CONTENTS

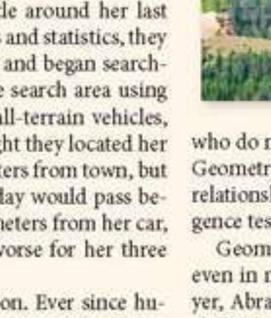
INTRODUCTION

Why Is Geometry Important?

On Monday Charlotte Burch drove off to visit family members who were camping at a nearby lake. She never arrived. Her concerned family contacted the authorities, and search and rescue crews were soon on the ground. They fixed the point where she was last seen and calculated how far she might have traveled. With this radius, they drew a circle around her last known location. Using probabilities and statistics, they prioritized regions within this area and began searching. For two days they combed the search area using dogs, global positioning systems, all-terrain vehicles, and helicopters. On Wednesday night they located her vehicle on a logging trail 35 kilometers from town, but Charlotte was not there. Another day would pass before Charlotte was located six kilometers from her car, a little dehydrated, but none the worse for her three days in the woods.

Stories like this are quite common. Ever since humans first fell into sin (Gen. 3), we have struggled to carry out the mission that God first gave to us—to subdue the earth and have dominion over it (Gen. 1:28). God placed humans in a position of authority over the world that He had just made. We should manage the earth's resources and develop them to their fullest potential. Over the years humans have developed many tools to help obey this Creation Mandate and to cope with problems created by the Fall. Engineers and architects make extensive use of geometry as they design and build bridges and skyscrapers. Search and rescue (SAR) teams use geometry to locate ships lost at sea or people lost in the wilderness. Geometry is a basic tool in exercising wise dominion in God's world and therefore is a significant tool for every Christian.

Geometry has many useful applications in science, navigation, surveying, building, engineering, trades, architecture, and many other occupations. If a picture is worth a thousand words, it is obvious that geometric representations will help a person solve problems. Mathematical training has become a filter to eliminate those who do not have the skills to enter many occupations. Geometry also develops the comprehension of spatial relationships, a skill evaluated in nearly every intelligence test.



Geometry trains us to think logically and clearly, even in nonmathematical situations. As a young lawyer, Abraham Lincoln worked diligently through Euclid's *Elements* to improve his mind for the practice of law. All of us will reason through problems and issues throughout our lives. The most "practical" courses are those that train us to come to the correct conclusions to complex or controversial problems.

For centuries, young men studied Latin and geometry in preparation for pastoral ministry. They might have wondered, "Why should preachers study geometry?" The famous preacher C. H. Spurgeon enjoyed doing geometry to keep his mind sharp. He found that the skills learned in geometry helped him to reason through issues in theology and to organize his sermons.

As Lincoln and Spurgeon found out, geometry can help you draw correct conclusions from what you read and hear, organize your own thoughts and ideas, and present them clearly to others. The geometric concepts and logical reasoning that you learn in this course will help you to solve problems, understand concepts in other courses, and prepare for college.

What Exactly Is Geometry?

All mathematics has its roots in human efforts to understand and describe the world around us. Since Creation, people have measured objects, described shapes, and used reason in order to exercise dominion over the earth. In Egypt the subject was especially important for building pyramids and reestablishing property boundaries along the changing floodplain of the Nile River.

The Greeks organized the ideas of several centuries into principles and properties. In fact, our word *geometry* comes from two Greek words meaning "earth measure." About 300 BC the Greek mathematician Euclid set forth the known principles in an orderly and systematic presentation. His system is called Euclidean geometry.

Euclid's system is a model. Some of you have built model cars. Just as model cars represent key features of real cars, geometry represents key features of God's creation. Model cars represent the shape and scale of real cars even though they are too small to be used for transportation. Likewise, geometry shows the relationships among measurements such as length and area as well as the reasons for such relationships. Thus geometry is a mathematical model of the world around us.

Geometry is also abstract. Something that is not a concrete, visible object is abstract. Numbers are abstractions describing quantities or "how much." For example, the idea of "fiveness" is exemplified by many things, such as the fingers on your hand, and is represented by many symbols, such as V, 5, *cinco*, and so on. The number five is an abstraction. Euclidean geometry is an advanced abstract model of our physical world. The creation of these abstract mathematical systems enables us to better exercise wise dominion over the world that God has given to us.

How Can I Succeed?

Think about other skills you have already learned, such as playing a musical instrument, building models, playing a sport, or even driving a car. You may have started out poorly when first attempting to perform these skills correctly. Others may have learned more quickly and may have performed better, but through diligent effort you were able to succeed. As your skills improved, the activity became more enjoyable. Like these skills, thinking logically and completing proofs are skills that everyone can learn when approaching them with an attitude of determination and diligence. A Christian who does geometry "to the glory of God" (1 Cor. 10:31) will acquire an awesome tool for glorifying God even more.

Discussion

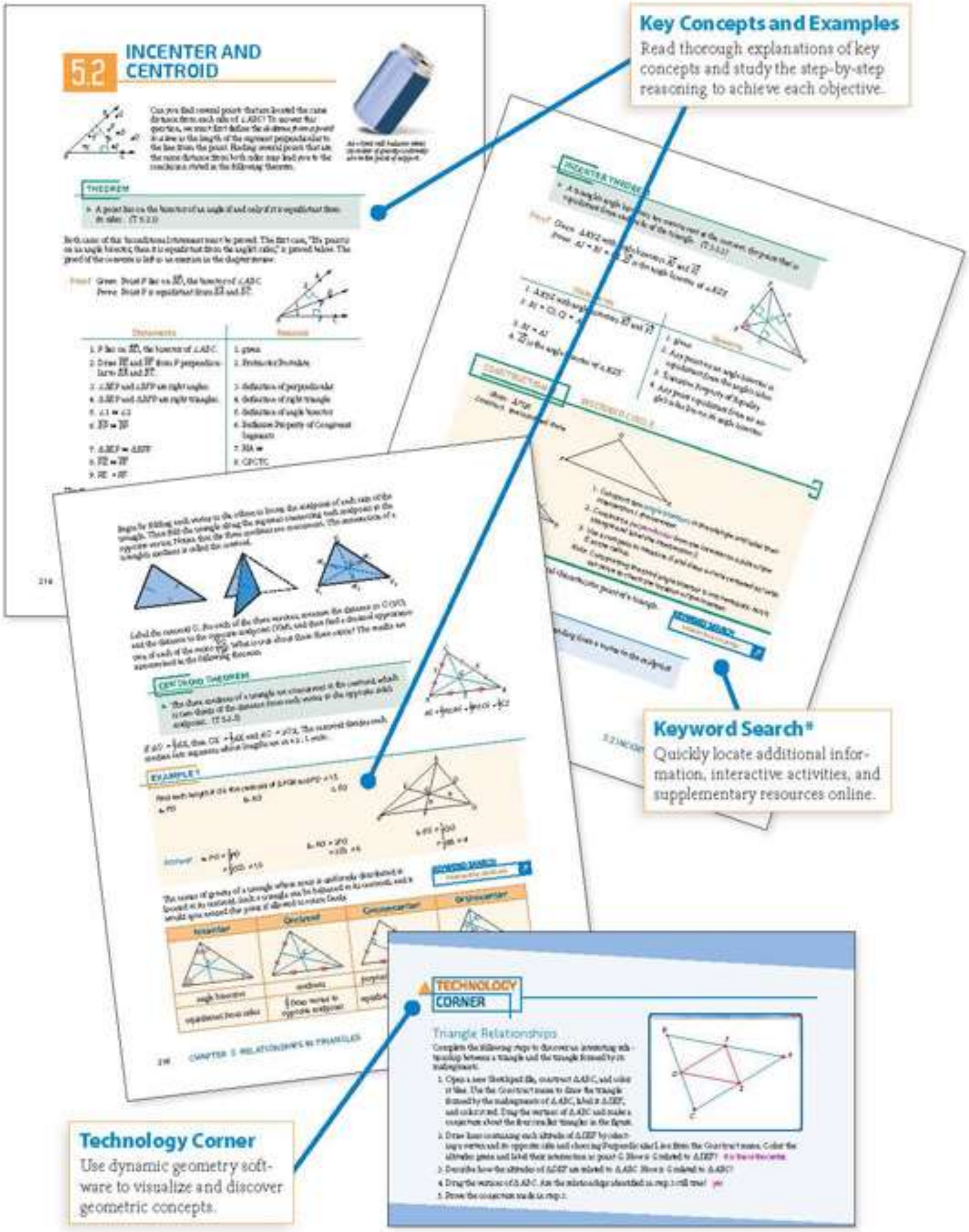
1. List two benefits of studying geometry.
2. What is the literal meaning of the word *geometry*?
3. Name a physical object and an abstract mathematical concept that can be used to model that object. (For instance, a homeowner's lot can be modeled by a rectangle.)
4. Name a skill that you have acquired that was initially difficult to learn.
5. Describe one way that the study of geometry can help a Christian better fulfill God's purpose for his life.

USING THIS BOOK

Chapter Opener
Read the opener to discover the chapter's learning objectives and connections to a biblical worldview.

Geometry Around Us
Discover and discuss the role of geometry in various careers and in your daily life.

Geometry in History
New historical-fiction narratives highlight key mathematical contributions and encourage discussion of the impact of a person's worldview.

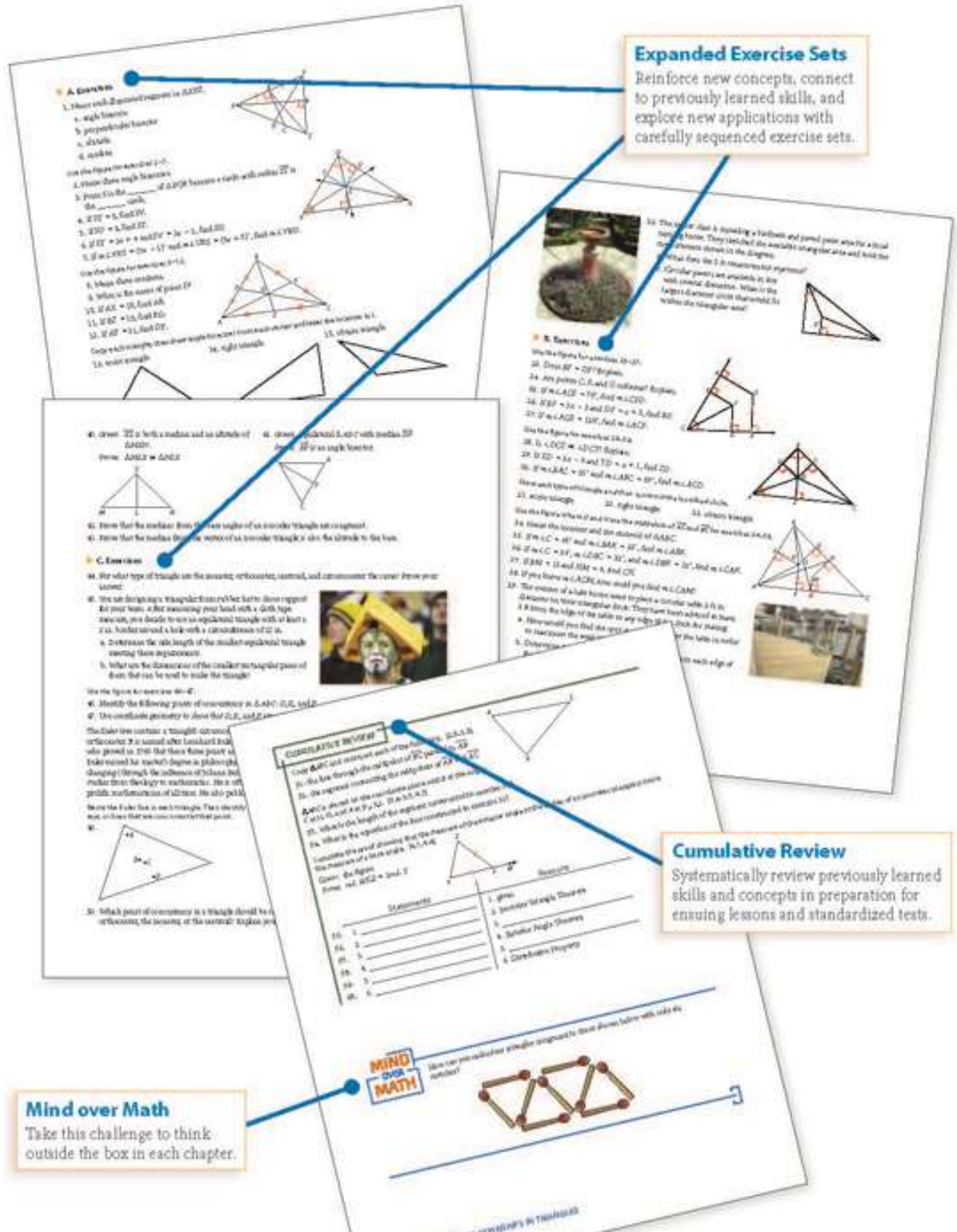


Key Concepts and Examples
Read thorough explanations of key concepts and study the step-by-step reasoning to achieve each objective.

Keyword Search*
Quickly locate additional information, interactive activities, and supplementary resources online.

Technology Corner
Use dynamic geometry software to visualize and discover geometric concepts.

*Use the Java exception site list to run blocked apps.



Expanded Exercise Sets

Reinforce new concepts, connect to previously learned skills, and explore new applications with carefully sequenced exercise sets.

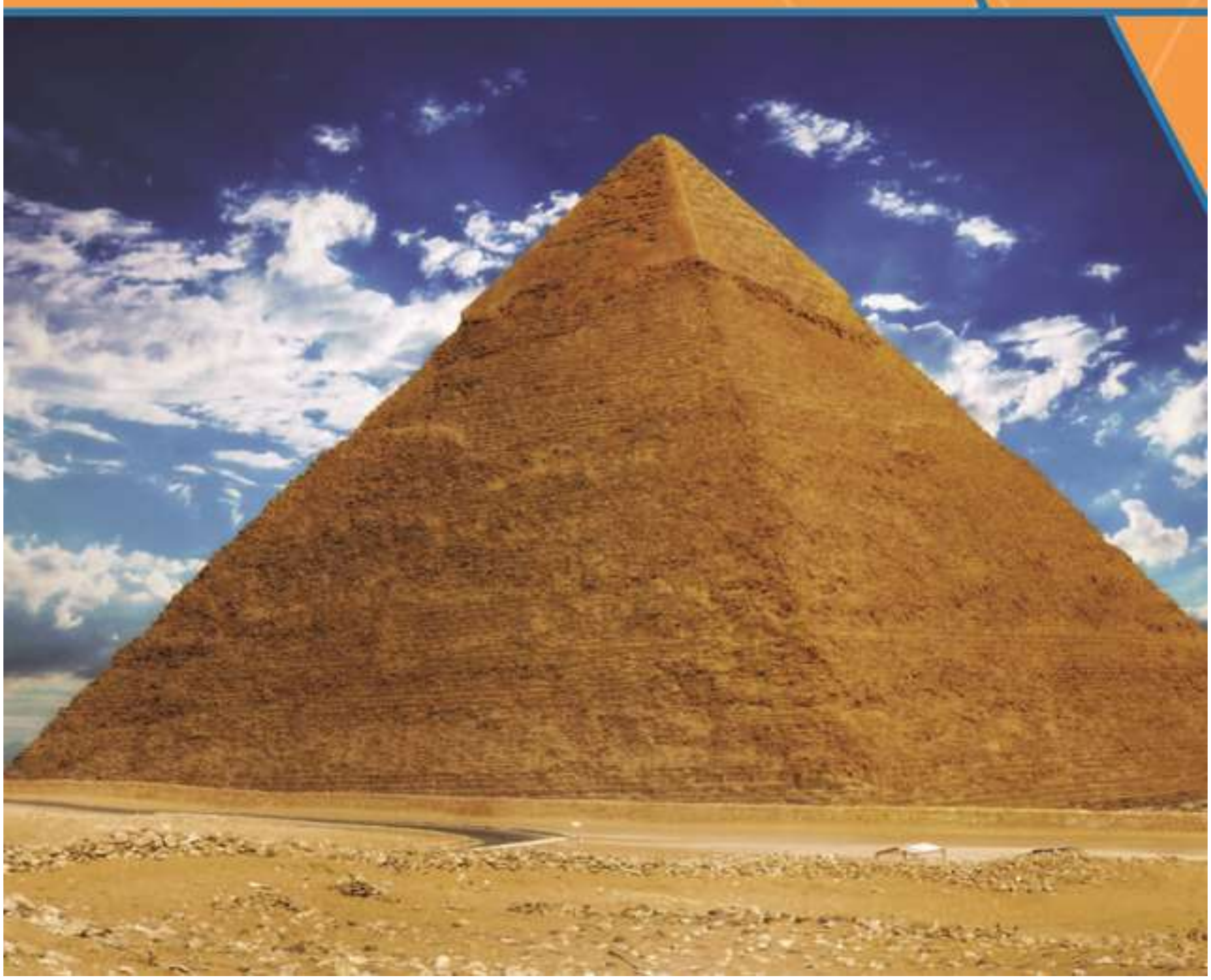
Cumulative Review

Systematically review previously learned skills and concepts in preparation for ensuing lessons and standardized tests.

Mind over Math

Take this challenge to think outside the box in each chapter.

1 Foundations of Geometry



The construction of the Pyramid of Khufu, the Great Pyramid, was one of the greatest architectural feats of history. How would you like the job of building the Great Pyramid? It has been estimated that it took 100,000 men thirty years to arrange the 2,300,000 blocks of stone averaging 2.5 tons each. The Egyptians had to be experts in geometry.

The head engineer built the 484.4 ft high pyramid on a base that covers 13.1 acres. The difference between the longest and shortest sides is only 7.9 in. The sides are aligned with the true compass points, forming almost perfect right angles. The exterior inclines are each 51°52′, and the inside corridors have almost exactly the same gradients.

Any structure that is able to stand the test of time must be built on a firm foundation. Wood, concrete, and steel may not appear to be related until you put them together and they form a building. This first chapter lays the foundation for your study of geometry. You'll learn about things like points, lines, planes, and angles. These are all "building blocks" that you'll use in later chapters to build more and more sophisticated structures.

After this chapter you should be able to

1. express relationships between sets and perform set operations.
2. describe the structure and characteristics of an ideal geometric system.
3. apply foundational definitions, postulates, and theorems.
4. identify subsets of lines and planes.
5. measure segments and angles.
6. identify and name circles, polygons, and related terms.
7. calculate the perimeter of a polygon and the circumference of a circle.
8. identify and name spheres, cones, cylinders, and polyhedra, including prisms and pyramids.
9. create accurate sketches and drawings.
10. construct congruent segments and angles.
11. recognize that a person's worldview affects every area of life, including mathematics.

1.1 SETS

You may wonder how sets relate to geometry. Sets can help us visualize, explore, and analyze mathematical relationships between different geometric objects. Set theory unifies the many branches of mathematics.

In mathematics a *set* is any collection of objects. Sets are typically named with a capital letter and designated using braces, { }. Objects in a set are called *elements* or *members* of the set. Sets are described by listing the elements or by stating a rule, especially when listing the members of the set is cumbersome. *Set builder notation* is often used to state the rule describing the elements in a set. For example, $C = \{x \mid x$ is a letter in the word *complement*$\}$ describes the set $C = \{$c, o, m, p, l, e, n, t$\}$. In set builder notation, x represents an arbitrary element of the set and $|$ is read "such that." Notice that the order in which the objects are listed is not important and that each member of a set is listed only once.

EXAMPLE 1

Describe set P pictured above using both a list and set builder notation.

Answer **a.** $P = \{$phone case, earphones, screen protector, car charger, battery, sync cable, armband$\}$

b. $P = \{x \mid x$ is one of Liam's cell phone accessories$\}$

The symbol $\in$ means "is an element of." The fact that the battery is an element of set P can be symbolized as

battery $\in P$.

A set containing no elements is called the *empty* or *null set* and notated by either { } or $\varnothing$. For example, $D = \{x \mid 2 < x < 3$, where x is an integer$\} = \varnothing$. Review the table summarizing set relationships.

	Equal Sets $=$	**Subset** $\subseteq$	**Proper Subset** $\subset$
Definitions	sets containing the same elements	Every element in B is also in A.	B is a subset of A, but B is not equal to A.
Examples	$A = \{1, 3, 5, 7, 9\}$, $B = \{3, 7\}$, and $C = \{x \mid 0 < x < 10$, where x is an odd integer$\}$		
	$A = C$	$B \subseteq A$ and $A \subseteq C$	$B \subset A$

The definitions imply that the empty set is a subset of every set and that every set is a subset (but not a proper subset) of itself.

Adding a slash to a symbol indicates that the relationship is "not" true.

$\neq$ not equal to

$\notin$ not an element of

$\not\subseteq$ not a subset of

$\not\subset$ not a proper subset of