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## LET'S GO EXPLORING!

Captain Frank Worsley's ship, the *Endurance*, was trapped and crushed by pack ice while carrying Ernest Shackleton's 1914 trans-Antarctic expedition. All 28 men escaped but were then stranded on Elephant Island. Their only hope of rescue was a whaling village on South Georgia Island, 800 nautical miles across some of the stormiest waters on earth.



Worsley, Shackleton, and four other men had no choice but to navigate gale-force winds and icy waves in a modified lifeboat in an attempt to reach this tiny speck in the southern ocean. Clinging to the mast of the *James Caird*, Worsley gripped his sextant in a cold-stiffened hand and attempted to get precise readings while the sun played hide-and-seek in an angry, overcast sky. The sun peeked through for a few seconds, and Worsley was able to obtain the critical readings.

Worsley's calculations using formulas such as

$$\cos(\text{zenith distance}) = \sin L \sin D + \cos L \cos D \cos A \text{ and}$$

$$\tan(\text{azimuth}) = \frac{\sin A \tan D}{\cos L} - \sin L \cos A,$$

(where  $L$  is the latitude,  $D$  is the declination, and  $A$  is the local hour angle), showed that they were on course! After reaching South Georgia Island and trekking over dangerous mountains to reach the whaling village, Shackleton's group was able to return and rescue the rest of the expedition.



A hundred years later, algebra still serves to help us explore the world and solve real-world problems. Look for the Biblical Worldview Connection box at the beginning of each chapter to see how algebra will be used to analyze data and solve problems within a Christian worldview. You will explore business and debt management; model motion in crash tests; and analyze sound, music, and pendulums. You will see how algebra can help us improve and save people's lives—people

who bear God's image. In this way, mathematics can be used to better fulfill God's commands to wisely care for and utilize His creation (the dominion mandate), love others, and glorify Him (Gen. 1:26–28; Matt. 22:35–40).

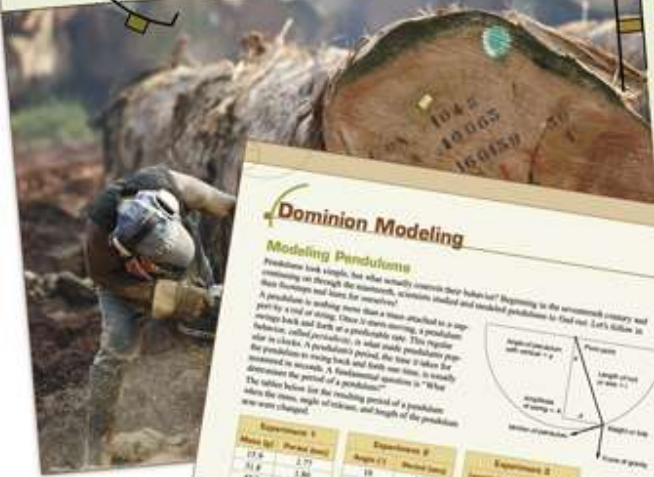
Mathematics can reflect the consistency and orderliness of God when it helps us to discover the regularity in His created universe. Apart from the regular, precise orbits of heavenly bodies, Worsley's navigational formulas would have been worthless. Our understanding of the complexities and magnitude of our universe is greatly enhanced by quantitative descriptions. The reflections of God's knowledge and power found in mathematics compel us to worship Him (Ps. 104:24).



### Chapter Openers

Read the opener to discover the chapter's Biblical Worldview Connection and learning objectives.

# 8 EXPONENTIAL AND LOGARITHMIC FUNCTIONS



As hearing disability handsaws 1 out of every 10 people in the United States—and you may be one of them. Hearing loss affects both young and old alike. Many times you encounter people who are wearing a soft cap, have headphones, earplugs, music converters, loud noise systems, communication, or hearing aids. The amount of hearing loss is affected by personally injury your hearing. It has also decreased from those around them. When people can't hear well, they feel disconnected from those around them. Helen Keller said, "Deafness separates us from things, but deafness separates us from people." The widespread hearing loss can have a significant effect on the country's function and culture.

It also can affect the economy, since many people encounter noise-induced hearing loss on the job. It is the most common workplace injury in the United States. Noise that is most valuable part of any business is its employees, employees often stress to make hearing protection a priority. The loss of our hearing could affect the way we work. How do we know what levels of sound are safe? That's where exponential and logarithmic functions come in. These functions help scientists, engineers, and doctors model sound levels and implement safety procedures to reduce harmful noise exposure.

**Biblical Worldview Connection**

**What?** Modeling sound

**How?** Using logarithmic functions

**Who?** Exponential functions model sound or businesses can protect the hearing of their most valuable asset, their employees. (Mark 11:8, Prov 23:15)

- After this chapter you should be able to:**
1. perform function operations, including composition.
  2. find and graph the inverse of a relation or function and determine whether it is a function.
  3. graph exponential functions.
  4. model exponential growth and decay.
  5. compare exponential and logarithmic functions.
  6. apply the properties of logarithms.
  7. apply logarithms to solve exponential and logarithmic equations.
  8. solve exponential equations and logarithms to solve problems.
  9. solve logarithmic equations and logarithms to solve problems.

## Dominion Modeling

### Modeling Pendulums

Pendulums look simple, but what actually occurs there? Beginning in the seventeenth century and continuing through the nineteenth, scientists studied and modeled pendulums to find out. Let's follow in their footsteps and look for ourselves!

A pendulum is nothing more than a mass attached to a string or rod and free to swing. Once it starts moving, a pendulum's behavior, called *periodicity*, is what makes pendulums possible to use. A pendulum's period, or the time it takes for the pendulum to swing back and forth one time, is usually measured in seconds. A fundamental question is "What factors have an effect on the period of a pendulum?"

The factors have an effect on the period of a pendulum are the mass, angle of release, and length of the pendulum.



Experiment 1		Experiment 2		Experiment 3	
Mass (g)	Period (sec)	Angle (°)	Period (sec)	Length (cm)	Period (sec)
15.6	2.77	10	1.77	20	0.81
31.2	3.80	20	1.79	40	1.23
47.8	4.79	30	1.80	60	1.54
64.4	5.77	40	1.80	80	1.77

- Use a graphing calculator or spreadsheet to analyze the data. Plot the period in the independent variable on an axis ranging from 0 to 2 sec.
1. Create a scatterplot of period versus mass using the data from Experiment 1.
  2. Does a constant, linear, or quadratic function appear to be the best fit for the data? Explain.
  3. Does a constant, linear, or quadratic function appear to be the best fit for the data? Explain.
  4. Does a constant, linear, or quadratic function appear to be the best fit for the data? Explain.

### MATH IN HISTORY

#### Francis Viète (1540–1603)

Francis Viète worked as a lawyer and also served as a member of the French parliament and as a private counselor to King Henry II. Mathematicians were not his work but his hobby, and he spent most of his leisure time exploring it. He was famous because he was the greatest French mathematician of the sixteenth century. In 1578 political events banished him from the royal court, and he devoted himself completely to mathematics for the next five years. As a rich attorney, he published his works on algebra, geometry, and trigonometry himself—or several printers for each trigonometry of the time.



When France was at war with Spain, Viète demonstrated a method using ternary (base 3) numerals that was used by the Spanish king. Realizing the value of his work but realizing that his plans could not be disclosed, King Philip II accused the French of being black agents. On another occasion, a Law Council under King Henry II by saying that Francis did not have a mathematician able to solve a particular three-fifth-degree equation. Viète quickly found twenty-three positive roots, and he had the king's head found the twenty-fourth root.

Viète is sometimes called the Father of Modern Algebra. Much of his symbolism and some concepts that he introduced are still used today. He

introduced the practice of using letters to represent unknowns. He was the first to use  $x$  and  $y$  as the standard formula that approximates the value of  $x$  and found a formula to solve the six trigonometric functions using the sine and cosine of a triangle.

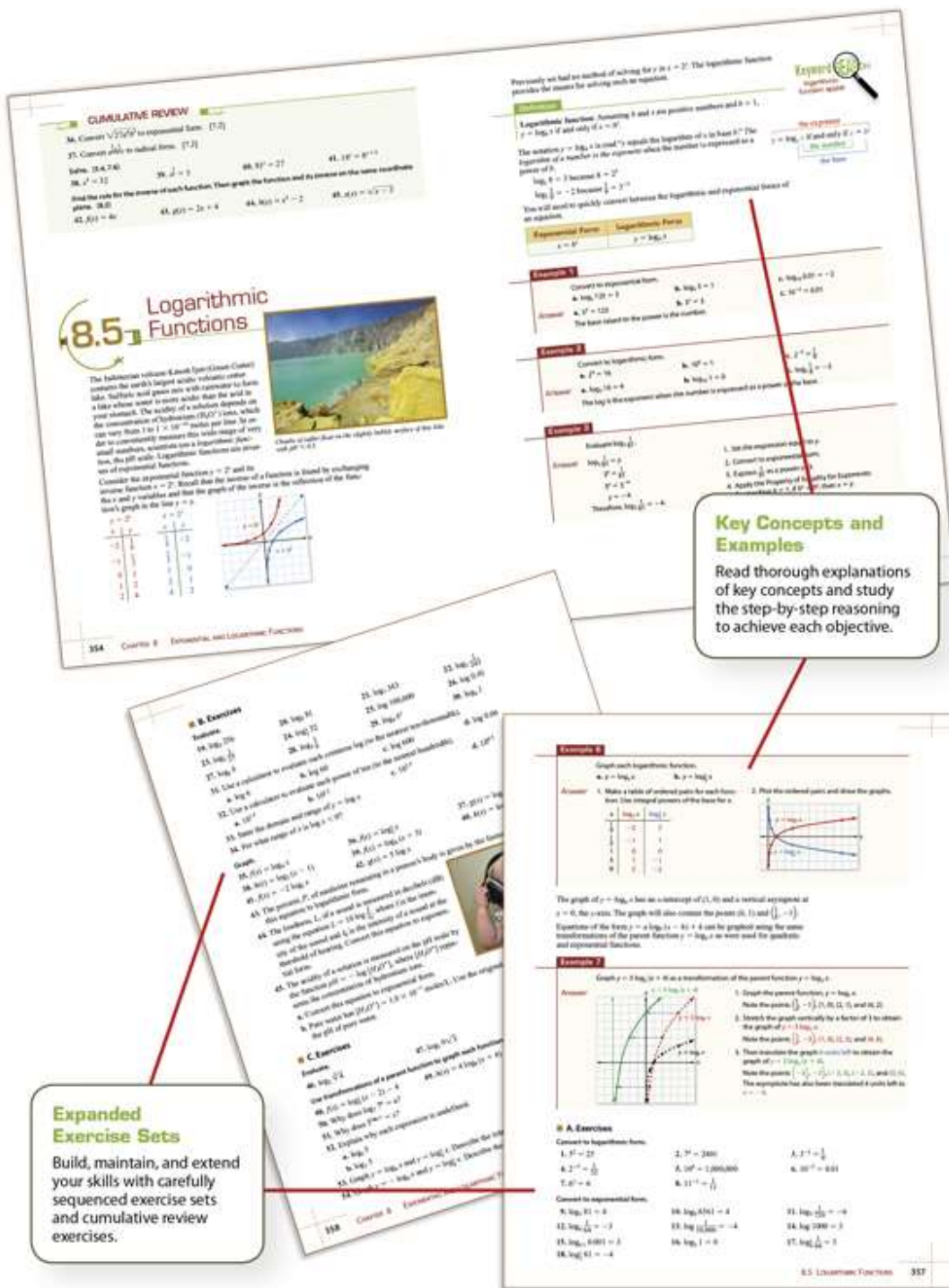
His main interest was with solving algebraic equations. His method is a method for approximating the solutions to equations as high as the sixth degree. His most famous work, *De aequationibus aequis*, divides algebra into three branches: translating problems into equations, solving equations, and proving discrete work equations.

### Dominion Modeling

Apply mathematical concepts to solve real-life problems within a Christian worldview.

### Biographies and History

Discover key mathematical contributions of individuals and cultures in the Math in History and Algebra Around the World features.



**CUMULATIVE REVIEW**

34. Convert  $\sqrt[3]{27}$  to exponential form. [1-2]  
 35. Convert  $27^3$  to radical form. [1-2]

Solve: 36.  $2^x = 8$       37.  $4^x = 2$       38.  $9^x = 3$       39.  $16^x = 4$   
 40.  $2^x = 16$       41.  $3^x = 9$       42.  $5^x = 25$       43.  $8^x = 64$

Find the rule for the inverse of each function. Then graph the function and its inverse on the same coordinate plane. 38.  $f(x) = 4x - 2$       39.  $f(x) = x^2 - 2$       40.  $f(x) = x^2 - 2$       41.  $f(x) = x^2 - 2$

**8.5 Logarithmic Functions**

The Indian volcano Mount Spurr (Lower Crater) pours the world's largest acidic volcanic crater lake. Unlike acid rain, which is water with a pH of 5.6 or lower, the acidity of a volcano depends on the concentration of hydrogen chloride (HCl) gases, which can vary from  $1 \text{ to } 10^{10}$  molecules per liter. In order to consistently measure the wide range of very small numbers, scientists use a logarithmic function, the pH scale. Logarithmic functions are inverses of exponential functions.

Consider the exponential function  $y = 2^x$  and the inverse function  $y = \log_2 x$ . Recall that the inverse of a function is found by exchanging the  $x$  and  $y$  variables and then the graph of the inverse is the reflection of the first graph in the line  $y = x$ .

$y = 2^x$        $y = \log_2 x$

$x = 1$        $x = 2$

$y = 2$        $y = 4$

$x = 2$        $x = 4$

$y = 4$        $y = 16$

$x = 4$        $x = 16$

$y = 16$        $y = 256$

$x = 16$        $x = 256$

$y = 256$        $y = 65536$

$x = 65536$        $x = 268435456$

$y = 268435456$        $y = 17179869184$

$x = 17179869184$        $x = 4294967296$

$y = 4294967296$        $y = 281474976710656$

Previously we had no method of solving for  $y$  in  $x = 2^y$ . The logarithmic function provides the means for solving such an equation.

**Logarithmic Function** Assuming  $b$  and  $a$  are positive numbers and  $b \neq 1$ ,  $y = \log_b a$  if and only if  $a = b^y$ .

The notation  $y = \log_b a$  is read "y equals the logarithm of a to the base b". The logarithm of a number is the exponent value the number is expressed as a power of  $b$ .

$\log_2 8 = 3$  because  $2^3 = 8$   
 $\log_3 27 = 3$  because  $3^3 = 27$

You will need to quickly convert between the logarithmic and exponential bases of an equation.

Exponential Form	Logarithmic Form
$a = b^x$	$x = \log_b a$

**Example 1**  
 Convert to exponential form.  
 a.  $\log_2 10 = 3$       b.  $\log_3 8 = 1$       c.  $\log_5 25 = -2$   
 Answer: a.  $2^3 = 10$       b.  $3^1 = 8$       c.  $5^{-2} = 25$   
 The base is the same as the number.

**Example 2**  
 Convert to logarithmic form.  
 a.  $2^3 = 8$       b.  $10^2 = 1$       c.  $2^7 = \frac{1}{2}$   
 Answer: a.  $\log_2 8 = 3$       b.  $\log_{10} 1 = 0$       c.  $\log_2 \frac{1}{2} = -1$   
 The log is the exponent when the number is expressed as a power of the base.

**Example 3**  
 Evaluate  $\log_2 8$ .  
 Answer:  $\log_2 8 = 3$   
 $2^x = 8$   
 $2^x = 2^3$   
 $x = 3$   
 Therefore,  $\log_2 8 = 3$ .

1. Set the exponent equal to  $x$ .  
 2. Convert to exponential form.  
 3. Express  $\frac{1}{2}$  as a power of 2.  
 4. Apply the Property of Equality for Exponents.  
 5. Solve for  $x$ .  
 Therefore,  $\log_2 \frac{1}{2} = -1$ .

**Key Concepts and Examples**  
 Read through explanations of key concepts and study the step-by-step reasoning to achieve each objective.

**B Exercises**

1.  $2^x = 8$       2.  $3^x = 27$       3.  $4^x = 16$       4.  $5^x = 25$   
 5.  $6^x = 36$       6.  $7^x = 49$       7.  $8^x = 64$       8.  $9^x = 81$   
 9.  $10^x = 100$       10.  $11^x = 121$       11.  $12^x = 144$       12.  $13^x = 169$   
 13.  $14^x = 196$       14.  $15^x = 225$       15.  $16^x = 256$       16.  $17^x = 289$   
 17.  $18^x = 324$       18.  $19^x = 361$       19.  $20^x = 400$       20.  $21^x = 441$   
 21.  $22^x = 484$       22.  $23^x = 529$       23.  $24^x = 576$       24.  $25^x = 625$   
 25.  $26^x = 676$       26.  $27^x = 729$       27.  $28^x = 784$       28.  $29^x = 841$   
 29.  $30^x = 900$       30.  $31^x = 961$       31.  $32^x = 1024$       32.  $33^x = 1089$   
 33.  $34^x = 1156$       34.  $35^x = 1225$       35.  $36^x = 1296$       36.  $37^x = 1369$   
 37.  $38^x = 1444$       38.  $39^x = 1521$       39.  $40^x = 1600$       40.  $41^x = 1681$   
 41.  $42^x = 1764$       42.  $43^x = 1849$       43.  $44^x = 1936$       44.  $45^x = 2025$   
 45.  $46^x = 2116$       46.  $47^x = 2209$       47.  $48^x = 2304$       48.  $49^x = 2401$   
 49.  $50^x = 2500$       50.  $51^x = 2601$       51.  $52^x = 2704$       52.  $53^x = 2809$   
 53.  $54^x = 2916$       54.  $55^x = 3025$       55.  $56^x = 3136$       56.  $57^x = 3249$   
 57.  $58^x = 3364$       58.  $59^x = 3481$       59.  $60^x = 3600$       60.  $61^x = 3721$   
 61.  $62^x = 3844$       62.  $63^x = 3969$       63.  $64^x = 4096$       64.  $65^x = 4225$   
 65.  $66^x = 4356$       66.  $67^x = 4489$       67.  $68^x = 4624$       68.  $69^x = 4761$   
 69.  $70^x = 4900$       70.  $71^x = 5041$       71.  $72^x = 5184$       72.  $73^x = 5329$   
 73.  $74^x = 5476$       74.  $75^x = 5625$       75.  $76^x = 5776$       76.  $77^x = 5929$   
 77.  $78^x = 6084$       78.  $79^x = 6241$       79.  $80^x = 6400$       80.  $81^x = 6561$   
 81.  $82^x = 6724$       82.  $83^x = 6891$       83.  $84^x = 7056$       84.  $85^x = 7225$   
 85.  $86^x = 7396$       86.  $87^x = 7569$       87.  $88^x = 7744$       88.  $89^x = 7921$   
 89.  $90^x = 8100$       90.  $91^x = 8281$       91.  $92^x = 8464$       92.  $93^x = 8649$   
 93.  $94^x = 8836$       94.  $95^x = 9025$       95.  $96^x = 9216$       96.  $97^x = 9409$   
 97.  $98^x = 9604$       98.  $99^x = 9801$       99.  $100^x = 10000$

**C Exercises**

1.  $2^x = 8$       2.  $3^x = 27$       3.  $4^x = 16$       4.  $5^x = 25$   
 5.  $6^x = 36$       6.  $7^x = 49$       7.  $8^x = 64$       8.  $9^x = 81$   
 9.  $10^x = 100$       10.  $11^x = 121$       11.  $12^x = 144$       12.  $13^x = 169$   
 13.  $14^x = 196$       14.  $15^x = 225$       15.  $16^x = 256$       16.  $17^x = 289$   
 17.  $18^x = 324$       18.  $19^x = 361$       19.  $20^x = 400$       20.  $21^x = 441$   
 21.  $22^x = 484$       22.  $23^x = 529$       23.  $24^x = 576$       24.  $25^x = 625$   
 25.  $26^x = 676$       26.  $27^x = 729$       27.  $28^x = 784$       28.  $29^x = 841$   
 29.  $30^x = 900$       30.  $31^x = 961$       31.  $32^x = 1024$       32.  $33^x = 1089$   
 33.  $34^x = 1156$       34.  $35^x = 1225$       35.  $36^x = 1296$       36.  $37^x = 1369$   
 37.  $38^x = 1444$       38.  $39^x = 1521$       39.  $40^x = 1600$       40.  $41^x = 1681$   
 41.  $42^x = 1764$       42.  $43^x = 1849$       43.  $44^x = 1936$       44.  $45^x = 2025$   
 45.  $46^x = 2116$       46.  $47^x = 2209$       47.  $48^x = 2304$       48.  $49^x = 2401$   
 49.  $50^x = 2500$       50.  $51^x = 2601$       51.  $52^x = 2704$       52.  $53^x = 2809$   
 53.  $54^x = 2916$       54.  $55^x = 3025$       55.  $56^x = 3136$       56.  $57^x = 3249$   
 57.  $58^x = 3364$       58.  $59^x = 3481$       59.  $60^x = 3600$       60.  $61^x = 3721$   
 61.  $62^x = 3844$       62.  $63^x = 3969$       63.  $64^x = 4096$       64.  $65^x = 4225$   
 65.  $66^x = 4356$       66.  $67^x = 4489$       67.  $68^x = 4624$       68.  $69^x = 4761$   
 69.  $70^x = 4900$       70.  $71^x = 5041$       71.  $72^x = 5184$       72.  $73^x = 5329$   
 73.  $74^x = 5476$       74.  $75^x = 5625$       75.  $76^x = 5776$       76.  $77^x = 5929$   
 77.  $78^x = 6084$       78.  $79^x = 6241$       79.  $80^x = 6400$       80.  $81^x = 6561$   
 81.  $82^x = 6724$       82.  $83^x = 6891$       83.  $84^x = 7056$       84.  $85^x = 7225$   
 85.  $86^x = 7396$       86.  $87^x = 7569$       87.  $88^x = 7744$       88.  $89^x = 7921$   
 89.  $90^x = 8100$       90.  $91^x = 8281$       91.  $92^x = 8464$       92.  $93^x = 8649$   
 93.  $94^x = 8836$       94.  $95^x = 9025$       95.  $96^x = 9216$       96.  $97^x = 9409$   
 97.  $98^x = 9604$       98.  $99^x = 9801$       99.  $100^x = 10000$

**Expanded Exercise Sets**  
 Build, maintain, and extend your skills with carefully sequenced exercise sets and cumulative review exercises.

**Example 1**  
 Graph each logarithmic function.  
 a.  $y = \log_2 x$       b.  $y = \log_3 x$

Answer: 1. Make a table of ordered pairs for each function. Use ordered pairs of the base for  $x$ .

$x$	$y = \log_2 x$	$y = \log_3 x$
1	0	0
2	1	
3		1
4	2	
9		2

2. Plot the ordered pairs and draw the graphs.

The graph of  $y = \log_2 x$  has an x-intercept of  $(1, 0)$  and a vertical asymptote at  $x = 0$ , the y-axis. The graph will also contain the points  $(2, 1)$  and  $(\frac{1}{2}, -1)$ . Equations of the form  $y = a \log_b (x - h) + k$  can be graphed using the same transformations of the parent function  $y = \log_b x$  as were used for the graphs and exponential functions.

**Example 2**  
 Graph  $y = 2 \log_3 (x - 4)$  as a transformation of the parent function  $y = \log_3 x$ .

Answer: 1. Graph the parent function,  $y = \log_3 x$ .  
 Note the points  $(\frac{1}{3}, -1)$ ,  $(1, 0)$ ,  $(3, 1)$ , and  $(9, 2)$ .  
 2. Stretch the graph vertically by a factor of 2 to obtain the graph of  $y = 2 \log_3 x$ .  
 Note the points  $(\frac{1}{3}, -2)$ ,  $(1, 0)$ ,  $(3, 2)$ , and  $(9, 4)$ .  
 3. Then translate the graph 4 units left to obtain the graph of  $y = 2 \log_3 (x - 4)$ .  
 Note the points  $(\frac{13}{3}, -2)$ ,  $(5, 0)$ ,  $(7, 2)$ , and  $(13, 4)$ . The asymptote has also been translated 4 units left to  $x = 4$ .

**D Exercises**  
 Convert to logarithmic form.  
 1.  $2^3 = 8$       2.  $3^4 = 81$       3.  $4^5 = 1024$   
 4.  $5^6 = 15625$       5.  $6^7 = 279936$       6.  $7^8 = 5764801$   
 7.  $8^9 = 134217728$       8.  $9^{10} = 3486784401$

Convert to exponential form.  
 9.  $\log_2 8 = 3$       10.  $\log_3 27 = 3$       11.  $\log_4 16 = 2$   
 12.  $\log_5 25 = 2$       13.  $\log_6 36 = 2$       14.  $\log_7 49 = 2$   
 15.  $\log_8 64 = 3$       16.  $\log_9 81 = 2$       17.  $\log_{10} 100 = 2$   
 18.  $\log_{11} 121 = 2$



### Technology Corners

Discover how to apply spreadsheets, graphing calculators, and interactive geometry software to create mathematical models and solve problems.

#### TECHNOLOGY CORNER

Spreadsheets are powerful tools for organizing numerical data. While specific details vary from one spreadsheet to another, the basic principles are fairly consistent for any spreadsheet. A student can use a spreadsheet to track a city's daily high temperature and to determine the weekly average high temperature in both Fahrenheit and Celsius.

Date	High Temperature (F)	Weekly Average
1	68	
2	72	
3	75	
4	70	
5	73	
6	71	
7	74	
8		72.14

Record the date and use formulas that calculate the average high temperature for the week in Fahrenheit and convert this average to degrees Celsius.

In cell D2 enter  $=95*(C2+C3+C4+C5+C6+C7)/7$ .

In cell D3 enter  $=5/9*(D2-32)$ .

Most spreadsheets include functions such as sum or average that can be used to determine the formula for D3.

These formulas can be copied into the cells for the remaining weeks by dragging the fill handle down. Repeat from the D2 reference.

As the formulas are adjusted relative to each cell's position in the spreadsheet when the formulas is replicated in this manner. Make spreadsheets after you can quickly generate spreadsheets from the historical data.

**Challenge** Make a spreadsheet that describes a local city's average high temperature in a weekly and construct a graph illustrating the data. Data may be

found in the Technology Corner.

**Keywords:** spreadsheet, average, formula, copy, paste, fill handle, drag, drop, weekly average, temperature, Fahrenheit, Celsius.

### Keyword Searches

Quickly locate additional information, interactive activities, and supplementary resources online.

### Chapter Reviews

Prepare for assessment with additional review exercises.

## COLLEGE ENTRANCE TEST PREPARATION

### Content: Plane Geometry

Approximately one-fourth of college entrance test questions involve plane geometry topics. Students taking these tests should have a good understanding of angles, lines, polygons, and circles. They should also review the special types of triangles and quadrilaterals and rectangles by area and volume formulas.

#### Exercises

1. If  $AD \perp BC$ ,  $AB = AC = 20$ , and  $AD = 14$ , find  $AC$ .

- A. 18
- B. 16
- C. 15
- D. 14
- E. 13



2. Find the area of the figure if the semicircle has a diameter of 16.

- A.  $64 + \frac{7}{2}\pi$
- B.  $64 + 16\pi$
- C.  $128 + 16\pi$
- D.  $128 + \frac{7}{2}\pi$
- E.  $128 + 16\pi$



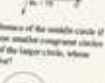
3. If line  $m$  is parallel to line  $n$ , find the measure of  $\angle ABC$ .

- A.  $20^\circ$
- B.  $40^\circ$
- C.  $50^\circ$
- D.  $110^\circ$
- E.  $111^\circ$



4. What is the circumference of the smaller circle if the radius of the three smaller congruent circles is one-fourth the diameter of the larger circle, whose circumference is 168π?

- A. 4π
- B. 8π
- C. 16π
- D. 32π
- E. 64π



5. Find the area of the shaded region.

- A. 63
- B. 75
- C. 76
- D. 81
- E. 86



6. If  $EF \perp PQ$  and  $RT = 20$ , find  $PT$ .

- A. 14
- B. 15
- C. 16
- D. 17
- E. 18



7. If the area of square  $WXYZ$  is 1 in<sup>2</sup>,  $V$  is the mid-point of  $WY$ , and  $U$  is the mid-point of  $XZ$ , what is the ratio of the area of  $UVWZ$  to the area of the square?

- A. 1:2
- B. 1:3
- C. 2:3
- D. 2:5
- E. 3:5



8. Given rectangle  $ABDC$  with  $P$  on circle  $C$ , which of the following must be true?

- A.  $AP = BP$
- B.  $AP = CP$
- C.  $AP = DP$
- D.  $AP = BP$
- E. cannot be determined

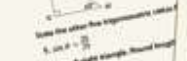


## CHAPTER 10 REVIEW

State the sine, cosine, and tangent values for each angle.



State the sine, cosine, and tangent values for each angle.



Solve each special triangle. Show your work.



11. Without using a calculator, find  $\sin 30^\circ$ .

12. Use a calculator to find  $\sin 45^\circ$ .

13. Use a calculator to find  $\cos 60^\circ$ .

14. A plane flying at an altitude of 21,000 feet is 30,000 feet from the ground. Find the angle of depression from the plane to the ground.

15. A ship leaves port heading  $15^\circ$  north of east and travels 500 mi. How far east and how far north is the ship from port?

16. If the ship changes course from this point and travels due east for 500 mi, find the distance from the ship to its starting point. Describe the ship's position relative to the port.

### College Entrance Test Preparation

Learn test-taking strategies and practice solving problems from key content areas for college entrance exams.

### Programming Projects

Extend your understanding by creating programs that automate tedious tasks.

## Programming Project

Programming Projects are activities for students with programming experience. These optional projects provide an opportunity to apply mathematical topics in computer science. Engineers, scientists, software developers, and mathematicians often create applications or programs to perform commonly used calculations. Programming can be done in the TI-84+ calculator or in any familiar computer language.

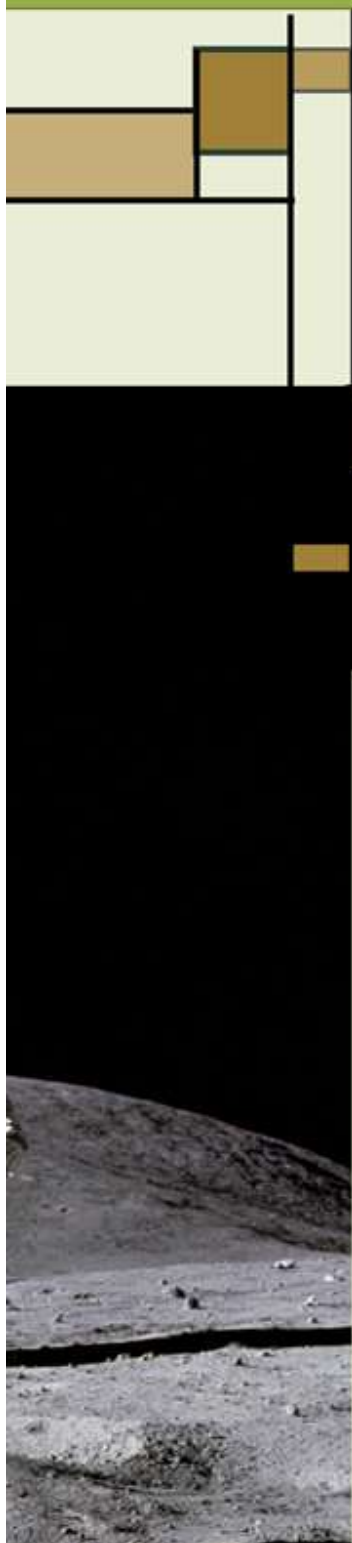
**Temperature Conversion**  
Write a temperature conversion program. The program should provide the user a choice of converting from Fahrenheit to Celsius or from Celsius to Fahrenheit. After accepting a temperature input from the user, the program should display the value of the temperature in the other scale.

# 4

## MATRICES







**“We never landed on the moon.”** That’s what some people really think! They say that the *Apollo* missions to the moon were a conspiracy—a great deception. They analyze images produced by these missions, looking for evidence of a grand hoax. These analyses have been debunked by many professionals, including living astronauts.

That’s not the only way people can misuse images. Viral photos circulate on the Internet, passing on urban legends. Many of these images have been heavily edited. But you’ve probably edited your own photos to get rid of red eye or to make the colors more vivid. What’s the difference? Editing can either enhance an image or cause it to deceive or mislead. We value truth because God is a God of truth (Deut. 32:4; John 4:23–24).

Digital technology and image editing have opened new doors for art, graphic design, and photography. It may surprise you to learn that many transformations of images are based on some elegant algebra. These tools can be used to transform images so that they better communicate truth (Eph. 4:25).

#### Biblical Worldview Connection

<b>What?</b>	Transforming images
<b>How?</b>	Using matrix operations
<b>Why?</b>	Matrix algebra is a powerful tool for making images that can better communicate truth (Deut. 32:4; Eph. 4:25).

#### After this chapter you should be able to

1. identify matrices and their elements.
2. organize data into matrices.
3. add and subtract matrices.
4. find scalar and matrix products.
5. represent a system of equations using a matrix equation.
6. find the determinant of a matrix.
7. find the inverse of a matrix.
8. solve matrix equations of the form  $AX = C$  using Cramer’s rule and inverse matrices.
9. use matrix operations to transform images.

# 4.1 Organizing Data with Matrices

A *matrix* is a mathematical tool used in many areas of business and science. In this chapter, you will see how matrices are used to quickly solve systems of equations. They also help graphic artists and computer programmers edit, render, and manipulate images. Matrices are the mathematics behind many amazing animations.

## Definitions

A **matrix** is a rectangular array of numbers. Each number in the matrix is called an *element* (or *entry*).

Matrices are named with capital letters. The elements in a matrix are arranged in *rows* (horizontal) and *columns* (vertical) and are enclosed in brackets.

$$M = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 7 & 5 \end{bmatrix}$$

The *dimensions* of a matrix are given in the form  $m \times n$ , where  $m$  is the number of rows and  $n$  is the number of columns. The fact that matrix  $M$  is a  $2 \times 3$  (read “two by three”) matrix is often notated as  $M_{2 \times 3}$ .



*It is unethical for photojournalists to manipulate photos to achieve a more dramatic effect.*

## Example 1

State the dimensions of each matrix.

a.  $A = \begin{bmatrix} 3 & 4 \\ 1 & 6 \end{bmatrix}$

b.  $B = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \\ j & k & l \end{bmatrix}$

c.  $C = [1 \ 4 \ 9 \ 2]$

d.  $D = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Answer a.  $2 \times 2$

b.  $4 \times 3$

c.  $1 \times 4$

d.  $3 \times 1$

Matrix  $C$  is a *row matrix* and matrix  $D$  is a *column matrix*. Matrix  $A$  is a *square matrix* because it has an equal number of rows and columns. Matrix  $A_{2 \times 2}$  is said to be of second order.

Subscripts are used to identify elements in a matrix. The element in row  $i$  and column  $j$  is expressed as  $a_{ij}$ .

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

