

CONTENTS

Introduction.....	v	CHAPTER 4: CONGRUENT TRIANGLES.....	146
Using This Book.....	vii	4.1 Angles in Triangles.....	148
CHAPTER 1: FOUNDATIONS OF GEOMETRY.....	1	4.2 Congruent Figures.....	156
1.1 Sets.....	2	4.3 Congruence Postulates.....	163
1.2 Undefined Terms and Definitions.....	6	4.4 Applying Congruence Postulates.....	169
1.3 An Ideal Geometry.....	10	Technology Corner —Exploring AAA and SSA.....	176
Geometry in History —Not Your Usual Math Club.....	16	4.5 Conditions for Congruent Triangles.....	177
1.4 Subsets of Lines and Planes.....	18	4.6 Right Triangle Congruence.....	185
1.5 Segment and Angle Measure.....	23	Geometry Around Us —Architects and Buildings.....	192
Technology Corner —		4.7 Coordinate Geometry of Triangles.....	193
Introduction to The Geometer's Sketchpad®.....	30	Chapter 4 Review.....	201
1.6 Two-Dimensional Figures.....	31	CHAPTER 5: RELATIONSHIPS IN TRIANGLES.....	204
1.7 Three-Dimensional Figures.....	39	5.1 Circumcenter and Orthocenter.....	206
1.8 Sketches, Drawings, and Constructions.....	45	5.2 Incenter and Centroid.....	214
Chapter 1 Review.....	50	Technology Corner —Triangle Relationships.....	221
CHAPTER 2: REASONING AND PROOF.....	52	5.3 Inequalities.....	221
2.1 Inductive Reasoning.....	54	5.4 Indirect Proofs.....	227
Geometry Around Us —Designers and Patterns.....	59	Geometry Around Us —Carpentry.....	233
2.2 Statements and Truth Values.....	60	5.5 More Inequalities.....	234
2.3 Conditionals and Biconditionals.....	65	Chapter 5 Review.....	243
2.4 Deductive Reasoning.....	71	CHAPTER 6: QUADRILATERALS.....	246
2.5 Algebraic Reasoning.....	78	6.1 Classifying Quadrilaterals.....	248
2.6 Proofs Using Segments.....	83	Technology Corner —Exploring Quadrilaterals.....	255
Technology Corner —Angle Conjectures.....	88	6.2 Characteristics of Parallelograms.....	256
2.7 Proofs Using Angles.....	89	6.3 Proofs of Parallelograms.....	263
2.8 Bisectors.....	97	6.4 Rectangles, Rhombi, and Squares.....	270
Chapter 2 Review.....	103	6.5 Trapezoids and Kites.....	277
CHAPTER 3: PARALLEL AND		Geometry Around Us —Sports.....	286
PERPENDICULAR LINES.....	106	6.6 Coordinate Geometry of Quadrilaterals.....	287
3.1 Parallel Lines and Transversals.....	108	Chapter 6 Review.....	293
3.2 Proving Lines Parallel.....	114	CHAPTER 7: AREA.....	296
3.3 Constructing Parallel and Perpendicular		7.1 Areas of Rectangles, Parallelograms,	
Lines.....	120	and Triangles.....	298
Geometry in History —A Revolutionary Museum.....	126	Technology Corner —Center of Gravity.....	305
3.4 Distances, Midpoints, and Slopes.....	129	7.2 Areas of Other Quadrilaterals.....	306
Technology Corner —Midpoints and Slopes.....	136	7.3 The Pythagorean Theorem.....	312
3.5 Equations of Lines.....	137	7.4 Special Right Triangles.....	318
Chapter 3 Review.....	143		

Geometry in History —Another Purpose.....	325
7.5 Areas of Regular Polygons.....	327
7.6 Circles, Sectors, and Segments.....	333
Chapter 7 Review	339
CHAPTER 8: CIRCLES	342
8.1 Circles and Chords.....	344
8.2 Tangents.....	351
Technology Corner —	
Constructing Common Tangents.....	359
8.3 Arc Measure and Length.....	360
Geometry Around Us —Transportation.....	369
8.4 Inscribed Angles.....	370
8.5 Angles and Circles.....	376
8.6 Circular Constructions.....	382
8.7 Coordinate Geometry of Circles.....	387
Chapter 8 Review	393
CHAPTER 9: SURFACE AREA AND VOLUME	396
9.1 Analyzing Three-Dimensional Figures.....	398
Technology Corner —Platonic Solids.....	405
9.2 Surface Areas of Prisms and Cylinders.....	406
9.3 Surface Areas of Pyramids and Cones.....	412
9.4 Volumes of Prisms and Cylinders.....	421
9.5 Volumes of Pyramids and Cones.....	428
Geometry Around Us —Engineering.....	435
9.6 Surface Areas and Volumes of Spheres.....	436
9.7 Geometry of Spheres.....	441
9.8 Drawing Three-Dimensional Figures.....	446
Chapter 9 Review	453
CHAPTER 10: TRANSFORMATIONS AND SYMMETRY	456
10.1 Reflections.....	458
10.2 Translations.....	465
10.3 Rotations.....	473
10.4 Isometries.....	481
Geometry in History —Computer Graphics.....	489
10.5 Symmetry.....	492
10.6 Dilations.....	498
10.7 Tessellations.....	505
Technology Corner —Drawing Tessellations.....	510
Chapter 10 Review	511
CHAPTER 11: SIMILARITY	516
11.1 Similar Figures.....	518
11.2 Proving Similar Triangles.....	524
11.3 Similarity Within Right Triangles.....	532
Technology Corner —	
Investigating Parallel Partitions.....	539
11.4 Proportional Partitions.....	540
11.5 Lengths, Areas, and Volumes of Similar Figures.....	549
11.6 Circles and Proportions.....	555
Geometry Around Us —Art.....	563
11.7 The Golden Ratio.....	564
Chapter 11 Review	571
CHAPTER 12: INTRODUCTION TO TRIGONOMETRY	574
12.1 Trigonometric Ratios.....	576
12.2 Solving Right Triangles.....	582
Geometry Around Us —Space Exploration.....	589
12.3 Area.....	590
12.4 Vectors.....	596
Technology Corner —	
Discovering Trigonometric Identities.....	602
12.5 Trigonometric Identities.....	603
12.6 Law of Cosines and Law of Sines.....	608
Chapter 12 Review	614
Symbols	617
Postulates, Theorems, and Constructions	618
Selected Odd Answers	624
Glossary	663
Index	669
Photo Credits	675
Quick Reference	678

INTRODUCTION

Why Is Geometry Important?

On Monday Charlotte Burch drove off to visit family members who were camping at a nearby lake. She never arrived. Her concerned family contacted the authorities, and search and rescue crews were soon on the ground. They fixed the point where she was last seen and calculated how far she might have traveled. With this radius, they drew a circle around her last known location. Using probabilities and statistics, they prioritized regions within this area and began searching. For two days they combed the search area using dogs, global positioning systems, all-terrain vehicles, and helicopters. On Wednesday night they located her vehicle on a logging trail 35 kilometers from town, but Charlotte was not there. Another day would pass before Charlotte was located six kilometers from her car, a little dehydrated, but none the worse for her three days in the woods.

Stories like this are quite common. Ever since humans first fell into sin (Gen. 3), we have struggled to carry out the mission that God first gave to us—to subdue the earth and have dominion over it (Gen. 1:28). God placed humans in a position of authority over the world that He had just made. We should manage the earth's resources and develop them to their full potential. Over the years humans have developed many tools to help obey this Creation Mandate and to cope with problems created by the Fall. Engineers and architects make extensive use of geometry as they design and build bridges and skyscrapers. Search and rescue (SAR) teams use geometry to locate ships lost at sea or people lost in the wilderness. Geometry is a basic tool in exercising wise dominion in God's world and therefore is a significant tool for every Christian.

Geometry has many useful applications in science, navigation, surveying, building, engineering, trades, architecture, and many other occupations. If a picture is worth a thousand words, it is obvious that geometric representations will help a person solve problems. Mathematical training has become a filter to eliminate those



who do not have the skills to enter many occupations. Geometry also develops the comprehension of spatial relationships, a skill evaluated in nearly every intelligence test.

Geometry trains us to think logically and clearly, even in nonmathematical situations. As a young lawyer, Abraham Lincoln worked diligently through Euclid's *Elements* to improve his mind for the practice of law. All of us will reason through problems and issues throughout our lives. The most "practical" courses are those that train us to come to the correct conclusions to complex or controversial problems.

For centuries, young men studied Latin and geometry in preparation for pastoral ministry. They might have wondered, "Why should preachers study geometry?" The famous preacher C. H. Spurgeon enjoyed doing geometry to keep his mind sharp. He found that the skills learned in geometry helped him to reason through issues in theology and to organize his sermons.

As Lincoln and Spurgeon found out, geometry can help you draw correct conclusions from what you read and hear, organize your own thoughts and ideas, and present them clearly to others. The geometric concepts and logical reasoning that you learn in this course will help you to solve problems, understand concepts in other courses, and prepare for college.



What Exactly Is Geometry?

All mathematics has its roots in human efforts to understand and describe the world around us. Since Creation, people have measured objects, described shapes, and used reason in order to exercise dominion over the earth. In Egypt the subject was especially important for building pyramids and reestablishing property boundaries along the changing floodplain of the Nile River.



The Greeks organized the ideas of several centuries into principles and properties. In fact, our word *geometry* comes from two Greek words meaning “earth measure.” About 300 BC the Greek mathematician Euclid set forth the known principles in an orderly and systematic presentation. His system is called Euclidean geometry.

Euclid’s system is a model. Some of you have built model cars. Just as model cars represent key features of real cars, geometry represents key features of God’s creation. Model cars represent the shape and scale of real cars even though they are too small to be used for transportation. Likewise, geometry shows the relationships among measurements such as length and area as well as the reasons for such relationships. Thus geometry is a mathematical model of the world around us.

Geometry is also abstract. Something that is not a concrete, visible object is abstract. Numbers are abstractions describing quantities or “how much.” For example, the idea of “fiveness” is exemplified by many things, such as the fingers on your hand, and is represented by many symbols, such as V, 5, *cinco*, and so on. The number five is an abstraction. Euclidean geometry is an advanced abstract model of our physical world. The creation of these abstract mathematical systems enables us to better exercise wise dominion over the world that God has given to us.

How Can I Succeed?

Think about other skills you have already learned, such as playing a musical instrument, building models, playing a sport, or even driving a car. You may have started out poorly when first attempting to perform these skills correctly. Others may have learned more quickly and may have performed better, but through diligent effort you were able to succeed. As your skills improved, the activity became more enjoyable. Like these skills, thinking logically and completing proofs are skills that everyone can learn when approaching them with an attitude of determination and diligence. A Christian who does geometry “to the glory of God” (1 Cor. 10:31) will acquire an awesome tool for glorifying God even more.



► Discussion

1. List two benefits of studying geometry.
2. What is the literal meaning of the word *geometry*?
3. Name a physical object and an abstract mathematical concept that can be used to model that object. (For instance, a homeowner’s lot can be modeled by a rectangle.)
4. Name a skill that you have acquired that was initially difficult to learn.
5. Describe one way that the study of geometry can help a Christian better fulfill God’s purpose for his life.

5.2 INCENTER AND CENTROID



Can you find several points that are located the same distance from each side of $\triangle ABC$? To answer this question, we start with the distance from a point to a line as the length of the segment perpendicular to the line from the point. Finding several points that are the same distance from both sides may lead you to the conclusion stated in the following theorem.

THEOREM

A point lies on the bisector of an angle if and only if it is equidistant from its sides. (T.5.2.2)

Both cases of the biconditional statement must be proved. The first case, "If a point is on an angle bisector, then it is equidistant from the angle sides," is proved below. The proof of the converse is left as an exercise in the chapter review.

Proof: Given: Point P lies on \overline{AD} , the bisector of $\angle ABC$.
Prove: Point P is equidistant from \overline{AB} and \overline{BC} .

Statements	Reasons
1. P lies on \overline{AD} , the bisector of $\angle ABC$.	1. given
2. Draw \overline{PE} and \overline{PF} from P perpendicular to \overline{AB} and \overline{BC} .	2. Perpendicular segments
3. $\angle BEP$ and $\angle CFP$ are right angles.	3. Definition of perpendicular
4. $\angle BPE$ and $\angle CPF$ are right angles.	4. Definition of right angle
5. $\angle BPE \cong \angle CPF$.	5. Definition of angle bisector
6. $\angle BPE \cong \angle CPF$.	6. Reflexive Property of Congruent Angles
7. $\angle BPE \cong \angle CPF$.	7. ASA
8. $PE \cong PF$.	8. CPCTC

Begin by filling each vertex to the center to form the midpoint of each side with the segments. Then fill the triangle with the segments intersecting each midpoint to form the medians. Then fill the triangle with the segments intersecting each midpoint to form the medians. Then fill the triangle with the segments intersecting each midpoint to form the medians.



Let the medians \overline{AD} , \overline{BE} , and \overline{CF} intersect at G . Then G is the centroid of the triangle. The centroid is the point of concurrency of the medians of a triangle.

THEOREM
The three medians of a triangle intersect at a point, which is the centroid of the triangle. This point is equidistant from each vertex. (T.5.2.3)

If AD is a median, then $AG = \frac{2}{3}AD$ and $GD = \frac{1}{3}AD$. The centroid divides each median into segments whose lengths are in a 2:1 ratio.

EXAMPLE 1
Find each length if G is the centroid of $\triangle ABC$ and $AG = 12$.

$AD = ?$
 $GD = ?$
 $BD = ?$
 $CE = ?$
 $EF = ?$

The ratio of the lengths of the medians of a triangle is 2:1. The ratio of the lengths of the medians of a triangle is 2:1. The ratio of the lengths of the medians of a triangle is 2:1.

Median	Centroid	Centroid	Centroid
AD	$AG = 2GD$	BE	$BG = 2GE$
BE	$BE = 3GE$	CF	$CF = 3GF$

CHAPTER 5 RELATIONSHIPS IN TRIANGLES

Technology Corner

Use dynamic geometry software to visualize and discover geometric concepts.

TECHNOLOGY CORNER

Triangle Relationships

Complete the following steps to discover interesting relationships between a triangle and the triangle formed by its medians.

1. Open a new Sketchpad file, construct $\triangle ABC$, and color it blue. Then the Centroid menu to draw the triangle formed by the midpoints of $\triangle ABC$, label it $\triangle DEF$, and color it red. Using the vertices of $\triangle ABC$ and make a conjecture about the relationship between the two triangles in the figure.
2. Draw lines connecting each vertex of $\triangle DEF$ to the opposite vertex of $\triangle ABC$ and check the Perpendicular tool. Color the vertices of $\triangle DEF$ and label their midpoints as point G . Draw a line from G to the centroid of $\triangle ABC$.
3. Describe how the centroid of $\triangle DEF$ is related to $\triangle ABC$. Show it is related to $\triangle ABC$.
4. Draw the vertices of $\triangle ABC$. Are the midpoints of $\triangle DEF$ the same as the vertices of $\triangle ABC$?
5. Prove the conjecture made in step 1.

Key Concepts and Examples

Read thorough explanations of key concepts and study the step-by-step reasoning to achieve each objective.

INCENTER AND CIRCUMCENTER

A triangle with vertices A , B , and C is shown. The incenter is the point of concurrency of the three angle bisectors. The circumcenter is the point of concurrency of the three perpendicular bisectors.

Proof: Given: $\triangle ABC$ with angle bisectors \overline{AD} , \overline{BE} , and \overline{CF} .
Prove: \overline{AD} , \overline{BE} , and \overline{CF} are concurrent at the incenter.

Proof: Given: $\triangle ABC$ with perpendicular bisectors \overline{DE} , \overline{FG} , and \overline{HI} .
Prove: \overline{DE} , \overline{FG} , and \overline{HI} are concurrent at the circumcenter.

Proof: Given: $\triangle ABC$ with angle bisectors \overline{AD} , \overline{BE} , and \overline{CF} .
Prove: \overline{AD} , \overline{BE} , and \overline{CF} are concurrent at the incenter.

Proof: Given: $\triangle ABC$ with perpendicular bisectors \overline{DE} , \overline{FG} , and \overline{HI} .
Prove: \overline{DE} , \overline{FG} , and \overline{HI} are concurrent at the circumcenter.

Proof: Given: $\triangle ABC$ with angle bisectors \overline{AD} , \overline{BE} , and \overline{CF} .
Prove: \overline{AD} , \overline{BE} , and \overline{CF} are concurrent at the incenter.

Proof: Given: $\triangle ABC$ with perpendicular bisectors \overline{DE} , \overline{FG} , and \overline{HI} .
Prove: \overline{DE} , \overline{FG} , and \overline{HI} are concurrent at the circumcenter.

Proof: Given: $\triangle ABC$ with angle bisectors \overline{AD} , \overline{BE} , and \overline{CF} .
Prove: \overline{AD} , \overline{BE} , and \overline{CF} are concurrent at the incenter.

Proof: Given: $\triangle ABC$ with perpendicular bisectors \overline{DE} , \overline{FG} , and \overline{HI} .
Prove: \overline{DE} , \overline{FG} , and \overline{HI} are concurrent at the circumcenter.

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Prove: \overline{AD} , \overline{BE} , and \overline{CF} are concurrent at the incenter.

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Prove: \overline{DE} , \overline{FG} , and \overline{HI} are concurrent at the circumcenter.

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Prove: \overline{DE} , \overline{FG} , and \overline{HI} are concurrent at the circumcenter.

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Prove: \overline{AD} , \overline{BE} , and \overline{CF} are concurrent at the incenter.

Proof: Given: $\triangle ABC$ with perpendicular bisectors \overline{DE} , \overline{FG} , and \overline{HI} .
Prove: \overline{DE} , \overline{FG} , and \overline{HI} are concurrent at the circumcenter.

Proof: Given: $\triangle ABC$ with angle bisectors \overline{AD} , \overline{BE} , and \overline{CF} .
Prove: \overline{AD} , \overline{BE} , and \overline{CF} are concurrent at the incenter.

Proof: Given: $\triangle ABC$ with perpendicular bisectors \overline{DE} , \overline{FG} , and \overline{HI} .
Prove: \overline{DE} , \overline{FG} , and \overline{HI} are concurrent at the circumcenter.

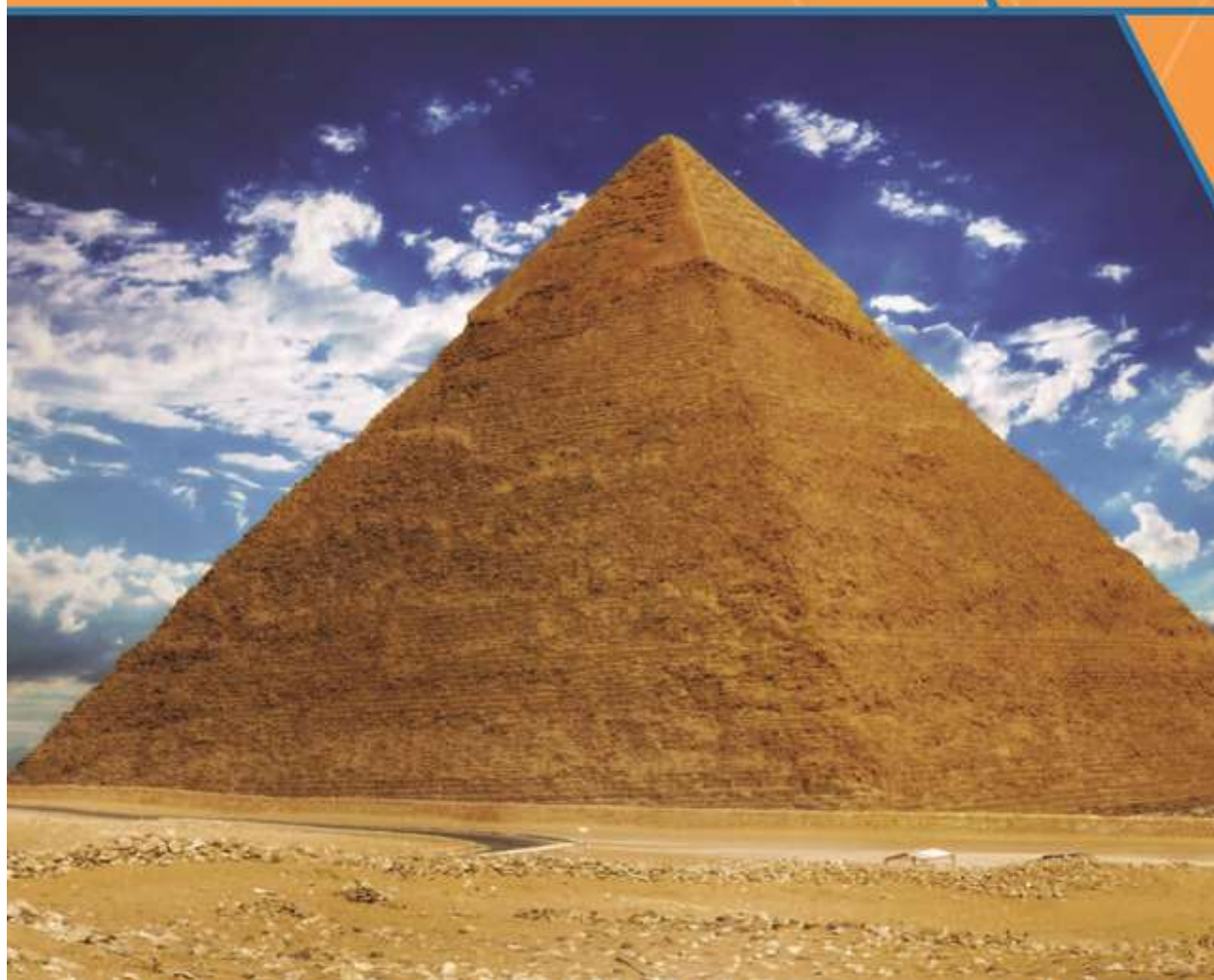
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Foundations of Geometry



The construction of the Pyramid of Khufu, the Great Pyramid, was one of the greatest architectural feats of history. How would you like the job of building the Great Pyramid? It has been estimated that it took 100,000 men thirty years to arrange the 2,300,000 blocks of stone averaging 2.5 tons each. The Egyptians had to be experts in geometry.

The head engineer built the 484.4 ft high pyramid on a base that covers 13.1 acres. The difference between the longest and shortest sides is only 7.9 in. The sides are aligned with the true compass points, forming almost perfect right angles. The exterior inclines are each $51^{\circ}52'$, and the inside corridors have almost exactly the same gradients.

Any structure that is able to stand the test of time must be built on a firm foundation. Wood, concrete, and steel may not appear to be related until you put them together and they form a building. This first chapter lays the foundation for your study of geometry. You'll learn about things like points, lines, planes, and angles. These are all "building blocks" that you'll use in later chapters to build more and more sophisticated structures.

After this chapter you should be able to

1. express relationships between sets and perform set operations.
2. describe the structure and characteristics of an ideal geometric system.
3. apply foundational definitions, postulates, and theorems.
4. identify subsets of lines and planes.
5. measure segments and angles.
6. identify and name circles, polygons, and related terms.
7. calculate the perimeter of a polygon and the circumference of a circle.
8. identify and name spheres, cones, cylinders, and polyhedra, including prisms and pyramids.
9. create accurate sketches and drawings.
10. construct congruent segments and angles.
11. recognize that a person's worldview affects every area of life, including mathematics.



1.1 SETS



You may wonder how sets relate to geometry. Sets can help us visualize, explore, and analyze mathematical relationships between different geometric objects. Set theory unifies the many branches of mathematics.

In mathematics a *set* is any collection of objects. Sets are typically named with a capital letter and designated using braces, $\{ \}$. Objects in a set are called *elements* or *members* of the set. Sets are described by listing the elements or by stating a rule, especially when listing the members of the set is cumbersome. *Set builder notation* is often used to state the rule describing the elements in a set. For example, $C = \{x \mid x \text{ is a letter in the word } \textit{complement}\}$ describes the set $C = \{c, o, m, p, l, e, n, t\}$. In set builder notation, x represents an arbitrary element of the set and \mid is read “such that.” Notice that the order in which the objects are listed is not important and that each member of a set is listed only once.

EXAMPLE 1

Describe set P pictured above using both a list and set builder notation.

- Answer**
- $P = \{\text{phone case, earphones, screen protector, car charger, battery, sync cable, armband}\}$
 - $P = \{x \mid x \text{ is one of Liam's cell phone accessories}\}$

The symbol \in means “is an element of.” The fact that the battery is an element of set P can be symbolized as

$$\text{battery} \in P.$$

A set containing no elements is called the *empty* or *null set* and notated by either $\{ \}$ or \emptyset . For example, $D = \{x \mid 2 < x < 3, \text{ where } x \text{ is an integer}\} = \emptyset$. Review the table summarizing set relationships.

	Equal Sets $=$	Subset \subseteq	Proper Subset \subset
Definitions	sets containing the same elements	Every element in B is also in A .	B is a subset of A , but B is not equal to A .
Examples	$A = \{1, 3, 5, 7, 9\}$, $B = \{3, 7\}$, and $C = \{x \mid 0 < x < 10, \text{ where } x \text{ is an odd integer}\}$		
	$A = C$	$B \subseteq A$ and $A \subseteq C$	$B \subset A$

The definitions imply that the empty set is a subset of every set and that every set is a subset (but not a proper subset) of itself.

Adding a slash to a symbol indicates that the relationship is “not” true.

- \neq not equal to
- \nsubseteq not a subset of
- \notin not an element of
- \subsetneq not a proper subset of