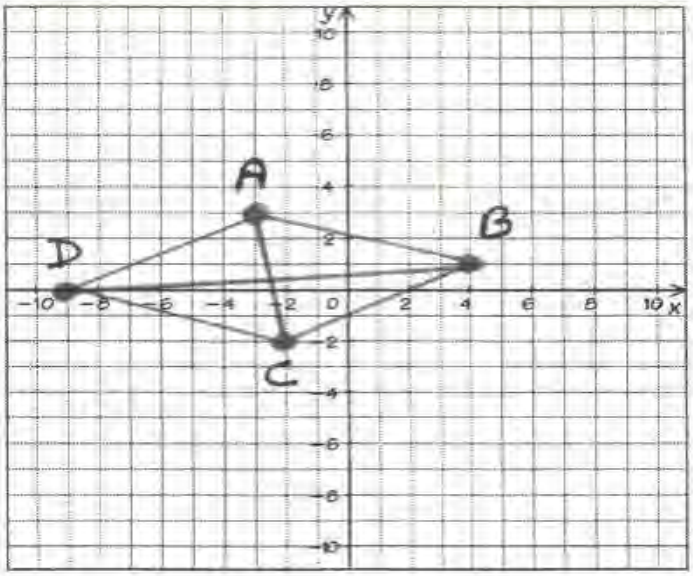
Midpoint of AC $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}) = (\frac{-3-2}{2}, \frac{3-2}{2}) = (\frac{-5}{2}, \frac{1}{2})$

Midpoint of BD $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}) = (\frac{4-9}{2}, \frac{1+0}{2}) = (-\frac{5}{2}, \frac{1}{2})$

The diagonals bisect each other at $(-\frac{5}{2}, \frac{1}{2})$

Solution by graphing:

Graph the points A (3, -3), B (4, 1), C (-2, -2), and D (-9, 0) to find the parallelogram. Then join the diagonals.



The diagonals bisect each other at $(-\frac{5}{2}, \frac{1}{2})$.

Exercises 1.2

1. Find the mid-point of each line segment with the given end-points.

   a. (4, 2) and (6, 8)

   b. (-3, -3) and (-1, -7)

   c. (-1, -2) and (-6, 5)

   d. (3x, -7x) and (x, -x)

2. One end-point and the mid-point of each line segment are given. Find the coordinates of the other end-point.

   a. End-point = (3, -4); Mid-point = (3, 6)

   b. End-point = (8, -1); Mid-point = (5, -5)

c. End-point = (-6,-4); Mid-point = (-3, -2)

3. The end-points of the diameter of a circle are (6, 4) and (-2, 0). Find the centre of the circle.

4. A triangle has vertices at (4, 4), (-6, 2) and (2, 0).

   a. Find the coordinates of the mid-points of each side.

   b. Find the lengths of the sides of the triangle formed by joining the mid-points.

5. A rectangle has vertices (-4, 4), (7, 4), (-4, -2), and (7, -2).

   a. Find the mid-point of each diagonal.

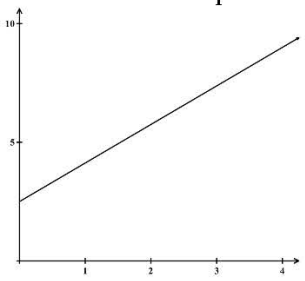
   b. Show that the diagonals bisect each other.

6. A map's numerical coordinates are in kilometres. Town A is at (16.3, 2.9) and town B is at (4.5, 6.3). A road is to be constructed on a direct line between the two towns. Each town is responsible for the construction up to the mid-point at a cost of $150 000 for each kilometre. Determine the cost for each town.

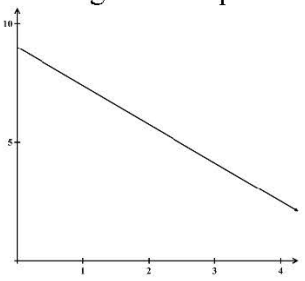
1.3 Slope of a Line

The slope is a measure of how steep a line is. The slope also describes the direction of the line.

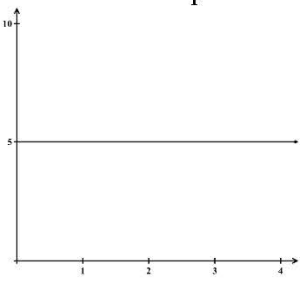
Positive Slope



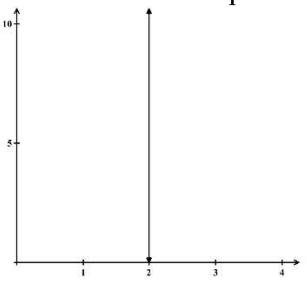
Negative Slope



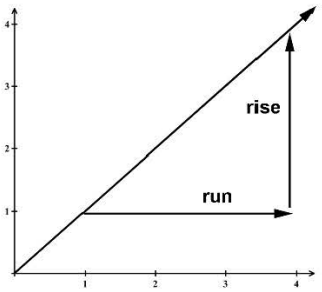
Zero Slope



Undefined Slope



The numerical value of a slope can be thought of as $\frac{Rise}{Run}$.



The slope of the line segment joining $P_1(x_1, y_1)$, and $P_2(x_2, y_2)$ is given by the formula (slope = $\frac{rise}{run}$).

Slope of $P_1P_2 = \frac{y_2 - y_1}{x_2 - x_1}$, $x_2 \neq x_1$ or $m = \frac{y_2 - y_1}{x_2 - x_1}$ (m = slope).

Examples:

1. Find the slope of the line segment joining each of the following points.

a. (-3, 4) and (5, -6)

$$m = \frac{y_2 - y_1}{x_2 - x_1}; \text{ (-3, 4) and (5, -6)}$$

$$= \frac{-6-4}{5--3} = \frac{-10}{8}$$

$$m = -\frac{5}{4} \quad \text{The slope is } -\frac{5}{4}.$$

b. (7, 2) and (-8, 2)

$$m = \frac{2-2}{-8-7} = \frac{0}{-15}$$

$m = 0$ The slope is zero (horizontal line).

c. (5, 3) and (5, -4)

$$m = \frac{-4-3}{5-5} = \frac{-7}{0}$$

m = undefined

The slope is undefined (vertical line).

2. The slope of a line is 2. The line passes through (-1, -2) and (4, y). Find the value of y.

$$m = \frac{y_2 - y_1}{x_2 - x_1}; \; m = 2 \quad \therefore 2 = \frac{y_2 - y_1}{x_2 - x_1}$$

(-1, -2) (4, y)

$x_1 y_1$ $x_2 y_2$

$$2 = \frac{y--2}{4--1} = \frac{y+2}{5}$$

y + 2 = 10 and y = 8

Exercises 1.3

1. Find the slop of the line passing through the following points.

a. (6, 8) and (2, 1)

b. (3, -1) and (-5, 4)

c. (3, 4) and (-6, 4)

d. (4, 6) and (4, -1)

e. (3.6, 5.2) and (-4.8, -7.5)

2. The slope of a line is $\frac{2}{5}$ and it passes though (-1, 3) and (x, 5). Find the value of x.

3. The slope of a line is -1 and it passes though (-2, 6) and (5, y). Find the value of y.

4. A ladder is leaning against a building. The bottom of the ladder is 1.85 m from the building. The ladder reaches up 2.1 m up the side of the building. What is the slope of the ladder?

5. A triangle has vertices (-2, -1), (-1, -6), and (5, 6). Determine the slope of each side.

6. Points are collinear if they lie on the same line. If the three points (-2, -1), (1, 1) and (7, y) are collinear, then find y.

ANSWERS TO EXCERCISES AND CHAPTER TESTS

CHAPTER 1

Exercises 1.1 (page 3)

1. a) $\sqrt{37}$ **b)** $\sqrt{117}$ **c)** $\sqrt{197}$ **d)** $\sqrt{26}$
2. a) Right b) Isosceles **c)** Equilateral
3. x = 4 **4. a)** (i) 18 units (ii) 18 square units (iii) 6.71 units **b)** (i) 32 units (ii) 64 square units (iii) 11.31 units
5. $\sqrt{149}$ or ≈ 12.21

Exercises 1.2 (page 6)

1. a) (5, 5) **b)** (-2, -5) **c)** $(-\frac{7}{2}, \frac{3}{2})$ **d)** (2x, -4x)
2. a) (3, 16) **b)** (2, -9) **c)** (0, 0) **3.** (2, 2)
4. a) (3, 2), (-2, 1), (-1, 3) **b)** $\sqrt{5}$, $\sqrt{17}$, $\sqrt{26}$
5. a) Midpoint $(\frac{3}{2}, 1)$ **b)** They have the same midpoint. The diagonals bisect each other.
6. $921 004.89

Exercises 1.3 (page 9)

1. a) $\frac{7}{4}$ **b)** $-\frac{5}{8}$ **c)** 0 **d)** Undefined **e)** $\frac{127}{84}$
2. x = 4 **3.** y = -1 **4.** $\frac{42}{37}$ **5.** Slopes are -5, 2, and 1. **6.** y = 5

Exercises 1.4 (page 12)

1. a) l1 // l2 **b)** l3 // l4 **c)** Not parallel **d)** l7 // l8 **e)** Not parallel **2.** Yes opposite sides are parallel with slopes $\frac{4}{3}$ and $-\frac{1}{5}$. **3.** x = -5
4. y = 9 **5.** y = 1 **6.** (-3, 0)

Exercises 1.5 (page 15)

1. a) m2 = $-\frac{8}{7}$ **b)** m2 = $\frac{5}{6}$ **c)** m2 = undefined **d)** m2 = 0 **e)** m2 = $-\frac{1}{10}$ **2.** mBC = $\frac{1}{2}$, mAB = -2; mBC x mAB = -1 and it is a right triangle.
3. ▲ABC is not a right triangle since no two sides are perpendicular. **4.** y = -4 **5.** 1
6. Yes. Opposite sides are parallel and adjacent sides are perpendicular. **7.** x = 7.5 **8.** (11.5, 0)

Chapter 1 Test (page 18)

1. $2\sqrt{85}$ **2.** (-1, 1) **3.** $-\frac{6}{7}$ **4.** Yes, AB//CD, Slopes = $\frac{4}{3}$ **5.** mCD = $-\frac{9}{8}$ **6. a)** x = -9 **b)** x = 4
7. a) (2, 0) **b)** $(0, -\frac{4}{3})$ **8.** $2\sqrt{34}$ or 11.66 block units **9.** $(\frac{3}{2}, \frac{5}{2})$, (0, 0), $(-\frac{3}{2}, -\frac{5}{2})$ **10.** mAB = 1; mBC = $-\frac{5}{6}$; mCD = -5; mDA = $\frac{1}{6}$. No slopes are parallel or perpendicular. It is a quadrilateral.

CHAPTER 2

Exercises 2.1 (page 23)

1. a) $-\frac{4}{3}$ **b)** $\frac{1}{3}$ **c)** $\frac{1}{2}$ **d)** $-\frac{2}{3}$ **e)** 5 **2. a)** y = 3x – 1 **b)** y = -2x + 13 c) y = $-\frac{2}{3}$x + 4 **3. a)** 2.24 **b)** 2.85 **c)** 4.47 **d)** 10.82 **4. a)** 6 **b)** 3 **c)** 4.24 **d)** 6.01

Exercises 2.2 (page 26)

1. a) 9 **b)** 6.4 **c)** $\frac{7}{8}$ **d)** 5.5 **e)** 5.8 **2. a)** 6 **b)** 7 **c)** $\frac{2}{11}$ **d)** 9

Exercises 2.3 (page 29)

1. a) 7 **b)** 3 **c)** 6 **d)** 3 **e)** 2 **f)** 7 **2. a)** 1.90 **b)** 2 **c)** 2.24 **d)** 3.64

Exercises 2.4 (page 31)

1. a)

The conjecture is not true.
b) (-2, 5) The conjecture is not true.
c)

The conjecture is not true.
d) There is no counter example, so it appears the conjecture is true.
e)

The conjecture is not true.
2. a) Use distance. AD = BC = 10, AB = CD = 4
b) Use distance. AC = BD = $\sqrt{116}$